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Order α_s^2 perturbative QCD corrections to the Gottfried sum rule

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Abstract

The order α_s^2 perturbative QCD correction to the Gottfried sum rule is obtained. The result is based on numerical calculation of the order α_s^2 contribution to the coefficient function and on the new estimate of the three-loop anomalous dimension term. The correction found is negative and rather small. Therefore it does not affect the necessity to introduce flavour-asymmetry between $\bar{u}$ and $\bar{d}$ antiquarks for the description of NMC result for the Gottfried sum rule.

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1. Introduction

One of the still actively discussed problems of deep-inelastic scattering (DIS) is related to the consideration of the Gottfried sum rule [1], namely

$$I_{\rm GSR}(Q^2) = \int_0^1 \left[F_2^{lp}(x,Q^2) - F_2^{ln}(x,Q^2)\right]\frac{dx}{x}$$
$$= \int_0^1 \left[\frac{1}{3}\left(u_v(x,Q^2) - d_v(x,Q^2)\right) + \frac{2}{3}\left(\bar{u}(x,Q^2) - \bar{d}(x,Q^2)\right)\right]dx$$
$$= \frac{1}{3} + \frac{2}{3}\int_0^1 \left(\bar{u}(x,Q^2) - \bar{d}(x,Q^2)\right)dx. \quad (1)$$

This sum rule is the *first* non-single (NS) moment of the difference of F_2 structure functions (SFs) of charged lepton–nucleon DIS which in general has the following definition:

$$M_n^{\rm NS}(Q^2) = \int_0^1 x^{n-2}\left[F_2^{lp}(x,Q^2) - F_2^{ln}(x,Q^2)\right]dx. \quad (2)$$

An extensive discussion of the current studies of this sum rule was given in the review of Ref. [2]. However, for the sake of completeness, we will remind the existing experimental situation,which is stimulating the continuation of the research of various subjects, related to the Gottfried sum rule.

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In fact if the sea is flavour symmetric, namely $\bar{u} = \bar{d}$, one should have

$$I_{\rm GSR} = \frac{1}{3}. \tag{3}$$

However, the most detailed analysis of muon–nucleon DIS data of NMC Collaboration gives the following result [3]:

$$I_{\rm GSR}^{\rm exp}\big(Q^2 = 4~{\rm GeV}^2\big) = 0.235 \pm 0.026. \tag{4}$$

It clearly indicates the violation of theoretical expression of Eq. (3) and necessitates more detailed investigations of different effects, related to the Gottfried sum rule. In this Letter we reconsider the question of studying the contributions of α_s^2, corrections to this sum rule previously raised in Ref. [4].

2. Available perturbative corrections

The status of the $O(\alpha_s)$ perturbative QCD corrections to $I_{\rm GSR}$ was summarised in Ref. [5]. Following this review, we will extend its presentation to the order α_s^2 level.

It should be stressed that the renormalisation group equation for $I_{\rm GSR}$ contains the anomalous dimension term:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(A_s)\frac{\partial}{\partial A_s} - \gamma_{I_{\rm GSR}}^{n=1}(A_s)\right]C_{I_{\rm GSR}}(A_s) = 0, \tag{5}$$

where $A_s = \alpha_s/(4\pi)$ and

$$\mu\frac{\partial A_s}{\partial\mu} = -2\sum_{i\geqslant 0}\beta_i A_s^{i+2}. \tag{6}$$

The first two scheme-independent coefficients in Eq. (6) are well known:

$$\beta_0 = \left(\frac{11}{3}C_A - \frac{2}{3}n_f\right) = 11 - 0.666667\,n_f, \tag{7}$$

$$\begin{aligned}\beta_1 &= \left(\frac{34}{3}C_A^2 - 2C_F n_f - \frac{10}{3}C_A n_f\right) \\ &= 102 - 12.6667\,n_f,\end{aligned} \tag{8}$$

where $C_F = 4/3$, $C_A = 3$ and n_f is the number of active flavours.

