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Chaos and the shadowing property

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Abstract

We present, as a simpler alternative for the results of [P. Kościelniak, On genericity of shadowing and periodic shadowing property, J. Math. Anal. Appl. 310 (2005) 188–196; P. Kościelniak, M. Mazur, On C^0 genericity of various shadowing properties, Discrete Contin. Dynam. Syst. 12 (2005) 523–530], an elementary proof of C^0 genericity of the periodic shadowing property. We also characterize chaotic behavior (in the sense of being semiconjugated to a shift map) of shadowing systems.

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1. Introduction

Let (X, d) denote a compact metric space and let $f : X \to X$ be a homeomorphism (a discrete dynamical system on X).

A sequence $(y_n)_{n\in\mathbb{Z}} \subset X$ is called a δ-pseudo-orbit ($\delta \geqslant 0$) of f if

$$d\big(f(y_n), y_{n+1}\big) \leqslant \delta \quad \text{for every } n \in \mathbb{Z}.$$

Note that 0-pseudo-orbit of f is simply its "real" orbit.

We say that the system f has the (periodic; weak, orbital) shadowing property if for every $\varepsilon > 0$ there exists $\delta > 0$ satisfying the following condition: given a (periodic) δ-pseudo-orbit $\mathrm{y} = (y_n)_{n\in\mathbb{Z}}$ we can find a corresponding (periodic) orbit $\mathrm{x} = (x_n)_{n\in\mathbb{Z}}$ which ε-traces y, i.e.,

$$\mathrm{y} \subset U_\varepsilon(\mathrm{x}) \quad \text{(weak shadowing);}$$

$$\mathrm{y} \subset U_\varepsilon(\mathrm{x}) \quad \text{and} \quad \mathrm{x} \subset U_\varepsilon(\mathrm{y}) \quad \text{(orbital shadowing);}$$

$$d(x_n, y_n) \leqslant \varepsilon \quad \text{for every } n \in \mathbb{Z} \quad \text{(shadowing).}$$

Here $U_\varepsilon(S)$ denotes the ε-neighborhood of the set $S \subset X$, i.e., the set of all $x \in X$ such that $\mathrm{dist}(x, S) \leqslant \varepsilon$.

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Let us note the following dependencies:

$$\text{shadowing} \Rightarrow \text{orbital shadowing} \Rightarrow \text{weak shadowing}.$$

The concept of shadowing was investigated by many authors (see [4,5,12,17,20,21,23,25] and references therein). It is of great importance to the problem of verifying, in a rigorous mathematical sense, the existence of complicated behavior detected via numerical experiments (see [10,11,28,29]).

In [26], making use of the topological transversality theorem, Pilyugin and Plamenevskaya proved that, as far as the C^0 topology is concerned, shadowing is a generic property in the space of discrete dynamical systems of a compact (boundaryless) topological manifold, admitting a handle decomposition. Subsequently, combining this result with Zgliczyński's and Gidea's idea of covering relations [7,8], the authors of this paper additionally obtained C^0 genericity of the periodic shadowing property (see [16]). Nevertheless, the proof still depended on both the topological transversality theorem and the existence of a handle decomposition of a manifold.[1] In this paper we give much simpler argumentation that involves none of the mentioned tools. The method presented here, based on [9,18], allow us to prove C^0 genericity of the periodic orbital shadowing property in the space of discrete dynamical systems of a compact topological manifold (with or without boundary) of the dimension at least 2.

Let Σ denote the compact shift space $\{0,1\}^{\mathbb{Z}}$, topologized by the metric $r(\mathrm{x},\mathrm{y}) = \sum_{i=-\infty}^{\infty} \frac{|x_i - y_i|}{2^{|i|}}$, and let $\sigma : \Sigma \to \Sigma$ be the corresponding shift map, i.e., $\sigma(\mathrm{x})_i = x_{i+1}$.

The system f is said to be semiconjugated to (Σ, σ) if there exist a closed, f-invariant set $\Lambda \subset X$ and a continuous surjection $p : \Lambda \to \Sigma$ such that $p \circ f = \sigma \circ p$.

The system f is called chaotic if some positive iteration of f is semiconjugated to (Σ, σ).

