Determining lot sizes in production areas – exact calculations versus research based estimation

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Abstract

Determining lot sizes in production areas is an essential task of production planning and control. Due to the growing number of product variants, calculating economical lot sizes is becoming increasingly important in the industrial practice. Although planning lot sizes plays an important role in industrial production, existing methods only partially consider the variety of impacts. This paper presents existing methods and discusses their impact on the multi-criteria objective achievement of industrial companies. Based on actual case studies the logistical and economical relevance of determining lot sizes is illustrated and the suitability of different approaches for industrial practice is discussed.

1. Introduction

Determining lot sizes in production areas is one of the key tasks of production planning and control [1]. Lot sizes need to be ascertained every time more than one product is to be manufactured on a single resource and setups are required. In order to avoid continual setups, a certain number of similar products are combined into a so-called ‘lot’ or ‘batch’. A lot can thus be defined as the number of products processed on a production system without interruptions from the processing of other products [2].

Determining economical lot sizes is an important administrative task in the industrial practice and is becoming all the more critical due to the increasing number of product variants. When controlling the production, the lot sizes set as a parameter of the ERP system have to be questioned since various lot sizing methods exist and since parameters required to calculating lot sizes (e.g. the annual demand of a product) change more and more often nowadays. The significance of this task is reflected by the vast amount of research conducted on it and by the number of methods developed to address it (see e.g., [3,4]). Nevertheless, the aptness of current methods is frequently debated. Moreover, there is a demand for one-piece-flow productions and a renunciation of purely cost-oriented lot sizing in order to meet the demands being placed on the production’s logistical performance.

With all this in mind, the following paper uses actual case studies to demonstrate the economical and logistical impact of lot sizing, examines the suitability of different methods in the industrial practice, and derives conclusions from there.

2. Lot size impact on costs

Generally, lot sizes are dimensioned to minimize costs. Traditionally this has meant focusing on the setup and storage costs influenced by the lot size [5].

The setup costs, which are also frequently referred to as ‘job change costs’ or ‘pre-production costs’, arise when a machine has to be reset between two lots. They
accrue with every lot change and thus increase when basic lot sizes are reduced. Usually these costs are comprised of the following components:

- material and wage costs for cleaning a system,
- wage costs for adjusting and mounting special equipment,
- tool change and transportation costs,
- administrative costs for generating production orders,
- ramp-up costs at the start of production e.g., due to more rejects,
- hourly rates for machines for the setup time.

As the second key component of the direct costs, the storage or holding costs increase along with the lot size, since the number of production units that cannot be sold immediately also climbs. Costs here include:

- interest on tied-up capital,
- depreciation, insurance, maintenance etc. costs for buildings and storage systems,
- administrative and maintenance costs for stored articles,
- risk related costs e.g., due to decreasing value,
- costs for depositing and removing products from storage.

The opposing effect of the lot size dependent setup and holding costs indicates that there is an optimal range for setting an economical lot size $x_0$ in regard to the considered costs. By implementing what we will refer to in the following as the base model, this can be calculated according to Harris [6] or Andler [7] as:

$$x_0 = \sqrt{\frac{2 \cdot S \cdot a}{h \cdot p}}$$

where

- $x_0$ is the economic-optimal lot size (base model) [units],
- $S$ is the setup costs [€],
- $a$ is the planned demand for article during period [units],
- $h$ is the holding rate [-],
- $p$ is the production costs per unit [€/unit].

The lot size decision is not very cost sensitive in regard to deviations from the optimum [8]. This is especially the case since lot size dependent costs only represent a single digit percentage of the total unit costs. Thus the sensitivity of the unit costs to a deviation in the economically optimal lot size is also rather low (Fig. 1). The curves depict the trend of the unit costs for three different articles in regard to material and production costs as a function of the deviation from the economical optimum lot size, calculated according to the base model. Despite the difference in the articles (visible in the different parameters), halving the lot size only led to additional costs of between 0.44% and 1.38%. The additional costs with a corresponding increase in the lot size are clearly even lower.

From this we can conclude that it is not even necessary to determine a supposed economically optimal lot size in the industrial practice and that instead, a sufficiently good estimation would be enough, especially when in return all of the relevant aspects are considered.

### 3. Key lot sizing models and their differences

A variety of methods are implemented to determine the size of production lots, the majority of which are oriented on costs. The first that should be discussed is the above mentioned model developed by Harris and a similar approach from Andler which results in Eq. 1 (base model).

It has also been extended a number of times – particularly with the aim of considering dynamic demands (see e.g., [9-11]). Despite various advantages and disadvantages for each of the models, research conducted by different authors has proven that the results of these methods differ minimally – even with diverse cost structures [12,13].

