Modelling failure of composite specimens with defects under compression loading

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A R T I C L E   I N F O

Article history:
Received 28 March 2012
Received in revised form 21 December 2012
Accepted 22 December 2012
Available online 5 January 2013

Keywords:
A. Carbon fibre
B. Defects
C. Damage mechanics
C. Finite element analysis

A B S T R A C T

Composite structures exhibit many different failure mechanisms, but attempts to model composite failure frequently make a priori assumptions about the mechanism by which failure will occur. Wang et al. [1] conducted compressive tests on four configurations of composite specimen manufactured with out-of-plane waviness created by ply-drop defects. There were significantly different failures for each case. Detailed finite element models of these experiments were developed which include competing failure mechanisms. The model predictions correlate well with experimental results – both qualitatively (location of failure and shape of failed specimen) and quantitatively (failure load). The models are used to identify the progression of failure during the compressive tests, determine the critical failure mechanism for each configuration, and investigate the effect of cohesive parameters upon specimen strength. This modelling approach which includes multiple competing failure mechanisms can be applied to predict failure in situations where the failure mechanism is not known in advance.

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1. Introduction

The thickness of structural members (e.g. wind turbine blades, wing spars) usually varies along their length. In composite structures, the thickness of the laminate can be varied by altering the number of plies in the lay-up, creating features called ply-drops. This can result in a localised waviness in the plies surrounding a ply-drop [2,3]. Waviness can also arise by other means such as drape around geometrical features [4], interaction between tooling and lay-up during cure [5], and variation in supplied prepreg material [6].

The effects of ply drops, waviness and other defects upon compressive strength of composites have been the subject of many studies. Soutis [7] conducted an extensive literature review of compressive behaviour of composites for both unidirectional and multidirectional laminates, which included descriptions of damage development in notched and unnotched composites. Subsequently, a review by He et al. [8] identified several works which demonstrate that ply-drop regions can act as initiation sites for compressive failure of fibre reinforced composite materials, such as Curry et al. [9], Varughese and Mukherjee [10] and Thomsen et al. [11]. In both reviews [7,8], many different mechanisms of failure are presented. For instance, failure can occur due to delamination initiating from the stress concentration at the ply-drops, as noted by Fish and Lee [12], Samborsky et al. [13] and Thomas and Weber [14] or due to a microbuckling failure induced by the local fibre waviness arising from the ply drop geometry as noted by Avery et al. [2] and Mandell et al. [3]. The effect of fibre waviness defects in the absence of other features such as ply drops has been widely studied, including work by Hsaio and Daniel [15], Liu et al. [16], Slaughter and Fleck [17], and Lemanksi and Sutcliffe [18]. Steeves and Fleck [19] tested compressive strength of composite laminates with internal ply drops and found that the failure mechanism was influenced by initial fibre misalignment angle induced in the continuous plies. Varughese and Mukherjee [10] evaluated design parameters for ply-drops in composite laminates to derive simple rule-of-thumb design guidelines for laminates with ply drops.

Many studies have modelled delamination failure in composites, and investigators have employed a variety of approaches to model delaminations at ply drops. For example Fish and Lee [12] investigated delamination using a Tsai-Wu strength criterion in a continuum resin layer surrounding a ply drop; Wisnom et al. [20] used a simple strain energy release rate equation to predict the static and fatigue strength of tapered laminates with ply drops; Tay et al. [21] used a novel element failure method (EFM) for in-plane propagation in conjunction with cohesive elements to model “delamination onset and propagation”. Yang and Cox [22] noted the use of cohesive zone approaches [23–25] in investigations of composite delamination.

It is thus seen that many earlier approaches to modelling composite failure make a priori assumptions about the mechanism by which failure occurs. However, the fractographic study of the
compressive failure of multidirectional laminates presented by Tsamapas et al. [26] highlights the differing progression of failure modes observed in different lay-ups for multidirectional laminates. A predictive modelling approach is therefore required which does not make a priori assumption about the mechanism by which failure will occur.

Wang et al. [1] performed experiments on a series of ply-drop specimens of different configurations to investigate the effect of geometric features upon compressive failure. For each configuration, the experiments gave markedly different results in which the failed specimens exhibited evidence of different failure mechanisms. Although the mechanisms of failure were determined by noting the overall damaged shape and examining the fracture surfaces of the failed specimens, the sequence of failure could not be directly observed in the experiments due to the sudden catastrophic nature of the experimental failure and a detailed fractographic analysis might not have been conclusive.