In [2] Akin et al. proved that a C^0 generic homeomorphism of a compact topological manifold is chaotic. (The proof presented there did not admit 2-dimensional manifolds with boundary. In [13,14] Kościelniak gave an independent argumentation that could be applied for this case, too.) In this paper we characterize such a chaotic behavior for systems with the shadowing property, in terms of the existence of unstable chain recurrent points.

2. Results

At first we complete notation and recall some known definitions.

Let $\mathcal{H}(X)$ denote the set of all homeomorphisms of X. Introduce in $\mathcal{H}(X)$ the complete metric

$$\rho_0(f,g) = \max\Big\{\max_{x\in X} d\big(f(x), g(x)\big), \max_{x\in X} d\big(f^{-1}(x), g^{-1}(x)\big)\Big\},$$

which generates the C^0 topology.

A point $x \in A \subset X$ is said to be stable for $f \in \mathcal{H}(X)$ in A if for any $\varepsilon > 0$ there exists $\delta > 0$ such that for every $y \in A$ satisfying $d(x,y) < \delta$ we have $d(f^n(x), f^n(y)) < \varepsilon$ for all $n \in \mathbb{N}$. Otherwise we call such a point unstable in A.

The set $CR(f)$ consisting of all chain recurrent points, i.e., such points $x \in X$ that for any $\delta > 0$ there exists a periodic δ-pseudo-orbit through x, is called the chain recurrent set of the system $f \in \mathcal{H}(X)$.

A property $\mathcal{P}$ of elements of a topological space S is said to be generic if the set of all $x \in S$ satisfying $\mathcal{P}$ is residual, i.e., it includes a countable intersection of open and dense subsets of S.

The goal of this paper is to prove the following theorems.

Theorem 1. *If M is a compact topological manifold* (*with or without boundary*) *of the dimension at least* 2, *then orbital shadowing is a generic property in $\mathcal{H}(M)$.*

Theorem 2. *Assume that $f \in \mathcal{H}(X)$ has the shadowing property. Then the following conditions are equivalent*:

(1) *there is a chain recurrent point which is unstable in $CR(f)$*;
(2) *there is an infinite number of chain recurrent points which are unstable in $CR(f)$*;

[1] This assumption was just relaxed by Kościelniak [13,15]. In his proof there is no need to use the topological transversality theorem any more, but covering relations are necessary as before.

(3) *f is chaotic.*

Additionally, if f possesses the periodic shadowing property, then in condition (3) *we also have $p(\Lambda \cap \mathrm{Per}(f)) = \mathrm{Per}(\sigma)$, where $p : \Lambda \to \Sigma$ denotes a semiconjugacy map.*

Let us note that an immediate consequence of Theorem 1 is so-called "the C^0 general density theorem" [9], providing genericity of the property of possessing by a system $f \in \mathcal{H}(M)$ a set of periodic points that is dense in $CR(f)$. The original statement of this theorem comes from [22]. However, the argumentation presented there contained some gaps that were indicated and partially corrected in [6,24].

3. Proof of Theorem 1

Fix any $\varepsilon > 0$. Let $\mathrm{U}^\varepsilon = \{U_i\}_{i=1}^k$ be a finite covering of M by open sets with diameters not greater than ε. Set $K = \{1, 2, \ldots, k\}$ and let $\mathcal{S}(T)$ denotes the collection of all subsets of a given set T. For $f \in \mathcal{H}(M)$ consider the family $\mathrm{I}_f \subset \mathcal{S}(K)$ consisting of these sets $L \subset K$ for which we can find a periodic orbit $\mathrm{x} = (x_0, \ldots, x_{n-1}, x_n = x_0)$, with $x_i \neq x_j$ for $i, j \in \{0, \ldots, n-1\}$, $i \neq j$, satisfying the following conditions:

(i) there exists a neighborhood V of x_0, homeomorphic to a closed Euclidean ball, such that

$$f^n(V) \subset \mathrm{Int}\, V \quad \text{and} \quad f^j(V) \subset \bigcap \{U_i \mid x_j \in U_i\} \quad \text{for } j \in \{0, \ldots, n\}; \tag{1}$$

(ii) $\mathrm{x} \cap U_i \neq \emptyset$ if and only if $i \in L$.