This is clarified in Fig. 2. In the upper part of the diagram an example of a demand curve for an article during a period of one year is plotted. The clearly identifiable seasonal fluctuations are also superimposed by sporadic daily demands. The total demand of 2,882 units is distributed over the individual shop calendar days with a mean of 7.9 units/SCD and a standard deviation of 11.5 units/SCD. There is thus no constant demand to speak of. Based on this a production lot size was calculated according to the base model and according to Silver and Meal’s method.
With Silver and Meal’s method, the demands for a number of periods continue to be combined together into one production lot as long as this adding of additional demands decreases the average costs accrued per period (i.e. the sum of setup and holding costs). As soon as the addition of further demands to the lot would increase the period costs, these demands are no longer produced in the current period. Instead, the demands are again combined together into a lot for the following period up until the ‘stop criterion’ is once again met.

The calculations are based on setup costs of 100€, production costs of 13€ and a holding rate of 15%. The resulting stock trends for both procedures are depicted in the middle part of Fig. 2; the lower part depicts the monetary impact. The allocation of the costs to the setup and storage cost components differs depending on the calculation method. Nevertheless, the manner in which the calculation is made barely has any influence on the total costs that are dependent on the lot size (Fig. 2 lower part). When we look at the total unit costs, which in addition to the lot dependent costs also include the fixed costs for materials, production processes etc., the base model results in additional costs of only 0.05% more than Silver and Meal’s method. This relativizes the apparent advantages of the dynamic method unless there are enormous demand fluctuations.

This is also not surprising when one realizes that just about all the methods are based on the same approach i.e., weighing the holding costs for the finished product against the costs for changing orders (setups).

Thus the actual method applied by an enterprise to determine the lot size is not as decisive as the fact that the method employed is economically oriented. Refraining from calculating lot sizes at least when there are strong deviations towards smaller or larger lots can lead to considerable greater costs (see Fig. 1).

4. Business survey on lot sizing in industrial practice

In order to support these results and the conclusions drawn from them for the industrial practice, a business survey was conducted, within which representatives from German industrial enterprises were questioned about the significance of this topic for their productions. The aim of the study was to gain a sense of the knowledge users have regarding lot sizing methods and which approaches are increasingly employed in the industry. Furthermore, information has to be ascertained about the criteria used in the practice for selecting a lot sizing method and the significance of the chosen lot size for the logistic targets has to be estimated.

For the purpose of the survey an online questionnaire was developed and approximately 230 business representatives from the production industry were invited to participate. With 70 participants, almost 30% of those invited chose to respond. This underlines the relevance of the topic in the industrial practice. The key findings in regard to the formulated goals of the survey are briefly summarized here:

- About 60% of the companies use a specific lot sizing method.
-Although 84% of the companies indicated to know several lot sizing methods, about half of the companies has implemented the base model.
- The reasons mentioned for not implementing other lot sizing methods are that most academic methods are time consuming to implement (38%), difficult to comprehend (33%) and based on assumptions irrelevant in practice (23%).
- The basic criteria for implementing a lot sizing method are the ease of use (48%) and its transparency (33%); less than 20% see the exact results of the lot sizing method as key criteria.
- The majority of respondents consider the choice of lot size as crucial for a consistently market-relevant logistic performance (85%). At the same time they generally do not find logistic dependencies taken into account in an appropriate methodology (77%).

Concluding the most important statements of the survey, the requirements for an application-oriented lot sizing method are less concerned with the precision of the results than with the clarity and transparency of the underlying logic and the consideration of the basic impact of the chosen lot size e.g., on managing the production’s logistics.
5. Relevant logistic aspects of lot sizing

In addition to the setup and storage costs already mentioned, the choice of lot size impacts other factors including e.g., the logistic performance of a production enterprise. Various logistics theories (queuing, logistic operating curves) prove that production lot sizes and the resulting work content of orders correlate with the production throughput times to a large extent [15].

The influence of the lot size on the throughput time is immediately obvious. With larger lots, more parts have to be manufactured for a production order, which leads to a greater mean work content. The mean and standard deviation of the work content (caused by different production orders on a workstation) determine the practical amount of work in process (WIP) necessary for ensuring the desired utilization of the corresponding resources. The broader the distribution of the work content – due to large production lots – the more WIP is required to prevent the production’s capacities from reaching an organizational standstill. Smaller lots in comparison, lead to reduced and generally harmonized work contents thus allowing WIP levels to be set lower.

The subsequent shorter queues in front of individual workstations result in shorter throughput times even with a higher utilization rate. This can be proven with the aid of Production Operating Curves [15].