This paper briefly describes the experimental set up of Wang et al. [1], and then presents details of a finite element model which has been created to allow these different mechanisms to compete against (and interact with) one another, thus modelling the compressive failure without making an a priori assumption about the failure mechanism. The predictions of the modelling work are compared both quantitatively and qualitatively with experimental results. These model predictions are examined in detail to provide physical insight into the sequence of failure and thus to identify the critical mechanism which leads to failure of each specimen. A parametric study was also carried out to investigate the sensitivity of the model to the cohesive material parameters.

2. Experimental configurations

This section summarises the experimental work of Wang et al. [1] who manufactured composite test pieces to investigate the effect of ply-drops and waviness defects on the compressive failure of composite laminates. Test specimens were manufactured from carbon-reinforced prepreg (AS4/8552 from Hexcel Composites [26]) and contained ply drops of different configurations. The nominal cured ply thickness was 0.13 mm and the fibre volume fraction was 57.42% based on manufacturers specifications [27]. Woven glass epoxy laminates of thickness 2 mm were made from prepregs containing CYCOM 977-2 epoxy [28] and were bonded to the cured laminate using Redux® 810 adhesive [29]. The specimen was then machined to the correct size. Fig. 1 shows a cross section of the specimen after machining to 90 mm length and 10 mm width (out-of-plane from the diagram).

Fig. 2 shows a representation of the gauge section and immediately surrounding features. The gauge section comprises straight plies on the outer surfaces, central blocks of discontinuous plies, and continuous wavy plies. The fibres in the discontinuous plies were either laid up longitudinally or transversely to the compressive loading direction as represented by the separate diagrams for “L-configuration” and “T-configuration”. Note the different material distributions in the ply drop layers for L and T configurations – the L configuration has a distinct end to the 0° dropped plies followed by a resin pocket as shown in Fig. 2, whereas the T configuration has 90° plies that taper to the end of the ply drop region. This is consistent with microscopy images of the cross section taken prior to the test procedure.

All test laminates were 16 plies thick in total. Four configurations were manufactured with discontinuous layers either 2 plies or 10 plies thick, and oriented either longitudinally or transversely to the compressive loading direction. The four configurations are referenced in the current modelling work as L2, T2, L10, and T10, with the letter indicating the orientation of fibres in the dropped plies and the number indicating the number of dropped plies.

The manufacturing process of lay-up, debulking and autoclave cure gave central wavy regions with a peak misalignment angle of around 8° (2 plies dropped) and 30° (10 plies dropped). Compression tests, using the fixture originally developed by Haberle and Matthews [30], were performed using a Zwick testing machine (100 kN capacity) at ambient temperature. Compressive strain was measured using an Imetrum non-contact video gauge [31] comprising a 15 Hz video camera and a telecentric lens. All samples were loaded to failure under displacement control at a rate of 1 mm/min crosshead speed. Specimens with L2, L10 and T10 configurations generally failed in the gauge section, although a few samples failed prematurely at the tab regions and were discarded as invalid tests. However, the T2 configuration repeatedly failed around the edge of the tabbed region, indicating that this was not an invalid failure mechanism for this test configuration. The manufacture, test methodologies and results are described in more detail by Wang et al. [1].

3. Finite element model

A finite element model was created and analysed using Abaqus/Explicit. Only part of the test specimen geometry was modelled.

Fig. 1. Cross-section of test specimen, showing end tabs bonded onto the top and bottom surfaces of a CFRP laminate with ply-drops. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
The section modelled is shown in Fig. 3 and includes the gauge section and part of the tabs. Modelling only part of the specimen in this way reduces the number of elements in the model and minimises the number of time steps required for the stresses arising from displacements applied at the end of the model to propagate to the gauge section. It also reduces the elastic energy stored in the model so that post-failure response is not completely dominated by elastic unloading (as it is in the tests), which in turn makes it easier to identify the sequence of failure events. Subsequent finite element analyses which modelled the entire specimen were performed for direct comparison with experimental stress–strain results.