It is easily seen that for any $f \in \mathcal{H}(M)$ the following holds:

$$\text{there exists a neighborhood } \mathcal{W} \text{ of } f \text{ such that } \mathrm{I}_f \subset \mathrm{I}_g \text{ for } g \in \mathcal{W}. \tag{2}$$

Indeed, taking $L \in \mathrm{I}_f$, corresponding a periodic orbit $\mathrm{x} = (x_0, \ldots, x_{n-1}, x_n = x_0)$ and a neighborhood V of x_0, we observe that (1) is also held for any $g \in \mathcal{H}(M)$ sufficiently close to f. Then by a simple application of the Brouwer fixed point theorem for g^n and V we obtain $L \in \mathrm{I}_g$ and, consequently, conclude (2). (Note that I_f is a finite set.)

Define the set $\mathcal{R}_{\mathrm{U}^\varepsilon}$ as the collection of such $f \in \mathcal{H}(M)$ that $\mathrm{I}_f = \mathrm{I}_g$ for g sufficiently close to f. Obviously, it is an open subset of $\mathcal{H}(M)$. To prove that $\mathcal{R}_{\mathrm{U}^\varepsilon}$ is dense in $\mathcal{H}(M)$, fix any open set $\mathcal{V} \subset \mathcal{H}(M)$. Observe that the set $\mathrm{I}_\mathcal{V} = \{\mathrm{I}_g \mid g \in \mathcal{V}\} \subset \mathcal{S}(\mathcal{S}(K))$ is a finite set, partially ordered by the relation of inclusion. Let I_f, corresponding to some $f \in \mathcal{V}$, be one of its maximal elements. Then, applying condition (2), we obtain a neighborhood $\mathcal{W} \subset \mathcal{V}$ of f such that $\mathrm{I}_f = \mathrm{I}_g$ for $g \in \mathcal{W}$. Thus, $f \in \mathcal{R}_{\mathrm{U}^\varepsilon} \cap \mathcal{V}$, which completes the proof of density of the set $\mathcal{R}_{\mathrm{U}^\varepsilon}$.

Take $f \in \mathcal{R}_{\mathrm{U}^\varepsilon}$ and choose $\beta > 0$ such that $\mathrm{I}_f = \mathrm{I}_g$ for $g \in \mathcal{H}(M)$ with $\rho_0(f, g) \leqslant 2\beta$. Since the dimension of M is greater than 1, the action $\mathcal{H}(M)$ is generalized homogeneous on $\mathrm{Int}\, M = M \setminus \partial M$ (see [1–3,19,27]), i.e., there exists $\gamma > 0$ (a β-modulus of homogeneity) such that if $\{x_1, \ldots, x_n\}, \{y_1, \ldots, y_n\} \subset \mathrm{Int}\, M$ is a pair of sets of mutually disjoint points satisfying $d(x_i, y_i) \leqslant \gamma$, $i \in \{1, \ldots, n\}$, then there exists $h \in \mathcal{H}(M)$ with $\rho_0(h, \mathrm{id}_M) \leqslant \beta$ and $h(x_i) = y_i$, $i \in \{1, \ldots, n\}$. Set $\delta = \gamma/2$. To make the proof complete it is sufficient to show that each periodic δ-pseudo-orbit of f has some 2ε-tracing periodic orbit.

Fix any periodic δ-pseudo-orbit $\mathrm{y} = (y_0, \ldots, y_{n-1}, y_n = y_0)$ and notice that we can easily find (see, e.g., the proof of Lemma 9 in [30]) a periodic 2δ-pseudo-orbit $\bar{\mathrm{y}} = (\bar{y}_0, \ldots, \bar{y}_{n-1}, \bar{y}_n = \bar{y}_0)$ such that $\bar{y}_i \in \mathrm{Int}\, M$, $d(y_i, \bar{y}_i) \leqslant \varepsilon$ and $\bar{y}_i \neq \bar{y}_j$ for $i, j \in \{0, \ldots, n-1\}$, $i \neq j$. Let $h \in \mathcal{H}(M)$, with $\rho_0(h, \mathrm{id}_M) \leqslant \beta$, be a homeomorphism connecting $f(\bar{y}_i)$ with $\bar{y}_{i+1}$ for $i \in \{0, \ldots, n-1\}$. Then $\bar{\mathrm{y}}$ is a periodic orbit of $h \circ f$. So, proceeding like in the second step of the proof of Proposition 3 in [9] (see also Lemma 3.1 in [22]), we can find a homeomorphism g such that $\rho_0(f, g) \leqslant 2\beta$ and $\bar{\mathrm{y}}$ is a periodic orbit of g satisfying (i)–(ii) with the set $L = \{i \in K \mid \bar{\mathrm{y}} \cap U_i \neq \emptyset\}$. Thus $L \in \mathrm{I}_g$. As $\rho_0(f, g) \leqslant 2\beta$ we have $\mathrm{I}_f = \mathrm{I}_g$, hence there exists a periodic orbit x of f that ε-traces $\bar{\mathrm{y}}$ and, consequently, 2ε-traces the δ-pseudo-orbit y. (Note that each of the sets $U_i \in \mathrm{U}^\varepsilon$, intersects either both x and $\bar{\mathrm{y}}$ or none of them.) □