The corresponding anomalous dimension function has the canonical expansion

$$\gamma_{I_{\rm GSR}}^{n=1} = \sum_{i\geqslant 0}\gamma_i^{n=1}A_s^{i+1}. \tag{9}$$

However, like in the case of the first moments of SFs of νN DIS, the first coefficient of the NS anomalous dimension function of the first moment $\gamma_0^{n=1}$ is identically equal to zero. The difference is starting to manifest itself from the two-loop level, where in order to get the corresponding result in case of anomalous dimension for $I_{\rm GSR}$ it is necessary to make analytical continuation and to use the so-called $(+)$ prescription (see, e.g., Ref. [6]). In the case of $\gamma_1^{n=1}$ this was done in Refs. [7] and [8] and results in the following analytical expression

$$\begin{aligned}\gamma_1^{n=1} &= -4\big(C_F^2 - C_F C_A/2\big)\big[13 + 8\zeta(3) - 2\pi^2\big] \\ &= +2.55755,\end{aligned} \tag{10}$$

where the numerical value of $\zeta(3) = 1.2020569$ was taken into account. The perturbative corrections to $I_{\rm GSR}$ can be obtained from the solution of the renormalisation group equation of Eq. (5):

$$I_{\rm GSR}(A_s) = AD(A_s) \times C(A_s), \tag{11}$$

where the anomalous dimension term is defined as

$$AD(A_s) = \exp\left[-\int_{\delta}^{A_s(Q^2)}\frac{\gamma_{I_{\rm GSR}}^{n=1}(x)}{\beta(x)}\,dx\right]. \tag{12}$$

Since the first coefficient of $\gamma_{I_{\rm GSR}}^{n=1}$ is identically zero (namely $\gamma_0^{n=1} = 0$), there is no singularity in $AD(A_s)$ and we can put in Eq. (12) the lower bound of integration $\delta = 0$. In this case we obtain the following expression for the expansion of $AD(A_s)$ up to $O(\alpha_s^2)$-corrections:

$$\begin{aligned}&AD\big(A_s(Q^2)\big) \\ &\quad = 1 + \frac{1}{2}\frac{\gamma_1^{n=1}}{\beta_0}A_s\big(Q^2\big) \\ &\qquad + \frac{1}{4}\left(\frac{1}{2}\frac{(\gamma_1^{n=1})^2}{\beta_0^2} - \frac{\gamma_1^{n=1}\beta_1}{\beta_0^2} + \frac{\gamma_2^{n=1}}{\beta_0}\right)A_s^2\big(Q^2\big).\end{aligned} \tag{13}$$

The only unknown term here is the third coefficient $\gamma_2^{n=1}$ of the anomalous dimension function $\gamma_{I_{\rm GSR}}^{n=1}(A_s)$, which in general is scheme-dependent.

In the cases of $n_f = 3$ and $n_f = 4$ the numerical versions of Eq. (13) read

$$AD(\alpha_s)_{n_f=3} = 1 + 0.0355\left(\frac{\alpha_s}{\pi}\right) + \left(-0.0392 + \frac{\gamma_2^{n=1}}{64\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2, \quad (14)$$

$$AD(\alpha_s)_{n_f=4} = 1 + 0.0384\left(\frac{\alpha_s}{\pi}\right) + \left(-0.0415 + \frac{\gamma_2^{n=1}}{64\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2, \quad (15)$$

where the scheme-dependent expression for $\gamma_2^{n=1}$ is still unknown. Its value will be fixed in the next section using the results of calculations in the $\overline{\text{MS}}$-scheme.

A few words should be added here on the perturbative theory expansion of $C(A_s)$. From general grounds it should have the following form:

$$C(A_s) = \frac{1}{3}\left[1 + C_1^{n=1} A_s\left(Q^2\right) + C_2^{n=1} A_s^2\left(Q^2\right)\right]. \quad (16)$$

As was found in Ref. [9] its first coefficient is zero, namely $C_1^{n=1} = 0$. However, as will be shown in the next section the non-zero perturbative theory contribution is appearing at the two-loop level.

3. Calculations and estimates of the α_s^2 contributions

We will start from the calculations of perturbative contribution to the coefficient function $C(A_s)$ at the α_s^2-level. It can be obtained after applying (+) prescription to the results of Ref. [10]. Indeed, the order α_s^2 correction to the coefficient function of I_{GSR} is defined by taking the first moment from the sum

$$C_2^{n=1} = \int_0^1 \left[C_2^{(2),(-)}(x,1) + C_2^{(2),(+)}(x,1)\right] dx, \quad (17)$$

where the expressions for the functions $C_2^{(2),(-)}(x,1)$ and $C_2^{(2),(+)}(x,1)$ were calculated in Ref. [10] and confirmed with the help of another technique in Ref. [11]. Integrating Eq. (17) numerically with arbitrary Casimir operators C_A and C_F, we obtain the following n_f-independent and scheme-independent result

$$C(A_s) = \frac{1}{3}\left[1 - 0\left(\frac{\alpha_s}{\pi}\right) + \left(3.695 C_F^2 - 1.847 C_F C_A\right)\left(\frac{\alpha_s}{\pi}\right)^2\right] = \frac{1}{3}\left[1 - 0.821\left(\frac{\alpha_s}{\pi}\right)^2\right]. \quad (18)$$