The specimen gauge section was 2.08 mm thick, so each ply of the composite was modelled with a 0.125 mm thickness using reduced-integration 2D plane stress elements which were separated from the adjacent plies by a 0.005 mm thick cohesive layer (attached to the solid elements by tied constraints). The entire thickness of the composite gauge section was built up in this way, with the out-of-plane waviness being explicitly modelled by the geometry of the parts created. Four elements were used through the thickness of each ply, and a mesh refinement study was performed which showed less than 1% change in predicted failure load when the mesh density was doubled. The meshes used for configurations with 2 discontinuous plies and 10 discontinuous plies are shown in Fig. 3.

An explicit analysis technique was used, as this captures the transient effects associated with post-failure response that cannot be modelled with an implicit method. An initial transverse displacement equivalent to 0.6% through thickness strain was applied to the flat (horizontal) faces of the tabs to mimic the clamping forces, and the compressive loading was then applied by prescribing equal and opposite velocity boundary conditions at both vertical ends which act to compress the specimen. Residual stresses from specimen manufacture were not considered.

Mass scaling factors were used in explicit analyses to increase the stable time-step and reduce the run time. Performing the analysis with a mass scaling factor of 1000 required over 2 h running on four cores of an Intel i7-2600K CPU running at 4.2 GHz. Increasing the mass scaling factor to 10,000 required only 45 min to run on the same system. At this level, the peak kinetic energy recorded
during failure remained below 5% of the strain energy, and the analysis predicted the same failure mechanisms and predicted the same failure load to within 0.5% of that predicted by the analysis with a mass scaling factor of 1000. The use of a large mass scaling factor is therefore considered to have a negligible effect upon the model behaviour for simulating these quasi-static experiments.

3.1. Material parameters

3.1.1. Unidirectional CFRP plies

The individual CFRP plies were modelled with homogeneous orthotropic linear-elastic, material properties. These elastic properties used are given in Table 1 based on values used by Wang et al. [1], derived from the product data sheet [27] and measurements by Ersoy et al. [32].

The post-elastic behaviour of the homogenised CFRP material has been represented as perfectly plastic, with the onset of yield determined by Hill’s quadratic failure criterion [33].

\[
F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L\tau_{23}^2 + 2M\tau_{13}^2 + 2N\tau_{12}^2 > 1
\]

where the coefficients \( F, G, H, L, M, \) and \( N \) are derived from uniaxial failure stress values as

\[
F = \frac{1}{2} \left( \frac{1}{S_{11}} + \frac{1}{S_{13}} - \frac{1}{S_{33}} \right)
\]

\[
G = \frac{1}{2} \left( \frac{1}{S_{11}} + \frac{1}{S_{22}} - \frac{1}{S_{33}} \right)
\]

\[
H = \frac{1}{2} \left( \frac{1}{S_{11}} + \frac{1}{S_{22}} - \frac{1}{S_{33}} \right)
\]

\[
L = \frac{1}{2S_{23}}
\]

The simplified continuum behaviour used has some limitations: most significantly (1) an equal tensile and compressive response, and (2) a perfectly-plastic kinematic response.

It is well established that compressive strength is lower than tensile strength in the axial direction, however, Budiansky’s plastic microbuckling analysis shows this is a result of buckling instability (i.e. geometric non-linearity). A geometrically non-linear analysis therefore captures this effect as easily demonstrated by simple single element tests. In the through-thickness direction, the equal tensile and compressive response of the Hill’s criterion is not resolved in this way, but when the continuum model is combined with an appropriate cohesive response, a good representation of through-thickness behaviour is obtained as different mechanisms are active in compressive and tensile failure modes.

Although the tensile behaviour of AS4/8552 would not be well modelled by an elastic-perfectly plastic kinematic response, this material behaviour provides sufficient physical representation of the non-linear material response in shear to predict and understand the failure mechanisms of the specimen structure under compression. It is acknowledged that a detailed user material which correctly follows the 3D failure envelope of CFRP may allow a more accurate solution, but the development of continuum failure criteria is an area of ongoing worldwide research [34,35] and is beyond the scope of the current work.