4. Proof of Theorem 2

Let $f \in \mathcal{H}(X)$ be a homeomorphism with the shadowing property.

Obviously, (2) implies (1). It remains to show that (1) implies (3) and (3) implies (2).

(1) $\Rightarrow$ (3) (The idea of the proof of this step comes from [29].) Take a point $x \in CR(f)$ which is unstable in $CR(f)$, i.e., we can find for it such a constant $e > 0$ that for any $\delta > 0$ there exist $y \in CR(f)$ and $N \in \mathbb{N}$ with $d(x, y) < \delta$ and $d(f^N(x), f^N(y)) > e$. Let $\delta(\varepsilon)$ denote a number $\delta > 0$ corresponding to a given $\varepsilon > 0$ by the shadowing property of f. Put $\delta = \delta(e/4)$ and $\delta' = \delta/2$. Take $y \in CR(f)$ and $N \in \mathbb{N}$ such that $d(x, y) < \delta'$ and $d(f^N(x), f^N(y)) > e$. Since f is continuous, there exists $\gamma \in (0, \delta')$ such that for any finite γ-pseudo-orbits $(x, x_1, \ldots, x_N)$ and $(y, y_1, \ldots, y_N)$ we have $d(f^N(x), x_N) < e/6$ and $d(f^N(y), y_N) < e/6$. Then, by the definition of $CR(f)$, there exist two periodic γ-pseudo-orbits $(x = x_0, x_1, \ldots, x_K = x_0)$ and $(y = y_0, y_1, \ldots, y_L = y_0)$ with $K, L > N$, $d(x_0, y_0) < \delta'$ and $d(x_N, y_N) > \frac{2}{3}e$.

Now, for a given $\mathrm{w} = (w_k)_{k\in\mathbb{Z}} \in \Sigma$ we define the δ-pseudo-orbit $\tilde{z}(\mathrm{w}) = (z(\mathrm{w})_k)_{k\in\mathbb{Z}}$ as follows:

$$z(\mathrm{w})_{KLi+j} = \begin{cases} x_{j \bmod K}, & \text{when } w_i = 0,\\ y_{j \bmod L}, & \text{when } w_i = 1 \end{cases}$$

for $i \in \mathbb{Z}$ and $j \in \{0, \ldots, KL - 1\}$. Put

$$\Lambda = \bigl\{z \in X \mid \exists \mathrm{w} \in \Sigma\ \forall i \in \mathbb{Z}\ d\bigl(f^i(z), z(\mathrm{w})_i\bigr) \leqslant e/4\bigr\}.$$

Note that the set Λ is nonempty since f has the shadowing property. Let $p : \Lambda \to \Sigma$ be the map given by the following formula:

$$p(z)_i = \begin{cases} 0, & \text{when } d(f^{iKL+N}(z), x_N) \leqslant e/4,\\ 1, & \text{when } d(f^{iKL+N}(z), y_N) \leqslant e/4 \end{cases}$$

for $i \in \mathbb{Z}$. Since $d(x_N, y_N) > 2e/3$, the function p is well defined. Note that p is a surjective map by the shadowing property of f.

From the above definitions it follows that Λ is invariant for f^{KL} and $p \circ f^{KL} = \sigma \circ p$. We prove now that Λ is a closed set and p is a continuous map.