Varying throughput times are difficult to manage within the frame of production planning and control since standard ERP systems often work with constant inter-operation and waiting times. This fact leads inevitably to a deviation between the planned and realized throughput time; it usually results in frequent and certainly in strongly dispersed lateness.

Due to a variety of reasons, when there are long queues in the industrial practice, it is necessary to change the sequence in which orders are processed. Consequently from a logistics perspective, orders that are pulled forward exhibit clearly shorter throughput times while orders pushed back lengthen their throughput times. The order lateness of those pushed back thus increases. In short, growing lot sizes not only lead to longer mean throughput times, but also to increased variance, which ultimately results in decreased flexibility and therefore to the production’s schedule reliability diminishing.

The poor logistics of an enterprise can lead to more work and additional costs for their customers. In a business environment characterized by a market economy, the enterprise would thus be taking a risk competitively and would have to, so to speak, ‘purchase’ the level of delivery service demanded by their customers by maintaining safety stock. Safety stock is held in addition to lot stock, in order to ensure the availability of articles despite deviations from the underlying replenishment time (planned throughput time of the production) as well as lateness in re-filling stores. Safety stock is thus directly dependent on the lot size induced objectives ‘throughput time’ and production ‘schedule reliability’. Consequently, this logistic interdependence is suitable for monetarily evaluating the choice of lot size.

The results of a discrete event oriented simulation study will be used here to verify these effects. With the aid of a simulation program called Plant Simulation® various production scenarios were generated for a production area. The only factor that varied between the scenarios was the selected lot size. For the production of eight products with different material flows on ten workstations, the lot sizes were varied from a sixth of the value determined using the base model up to double it. In accordance with the resulting throughput times and lateness the required safety stocks for ensuring the delivery capability were dimensioned. The associated costs are shown in Fig. 3: along with the stock costs caused by the WIP, the setup costs and the holding costs for the lot stock in the finished goods store of an article are depicted as a function of the lot size (in relation to the lot size determined by the base model).

It turns out that the safety stock required to be able to always serve customer demands with a constant demand rate clearly increases with larger lot sizes.

![Lot Size vs Costs](image)

**Lot Size vs Costs**

<table>
<thead>
<tr>
<th>Lot Size</th>
<th>Logistic</th>
<th>BM</th>
</tr>
</thead>
<tbody>
<tr>
<td>setup</td>
<td>16,500 €</td>
<td>5,200 €</td>
</tr>
<tr>
<td>finished goods*</td>
<td>1,500 €</td>
<td>5,200 €</td>
</tr>
<tr>
<td>evaluation basis BM</td>
<td>8,600 €</td>
<td>10,000 €</td>
</tr>
<tr>
<td>WIP*</td>
<td>8,500 €</td>
<td>18,900 €</td>
</tr>
<tr>
<td>safety stock*</td>
<td>6,500 €</td>
<td>8,900 €</td>
</tr>
<tr>
<td>evaluation basis logistics</td>
<td>33,000 €</td>
<td>33,300 €</td>
</tr>
</tbody>
</table>

* corresponding tied-up capital costs
** in relation to optimal lot size according to base model

* simulated operating point

Fig. 3. Comparison of lot size dependent costs (Simulation Study, see [16])
If only the traditional types of costs are taken into account, an optimum lot size results with lot size dependent costs equal to 10,400 euro. Nevertheless, setting this lot size also entails additional costs of approximately 27,800 euro for the lot and safety stock. If these indirect lot size dependent costs are also taken into consideration when determining the lot size, an optimum results which is almost a third of that determined with the base model. Moreover, this new optimum causes lot size dependent costs of only 33,000 euro in comparison to the 38,200 euro for the base model’s optimum lot size. The actual optimum when considering the perspective of multiple criteria is therefore considerably smaller.

6. Practice oriented approach to lot sizing

With the aim of being able to easily model and compare different scenarios, a method will be introduced here that expresses the sum of the costs that increase with the lot size in the form of a logistics cost factor LF and that estimates it with sufficient precision. The logistics cost factor stands for the relationship between all of the relevant and positive lot size induced costs such as costs resulting on long throughput time, low schedule reliability etc. and the traditionally used holding costs, all of which can be fairly simply determined.

Base on Eq. 1, Eq.2 would then result as the actual optimal lot size \(x_{opt}\). While having a similar structure as the basic model, it is extended with the described logistic cost factor.

\[
x_{\text{opt,log}} = \frac{2 \cdot S \cdot a}{\sqrt{LF \cdot h \cdot p}}
\]

\(x_{\text{opt,log}}\) economic-optimal lot size according to logistics oriented model [units]

LF logistics cost factor [-]

With increasing logistics induced costs due to the greater logistics relevance and thus a growing logistics cost factor, more costs are considered in decision making. The result is a shrinking optimal lot size with costs quickly increasing on both sides of the optimal value. It should be noted here that considering the logistic induced costs does not truly cause any higher unit costs than the base model. They clearly exist--; the base model simply fails to report them.