The input parameters for this failure criterion are given in Table 2. The uniaxial tensile strength is used in the fibre direction \( (S_{11}) \) and the in-plane shear strength is used for all shear strengths \( (S_{12}, S_{13}, S_{23}) \). These values are obtained from the product data sheet [27]. The transverse strength parameters \( (S_{22}, S_{33}) \) are each taken as \( S_{23}/\sqrt{3} \), i.e. assuming transverse isotropy and von Mises yield behaviour in the plane perpendicular to the fibre direction.

3.1.2. Cohesive interfaces

The cohesive interfaces were modelled using the traction–separation implementation in Abaqus, using the parameters given in Table 3.

The stiffness parameter inputs \( (K_{nn}, K_{tt} \) and \( K_{nt} \) ) required by Abaqus are the moduli of the cohesive material divided by its thickness [36]. The actual value to use is subject to considerable uncertainty, as experimental measurement of the stiffness of a cohesive layer a few micrometres thick is practically impossible. For CFRP-to-CFRP cohesion, the stiffness was calculated by assuming neat resin properties \( (E = 4.67 \text{ GPa}, G = 1.67 \text{ GPa}[27,37]) \) and a cohesive layer thickness of 0.005 mm. A parametric study is presented in Section 6 to show that the results are relatively insensitive to cohesive layer stiffness.

Damage in the cohesive layer is initiated subject to a quadratic stress criterion of the form

\[
\left( \frac{\sigma_{nn}}{S_{nn}} \right)^2 + \left( \frac{\sigma_{ss}}{S_{ss}} \right)^2 + \left( \frac{\sigma_{tt}}{S_{tt}} \right)^2 = 1
\]

where \( \sigma_{nn} \) is the normal component of stress in the cohesive layer, \( \sigma_{ss} \) and \( \sigma_{tt} \) are two perpendicular shear components of stress in the cohesive layer, and \( S_{nn}, S_{ss} \) and \( S_{tt} \) are the corresponding normal and shear strengths in the cohesive layer.

Normal compressive stress does not contribute towards cohesive damage. For CFRP-to-CFRP cohesion the damage initiation stresses were taken as the through thickness (81 MPa) and in-plane shear (114 MPa) strengths from the product data sheet [26], while for CFRP-to-tab cohesion, the tensile strength value (120 MPa) was estimated to be around that of neat 8552 resin [27] and the shear strength (69.3 MPa) was estimated by assuming neat resin behaves in an isotropic manner with a von Mises yield criterion.

Energy based cohesive damage evolution is defined using the Benzegagh–Kenane criterion [38], with a linear softening law. Fracture energies of \( G_f = 200 \text{ J/m}^2 \) and \( G_{II} = 1000 \text{ J/m}^2 \) are used for normal (Mode I) and shear (Mode II) cohesive failures respectively for the CFRP-to-CFRP adhesion as used by Kawashita et al. [39]. Fracture energy values of \( G_f = 400 \text{ J/m}^2 \) and \( G_{II} = 2000 \text{ J/m}^2 \) were used for the CFRP-to-tab cohesive layer, which experimental results suggested was not a critical parameter.

3.1.3. Glass tabs and adhesive wedges

As neither the glass tabs, nor the adhesive wedges failed during any tests these materials are of lesser interest and are modelled with generic representative properties. The glass tabs have been
modelled as isotropic linear elastic material, and the adhesive wedges modelled as isotropic linear elastic perfectly-plastic material.

4. Results

A comparison of experimental results and finite element predictions is presented. This firstly compares the location of failure and overall appearance of the failed specimen, then compares the predicted and observed failure loads for FE analysis, and finally describes the failure mechanisms in detail.

4.1. Qualitative comparison of post-failure geometry

Any comparison must first determine whether the modelling predicts the correct failure mechanisms. If the incorrect failure mechanism is predicted, any correlation between predicted and observed failure loads is likely to be coincidental.

Due to the sudden catastrophic nature of the compressive failure in the experiments, it was generally not possible to observe the mechanisms of failure directly. The failed test specimens show the location and types of failure that occur but do not conclusively show the order in which they occur – not least because the elastic unloading post-failure causes significant additional damage to the specimens. These failed specimens are compared with numerical predictions in Fig. 4 which shows images of failed test specimens and the results of the finite element modelling for each configuration, with the failure locations highlighted. Although the in situ photographic image for the T2 configuration in Fig. 4 only shows a failure on the lower half of the specimen, a physical examination of all the failed T2 specimens indicates that microbuckling failure had occurred through the full thickness of the composite, as shown in Fig. 5 which also shows an unusual example of multiple kink band formation within the end tabbed region for this specimen.