Take a sequence $(z_i)_{i=1}^{\infty} \subset \Lambda$ which tents to some point $z_0 \in X$. Then $z_0 \in \Lambda$. Indeed, each z_i $e/4$-traces $\tilde{z}(\mathrm{w}_i)$ for some $\mathrm{w}_i = (w_{ij})_{j\in\mathbb{Z}} \in \Sigma$. Then there exist subsequence $(i_k)_{k=1}^{\infty}$ and $\mathrm{w}_0 = (w_{0j})_{j\in\mathbb{Z}} \in \Sigma$ such that for all $l \in \mathbb{Z}$ we have $w_{i_k l} \to w_{0l}$ as $k \to \infty$. For a given $l \in \mathbb{Z}$ we obtain $d(f^l(z_{i_k}), z(\mathrm{w}_{i_k})_l) \leqslant e/4$ and hence

$$d\bigl(f^l(z_0), z(\mathrm{w}_0)_l\bigr) \leqslant e/4,$$

which proves that z_0 $e/4$-traces $\tilde{z}(\mathrm{w}_0)$. The continuity of p is a simple consequence of the definition of p and the continuity of f^{iKL+N} for all $i \in \mathbb{Z}$.

Now, let us note that $p(\Lambda \cap \mathrm{Per}(f)) \subset \mathrm{Per}(\sigma)$. In the case when f additionally possesses the periodic shadowing property we also have $p(\Lambda \cap \mathrm{Per}(f)) = \mathrm{Per}(\sigma)$. It finishes the proof of the implication (1) $\Rightarrow$ (3).

(3) $\Rightarrow$ (2) Let f^K, with some $K \in \mathbb{N}$, be semiconjugated to (Σ, σ) and let $p : \Lambda \to \Sigma$ denote a semiconjugacy map. It will not result in the loss of generality if we assume that $\Lambda \subset CR(f)$ (see [2]). Since a point unstable for f^K is also unstable for f, it suffices to prove that for all $\mathrm{x} \in \Sigma$ there exists a point $x \in p^{-1}(\mathrm{x})$ which is unstable for f^K in Λ.

Fix $\mathrm{x} \in \Sigma$. Since x is unstable in Σ with the constant 1, for all $n \in \mathbb{N}$ there exist $\mathrm{y}_n \in \Sigma$ and $N_n \in \mathbb{N}$ such that $r(\mathrm{x}, \mathrm{y}_n) < 1/n$ and $r(\sigma^{N_n}(\mathrm{x}), \sigma^{N_n}(\mathrm{y}_n)) > 1$. The map p is a surjection, so for any $n \in \mathbb{N}$ we can find $y_n \in \Lambda$ satisfying $p(y_n) = \mathrm{y}_n$. By the compactness of Λ we can assume that there exists a point $x \in \Lambda$ such that $y_n \to x$. Then $p(x) = \mathrm{x}$ since p is continuous.

Assume that x is stable for f^K in $CR(f)$. By the continuity of p there exists $\delta > 0$ satisfying

$$d(x, y) < \delta \quad \Rightarrow \quad r\bigl(p(x), p(y)\bigr) < 1.$$

Since x is stable, we can find $\delta' > 0$ such that for any $y \in CR(f)$ and $n \in \mathbb{N}$ we have

$$d(x, y) < \delta' \quad \Rightarrow \quad d\bigl(f^{nK}(x), f^{nK}(y)\bigr) < \delta.$$

Take a number n for which $d(x, y_n) < \delta'$. Then $d(f^{N_nK}(x), f^{N_nK}(y_n)) < \delta$ and hence

$$r\bigl(p\bigl(f^{N_nK}(x)\bigr), p\bigl(f^{N_nK}(y_n)\bigr)\bigr) < 1.$$

On the other hand,

$$p\bigl(f^{N_nK}(x)\bigr) = \sigma^{N_n}\bigl(p(x)\bigr) = \sigma^{N_n}(\mathrm{x})$$

and

$$p\left(f^{N_n K}(y_n)\right) = \sigma^{N_n}\left(p(y_n)\right) = \sigma^{N_n}(\Upsilon_n),$$

which makes a contradiction. It finishes the proof of the implication (3) ⇒ (2) and, in consequence, Theorem 2. □

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