Based on Fig. 4, we can draw important conclusions that are helpful in assessing the extent to which an estimated LF represents a practical solution. Here, the unit costs curves for an exemplary article from an electronics manufacturer are depicted using various values for LF. In addition, the respective optimum lot sizes are also identified.

It can be seen that the greater the chosen logistics cost factor, the closer the optima are to one another and the more steeply they climb towards the left. We can thus conclude that on the left side, there is a setup cost induced limit, which should not be undercut from a monetary perspective. This limit is dependent on the specific article data, but is generally below 25% of the lot size determined by the base model (LF = 16). We do not know this limit explicitly, but this is not necessary.

We can also deduce that overestimating the LF entails clearly less additional costs than underestimating it. If we assume an actual LF of five (Point A, \(x_{opt} = 950\)), the in this case wrong – assumption of an LF of nine and thus a lot size of 700 units creates, in this example, additional costs of 0.15% per unit (Point B). In comparison, when using the base model with LF of one and thus a lot size of 2,100 units, the additional costs are 1.13% (Point C). Even when the actual value of LF is only three, with an LF of nine a similar cost error would result, as with the use of the base model. Generously dimensioning LF is only connected to minimal cost risks, however results in logistical advantages, as the above discussion clearly demonstrates.

In order to offer a concrete approach for industrial practice, it seems logical to calculate the part of the logistic costs that can be determined analytically. This would be the case with the WIP costs, which with increasing lot sizes and the resulting longer queues is also positively correlated with the lot size. Throughput Oriented Lot Sizing [12] determines these costs and integrates them into the lot size decision. Numerous years of collaboration between IFA and industry has confirmed that taking into account the WIP costs almost always leads to lot sizes that are more than 50% smaller than those determined with the base model. This would amount to a logistics cost factor of 4 or higher, which can be applied as a lower limit for LF. As an upper limit a value of LF = 16 should be implemented, since the pure estimation of LFs with higher values also entail noticeably higher unit costs.

For application in the operational practice, it is thus advisable to identify articles that are relevant for competition (generally A-parts) and to determine a logistics cost factor for them corresponding to existing setup costs and acceptable unit surcharges. This logistics cost factor, generally entails distinctly lower additional costs than it is the case with the very common base model. The determined LF should be adopted for as much of the entire article spectrum as possible in order to be able to realize the logistical advantages in the order throughput. Current work, based on simulation studies and research in the practice, suggest a general orientation on logistic factors between six and twelve, which corresponds to lot sizes that are between 30% and 40% of that calculated with the base model.
Fig. 4. Unit costs curves for different logistics cost factors [14]

Additional logistical advantages can also be attained when the logistic costs factor is practically adapted to specific articles. Since ultimately, strongly dispersed work content results in higher WIP levels and thus longer and varying throughput times, harmonizing the work content is absolutely advisable from a logistics perspective. The LF should thus be varied according to the process-intensity of an article. Instead of just using the same LF for articles, more potential can be created by setting the LF for process-intensive articles at 12 and the LF for the least process-intensive articles at 6. All other LF values can be defined accordingly between these two values. Münzberg discusses the estimation of the LF extensively [14,17].

7. Conclusions

Existing lot sizing methods provide exact solutions but only partially consider the vast implications of the chosen lot size. The impact of lot sizing on the multi-criteria target attainment found in today’s industry is not taken into consideration holistically.

Based on the selected lot size, production logistics related interactions were outlined and integrated into a practice oriented approach bases on the base model which allows users to easily determine lot sizes in consideration of both direct and logistics-induced indirect costs.

The paper showed that even when the procedure still does not represent a closed mathematical approach, it should still support the decision maker in the production, to argue the necessity of smaller lot sizes and to be able to pragmatically determine lot sizes in consideration of additional indirect costs. So, not an exact solution considering only some aspects but a good solution considering all relevant aspects is offered for industry.

This approach is already implemented in several companies in the automotive industry, the pharmaceutical industry and the electronic industry. In all business cases the method was easy to implement since all users understood it easily accepted its logic and since no further data than those already required by the base model is needed. Additionally, a huge programming effort in the company’s ERP system is not necessary. In every company the results of the new method were very promising. The costs reacted as calculated and the positive effects on logistic key performance indicators – such as throughput times or delivery reliability – could be measured after some month.

References