There is some scatter in the experimental results, but nonetheless, there is a good qualitative correlation between the end state of the model and the failed specimens in each case, which suggests that the finite element modelling has predicted the correct location and type of failure for each test configuration.

4.2. Comparison of failure loads

The failure loads predicted by the modelling are compared with the average experimentally recorded failure loads of Wang et al. [1] in Table 5. In general, there is good agreement between the values, further confirming that the models are accurately describing experimental behaviour. The numerical modelling can therefore be used to investigate the details of the mechanisms by which compressive failure of the specimen may have initiated and developed in the experiments.

The numerically predicted load–displacement curves were compared with those obtained experimentally. In each case, the load–displacement curves were similar up to peak load, but the response after peak load differed. In the experiments, there is a large amount of elastic strain energy stored in the tab regions which causes catastrophic failure after peak load. These tabbed regions are not modelled in the finite element analysis, so the elastic unloading after peak load does not cause complete catastrophic failure. This was an intended effect of the modelling approach taken and allowed the progression of failure mechanisms to be observed more clearly than would otherwise have been possible.

4.3. Detailed description of failure mechanisms

Since the failure mechanism and sequence of failure is unique for each test configuration, the results are examined on a case-by-case basis.

4.3.1. L2 configuration

Fig. 6a plots the predicted load displacement response for the L2 specimen. The displacement plotted in the figure is the relative displacement of the two ends of the model in the compressive loading direction. Key failure events are identified on the load response and on a schematic of the gauge section, Fig. 6b. The numbers used in the figures to identify each event are referred to in the following description of the sequence of failure. There is initial linear elastic behaviour, followed by the onset (1) of cohesive damage (in shear mode) between the ply-drop and wavy layers which develops into a full delamination. (Note there is also cohesive damage between the ply-drop layer and the straight layer, but this initiates later and propagates less far). At peak load these delaminated sections begin to bend outwards (2) as an Euler buckling response to the compressive loading. A microbuckle-type failure (3) is then seen in the wavy layer, which does not cause a significant load reduction because the wavy layer is not carrying much load. However, the combination of compression and bending of the straight layers creates high compressive stresses on the inside surfaces of these layers which causes a microbuckle-type failure which propagates across these layers from the inside out, causing a large reduction in load (4). Subsequent delamination is seen between the plies in the straight layers (5).

This failure is delamination driven because, although the large loss of strength is due to the compressive microbuckle-type failure, this mechanism can only occur as a result of delamination.

This numerical prediction correlates well with experimental results (i.e. failure load, location of different failure mechanisms in the failed specimens, and high speed video of the experimental test). The positions where different failure mechanisms occurred in the test specimens are illustrated in Fig. 6c. In the experiments a compressive failure occurs across the thickness of the specimen at the location where the ply drop layer ends, and there are as several delaminations between the wavy layer and the ply drop layers, between the straight layers and the ply drop layers, and within the blocks of straight plies. In a small minority of tests, the compressive failure occurred where the curve of the wavy layer meets the straight layer.

4.3.2. T2 configuration

The predicted failure progression for the T2 specimen is shown in Fig. 7a and b. There is initial linear elastic behaviour. This is followed by formation of a microbuckle (1), which initiates at the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CFRP to CRFP cohesive properties</th>
<th>CFRP to tabs cohesive properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal stiffness K_n</td>
<td>$9.34 \times 10^5$ GN/m$^3$</td>
<td>$5.00 \times 10^5$ GN/m$^3$</td>
</tr>
<tr>
<td>Shear stiffness K_s</td>
<td>$3.34 \times 10^5$ GN/m$^3$</td>
<td>$1.923 \times 10^5$ GN/m$^3$</td>
</tr>
<tr>
<td>Normal strength S_n</td>
<td>81 MPa</td>
<td>120 MPa</td>
</tr>
<tr>
<td>Shear strength S_s</td>
<td>114 MPa</td>
<td>69.3 MPa</td>
</tr>
<tr>
<td>Mode I fracture energy G_I</td>
<td>$200$ J/m$^2$</td>
<td>$400$ J/m$^2$</td>
</tr>
<tr>
<td>Mode II fracture energy G_II</td>
<td>$1000$ J/m$^2$</td>
<td>$2000$ J/m$^2$</td>
</tr>
</tbody>
</table>
outside surface of one straight layer close to where the adhesive tab joins the gauge section due to the local stress concentration at the tab. This microbuckle propagates inwards through the thickness of this block of straight layers, and reaches the subsequent block of ply-drop and wavy layers (2). The microbuckle then continues to propagate into the second block of straight layers (3) at the same time as a delamination appears between the wavy layer and ply drop-layer (4), which is followed very closely by delamination between the ply-drop layer and straight layer (5).