Combining now Eqs. (14) and (15) with Eq. (18) we find the following expressions for I_{GSR}:

$$I_{\text{GSR}}\left(Q^2\right)_{n_f=3} = \frac{1}{3}\left[1 + 0.0355\left(\frac{\alpha_s}{\pi}\right) + \left(-0.862 + \frac{\gamma_2^{n=1}}{64\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2\right], \quad (19)$$

$$I_{\text{GSR}}\left(Q^2\right)_{n_f=4} = \frac{1}{3}\left[1 + 0.0384\left(\frac{\alpha_s}{\pi}\right) + \left(-0.809 + \frac{\gamma_2^{n=1}}{64\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2\right], \quad (20)$$

where $\alpha_s = \alpha_s(Q^2)$ is the NLO expression for $\overline{\text{MS}}$ coupling constant.

In order to get the feeling what might be the contribution of the terms proportional to $\gamma_2^{n=1}$ we will avoid extrapolation procedure of the values of γ_2^n used in Ref. [4], calculated analytically for even $n = 2, 4, \ldots, 14$ in the works of Ref. [12]. Indeed, performing extrapolation from the even values of n for the NLO terms γ_1^n of the corresponding anomalous dimension function, we are obtaining the following estimate $\gamma_1^{n=1} = 28.23$, which is 10 times larger that the real value given in Eq. (10). Therefore, the extrapolation procedure used in Ref. [4] is considerably overestimating the value of the coefficient $\gamma_1^{n=1}$. The similar situation can occur in the case of using extrapolation procedure for fixing the value of $\gamma_2^{n=1}$. Indeed, following the ideas of Ref. [4] we get from extrapolation of the known even values for γ_2^n the following estimates: $\gamma_2^{n=1} \approx 361$ for $n_f = 3$ and $\gamma_2^{n=1} \approx 283$ for

$n_f = 4$, which to our point of view might be unrealistically large.

Keeping in mind that only direct calculation of $\gamma_2^{n=1}$ can give the real numerical value of this term, we nevertheless are proposing the following way of fixing uncalculated contribution to $\gamma_{I_{\rm GSR}}^{n=1}$ function. We noticed the following numerical pattern of the behaviour of anomalous dimension function for $n \geqslant 2$: $\gamma_1^n/\gamma_2^n \sim 0.12$ for $n_f = 4$ (see Refs. [4] and [13] especially). We have checked that for $n_f = 3$ the similar relation is $\gamma_1^n/\gamma_2^n \sim 0.09$ for $n \geqslant 2$. Hoping that these relations are also valid in case of $n \geqslant 1$, we estimate the values for $\gamma_2^{n=1} = (1/0.12)\gamma_1^{n=1} \approx 21.3$ in the case of $n_f = 4$ and $\gamma_2^{n=1} = (1/0.09)\gamma_1^{n=1} \approx 28.4$ in the case of $n_f = 3$. Substituting them into Eqs. (19) and (20) we get:

$$I_{\rm GSR}\big(Q^2\big)_{n_f=3} = \frac{1}{3}\left[1 + 0.0355\left(\frac{\alpha_s}{\pi}\right) - 0.811\left(\frac{\alpha_s}{\pi}\right)^2\right], \qquad (21)$$

$$I_{\rm GSR}\big(Q^2\big)_{n_f=4} = \frac{1}{3}\left[1 + 0.0384\left(\frac{\alpha_s}{\pi}\right) - 0.822\left(\frac{\alpha_s}{\pi}\right)^2\right]. \qquad (22)$$

Taking now $\alpha_s(Q^2) \approx 0.35$ we arrive at the following numerical versions of Eqs. (21) and (22):

$$I_{\rm GSR}\big(Q^2\big)_{n_f=3} = \frac{1}{3}[1 + 0.0039 - 0.0101] = 0.3313, \qquad (23)$$

$$I_{\rm GSR}\big(Q^2\big)_{n_f=4} = \frac{1}{3}[1 + 0.0042 - 0.0102] = 0.3313. \qquad (24)$$

Therefore, in presented expression for the order α_s^2 correction to the Gottfried sum rule is larger than the order α_s-term.

Theoretical errors to the presented third terms in Eqs. (21)–(24) are coming from the errors of $\gamma_2^{n=1}$ terms in Eqs. (19), (20), which are impossible to estimate without their direct theoretical calculations. In any case these terms are damped by huge numbers ($64\beta_0$) and it is unlikely that the direct calculations of $\gamma_2^{n=1}$ terms will change the results of Eqs. (23), (24) substantially. One can check this conclusion using the to our point of view overestimated results of application of the extrapolation procedure. Moreover, the main contributions to the α_s^2-term in Eqs. (21)–(24) come from the α_s^2 term of the coefficient function of the Gottfried sum rule calculated by us.