In the model, the overall bending induced by the asymmetry of the wavy layer leads to a larger stress concentration on the top surface in Fig. 7b, and the microbuckle is therefore predicted to initiate from this point. However, the stress difference between the corresponding points on the top and bottom surfaces is small (~1%) and thus small geometric defects may in some cases lead to initiation from the bottom surface in practice.

The microbuckling failure initiates due the combination of shear and compressive stresses at this location. This microbuckle formation causes a delamination to form between the wavy layer and the ply drop layer, but this is incidental to the initial failure, i.e. the compressive strength is lost due to the formation of the microbuckle. This failure is therefore microbuckle-driven.

Fig. 4. Experimental photographs and finite element predictions of failed specimens with failure locations highlighted. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
This numerical prediction correlates well with the experimental results. The positions where different failure mechanisms occurred in the test specimens are illustrated in Fig. 7c, and show an inclined failure plane at the very end of the gauge section where the gauge section meets the tip of the adhesive triangle. This indicates a microbuckling/shear instability failure has occurred. Failed test specimens also have delaminations either side of the ply-drop layers, as predicted by the finite element model.

### 4.3.3. L10 configuration

The predicted failure progression for the L10 specimen is shown in Fig. 8a and b. Although the initial response is linear elastic, a non-linearity of specimen response gradually becomes apparent due to the onset and propagation (1) of shear damage in the cohesive layer between the ply-drop layer and wavy layer up to peak load (note that there is also some cohesive damage between the ply drop layer and non-wavy layer, but less extensive and with a later onset). After peak load, the ply-drop layer begins to separate from the wavy layer and there is a global deformation of the separated parts, giving an initial reduction and subsequent increase in load (2). This eventually leads to microbuckling in one of the straight layers (3), close to the ply drop location, which leads to a further reduction in load. There is subsequent delamination between the ply drop layer and straight layers (4), which eventually leads to microbuckling of the other straight layer (5) and the wavy layer a short time after that.

Although the loss of strength arises due to the compressive microbuckle-type failure that occurs in the non-wavy layers, the mechanism for this to occur is only possible after delamination. This failure is therefore delamination driven.

The exact sequence and mechanism of failure is not clear from the experimental results, but the test specimen failed around the region where plies are dropped and there was a significant delamination between the wavy plies and ply drop layers, as shown schematically in Fig. 8c. These experimental observations are generally consistent with the sequence and mechanisms of failure predicted by the finite element model.

### 4.3.4. T10 configuration

The predicted failure progression for the T10 specimen is shown in Fig. 9a and b. After initial linear elastic behaviour, there is a non-linearity in the load–displacement response due to bending in-
duced by the wavy layer. Note that there is no material failure or cohesive damage at this point, and this is an elastic deformation. At peak load a microbuckle forms in the straight layers leading to a large reduction in load (1). This is immediately followed by extensive delaminations within the wavy layer and straight layers and around the ply-drop layer (2) and followed almost immediately by microbuckling in the wavy layer close to the ply-drop (3).

The predicted nonlinearity in load displacement response prior to failure is also seen experimentally, although the effect is less pronounced than that predicted by the modelling.

Although there is some evidence of cohesive damage within the wavy layer prior to failure, removal of this cohesive interface has no effect upon the predicted failure load indicating that this failure is driven by microbuckling of the non-wavy layers.

The exact sequence and mechanism of failure is unclear from experimental data, but examination of the failed specimen showed that compressive failure occurred in the straight and wavy plies at an angle inclined to the compressive loading direction, as shown in Fig. 9c. These experimental observations are consistent with the finite element predictions.

4.4. Comparison of stress–strain curves

The finite element results of the full model analyses (i.e. including the full length of the end tabs) are compared with the experimental results and the predicted failure mechanisms are unchanged from previous results, justifying the earlier approach to model only a section of the specimen. Fig. 10 shows that there is generally good agreement between the stress–strain curves obtained experimentally and those predicted by the full-model finite element analysis.

4.5. Discussion

The comparison of experimental results and finite element predictions shows similar appearance of failed specimens, a good agreement of predicted failure loads, and consistency between numerically predicted failure mechanisms and damage observed...
from the failed test specimens. This demonstrates that the finite element formulation and material failure models adopted are able to correctly capture the experimentally observed failure mechanisms.

The values used for the cohesive parameters used in the modelling were not measured experimentally, and were subject to come uncertainty. Parametric studies were therefore performed to investigate the effect of varying these parameters.

5. Parametric studies of cohesive parameters

The failure mechanisms identified in the previous section show that the compressive failure of the T specimens is governed by the strength of the CFRP plies while the compressive failure of the L specimens is governed by delamination of the cohesive layers.

A good agreement between experimental results and finite element model was obtained even though the cohesive zone parameters (e.g. stiffness, strength) were estimated as they were not measured directly. A parametric study was therefore undertaken to investigate the effect of these parameters upon the predicted response, and thus demonstrate the robustness of this type of analysis, and also to determine whether the failure mechanisms that occur in specimens with these ply-drop configurations can be controlled by altering the cohesive properties of the composite (e.g. by using tougher resin systems).

5.1. Effect of fracture energy

Different composite systems can have significantly different varying fracture energies [10], and so it is appropriate to investigate whether variation of this parameter will affect the predicted failure mechanisms and loads. The fracture energies $G_I$ and $G_{II}$ were varied (but the ratio of $G_I$ to $G_{II}$ was kept constant). Other parameters were fixed at the baseline values given in Tables 1–5. Fig. 11 summarises the effect of fracture energy on predicted compressive failure load.

Fig. 11 shows that the T configurations are insensitive to fracture energy, because the failure of this configuration is controlled by the strength of the CFRP plies. In contrast, the failure of the L configuration is controlled by delamination, and so is sensitive to fracture energy. However, the specimen strength only increases with fracture energy up to the point at which the failure becomes controlled by the strength of the CFRP plies as for T configurations.

5.2. Effect of cohesive zone strength

The effect of cohesive zone strength upon specimen compressive strength is investigated by varying the values of $S_{nn}$ and $S_{tt}$ in the cohesive material definition, i.e. both $S_{nn}$ and $S_{tt}$ were varied simultaneously to keep the ratio between them constant. The fracture energy is held constant at the baseline values ($G_I = 200$ J/m$^2$, $G_{II} = 1000$ J/m$^2$). The variation of compressive strength of the specimen with cohesive strength is shown in Fig. 12.

For the T2 configuration, the estimated cohesive strength of 81 MPa is well within the plastic-microbuckling failure range. Varying the cohesive strength ($S_{nn}$) from the estimated value of 81 MPa therefore has little effect upon the compressive strength of the specimen unless the compressive strength is reduced below 20 MPa at which point the failure mechanism changes to one of global (Euler) buckling of the individual plies.

For the T10 configuration, the estimated cohesive strength of 81 MPa results in a plastic microbuckling failure mechanism. If the cohesive strength is reduced, the failure becomes delamination-initiated. As with the T2 configuration, if the cohesive strength is reduced to a very low value (i.e. below 20 MPa) then failure occurs by global buckling of the individual plies.

At the expected cohesive strength of 81 MPa, the failure of L2 and L10 configurations is initiated by delamination between the ply-drop layer and wavy layer which occurs due to the large stress concentrations at the ply drops. This delamination allows plastic microbuckling of the wavy layer, and subsequent Euler buckling of the surrounding blocked straight layers, which leads to plastic microbuckling failure propagating from the compressive side. The L2 and L10 configurations can also exhibit plastic microbuckling for a high cohesive strength and Euler buckling of individual plies for a low cohesive strength. At the estimated cohesive strength of 81 MPa, the predicted failure load is delamination dependent, so increasing the cohesive strength increases the strength of the specimen, up to a cohesive strength value of around 200 MPa (for the L2 configuration) and 100 MPa (for the L10 configuration). Beyond these values, there is no further increase in compressive strength since the failure is then governed by plastic microbuckling.

In cases where there are stress singularities, such as the discontinuity at a ply drop, the fracture toughness is usually the critical parameter in determining delamination behaviour and cohesive strength is largely irrelevant. However, the results above show that cohesive strength is also important in the problem considered in this paper. A mesh refinement study confirmed that this was not a mesh effect, and the following explanation is proposed.

The specimens are end-loaded in compression. In the absence of any inter-laminar cohesion, the slender individual plies would buckle in compression (Euler buckling). The addition of cohesive

### Table 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Glass tabs</th>
<th>Adhesive wedges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$</td>
<td>30 GPa</td>
<td>2.4 GPa</td>
</tr>
<tr>
<td>Poisson ratio, $\nu$</td>
<td>0.3</td>
<td>0.49</td>
</tr>
<tr>
<td>Yield stress, $\sigma_y$</td>
<td>~</td>
<td>80 MPa</td>
</tr>
</tbody>
</table>
behaviour between the plies strengthens the specimen against this failure mode by causing the plies to act as a single, non-slender structure. However, since the cohesive behaviour has been defined as linear elastic with subsequent linear damage degradation, the cohesive layer only provides increasing cohesion up to the onset of damage. Beyond the onset of cohesive damage, the cohesion between plies begins to degrade and this weakening rapidly leads to overall failure. Thus the compressive failure of the specimen is more significantly affected by the strength of the cohesive layer than its fracture energy.

5.3. Effect of cohesive stiffness

The effect of the cohesive zone stiffness on compressive strength is explored, setting the cohesive failure stress to a very high value, so that the cohesive layer effectively behaves in a linear elastic manner and variations in behaviour are due solely to stiffness effects. The variation of compressive strength with cohesive stiffness is shown in Fig. 13 for the four specimen configurations. These results show little change in predicted strength with cohesive stiffness unless a very compliant cohesive layer is used. This is because a very compliant cohesive layer will permit larger transverse displacements of the specimen at a given overall compressive strain than would a stiff cohesive layer. Hence a very low stiffness in the cohesive layer leads to an earlier onset of geometric instability and a reduced failure load. However, for reasonable estimates of the cohesive zone stiffness calculated as described in Section 3.1.2, the predicted failure load is insensitive to the assumed value.

6. Conclusions

Compression tests were carried out by Wang et al. [1] on composite specimens with out-of-plane waviness arising from plydrops. The experimental results showed different mechanisms of failure despite the superficially similar configuration of the test specimens. A detailed finite element analysis has been presented to model these experiments allowing for failure of the plies by continuum failure within the plies and for delamination failure between plies. Results showed that both mechanisms of failure occurred in each configuration, but that the critical failure mechanism and predicted sequence of failure depends on the details of the configuration modelled. The critical mechanism was found to be compressive microbuckling for the transverse configurations, and delamination failure for the longitudinal configurations. The model correlates very well with each of the test configurations, both in terms of the failure loads and details of the failure mechanisms. The model predicts the sequence of failure in a way that is difficult to capture during the sudden catastrophic failure of the experimental specimens and thus provides valuable physical insight into the critical failure mechanisms for these tests.

Parametric studies have been carried out which indicate that both the cohesive zone strength and the fracture energy can affect the predicted mechanism of failure and consequently the predicted strength of the specimen. The dependence upon cohesive strength is due to additional geometric instability that is introduced by the corresponding change in the size of the process zone. The predicted specimen strength is relatively insensitive to cohesive stiffness for values within a reasonable range.

This novel approach to failure modelling can be used to predict the failure response of laminates containing a wide range of waviness or ply drop defects, where multiple possible failure mechanisms exist and the actual mechanism of failure is not known a priori.

Acknowledgements

The authors acknowledge EPSRC/DSTL for funding this work (Grant EP/G012938/1). The authors would like to thank Wilfried Liebig (visiting research fellow at Sheffield University), and industrial partners for their input into this research.

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