4. Comments on violation of the Gottfried sum rule

In the previous section we found that order α_s^2 perturbative QCD corrections to the Gottfried sum rule are really small and cannot describe violation of the theoretical prediction from its NMC experimental value. This, in turn, leads to the necessity of introduction of the effect of flavour asymmetry of antiquark distributions in the nucleon [3], namely,

$$\int_0^1 dx\big[\bar{d}\big(x, 4~{\rm GeV}^2\big) - \bar{u}\big(x, 4~{\rm GeV}^2\big)\big]_{\rm NMC} = 0.147 \pm 0.039. \qquad (25)$$

This phenomenological result is important for fixing the corresponding $\bar{d}/\bar{u}$ ratio in different sets of parton distribution functions, which are relevant to the LHC physics (for a review see, e.g., Ref. [15]). On the other side the consideration of available E866 data for the Drell–Yan production in proton–proton and proton–deuteron scattering has confirmed the effects of flavour asymmetry. Indeed the analysis of Ref. [14] gave the following number

$$\int_{0.015}^{0.35} dx\big[\bar{d}\big(x, 54~{\rm GeV}^2\big) - \bar{u}\big(x, 54~{\rm GeV}^2\big)\big]_{\rm E866} = 0.0803 \pm 0.011. \qquad (26)$$

It was also noted in Ref. [14] that it is unlikely to receive additional contribution to Eq. (26) from the region above $x = 0.35$, since the sea is rather small in this region. However, the contribution to this whole integral from the unmeasured region $x \leqslant 0.015$ is missed. The attempt to fix it was made in Ref. [16] using the extrapolation to small x region. As the result the authors of Ref. [16] suggested the manifestation of substantial contribution of twist-4 $1/Q^2$-effects in

Eq. (1). Note, that final E866 result is

$$\int_0^1 dx\big[\bar{d}\big(x, 54\ \mathrm{GeV}^2\big) - \bar{u}\big(x, 54\ \mathrm{GeV}^2\big)\big]_{\mathrm{E866}} = 0.118 \pm 0.012 \tag{27}$$

is closer to NMC result, than obtained in Ref. [16] extrapolated value, namely,

$$\int_0^1 dx\big[\bar{d}\big(x, 50\ \mathrm{GeV}^2\big) - \bar{u}\big(x, 50\ \mathrm{GeV}^2\big)\big]_{\mathrm{Ref.\ [16]}} = 0.09 \pm 0.02. \tag{28}$$

Therefore, in order to understand the status of their conclusion on the possibility of existence of substantial contribution of the $1/Q^2$-corrections to the Gottfried sum rule it is necessary to be more careful in performing extrapolations to low x-region. It is highly desirable to estimate the effects of higher-twist contributions to the Gottfried sum rule using any concrete model. However, one should keep in mind that there are also some other explanations of the observed deviation from the canonical value $1/3$ for the Gottfried sum rule (see Refs. [17] and [2] for the review of other works on the subject).

5. Conclusions

In summary: we found non-zero $O(\alpha_s^2)$ perturbative QCD contributions to the coefficient function of the Gottfried sum rule. We also estimated the effect due to non-zero value of the three-loop contribution to NS anomalous dimension function for $n = 1$ moment, which we think is rather small. More detailed result can be obtained after completing the analytical calculations of the three-loop corrections to the NS kernel of DGLAP equation. This work is now in progress (see, e.g., Ref. [18]). In any case the value of the α_s^2-correction is dominated by the contribution to the coefficient function calculated in this Letter, which is negative, but also small. Therefore, the existing NMC observation of the flavour asymmetry between $\bar{d}$ and $\bar{u}$ antiquarks survives. We hope that the possible future new HERA data might be useful for more detailed measurement of the Gottfried sum rule and for further studies of the effect of flavour asymmetry in the $\bar{d}/\bar{u}$ ratio.

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References

[1] K. Gottfried, Phys. Rev. Lett. 18 (1967) 1174.

[2] S. Kumano, Phys. Rep. 303 (1998) 183, hep-ph/9702367.

[3] M. Arneodo, et al., New Muon Collaboration, Phys. Rev. D 50 (1994) 1.

[4] A.L. Kataev, A.V. Kotikov, G. Parente, A.V. Sidorov, Phys. Lett. B 388 (1996) 179, hep-ph/9605367.

[5] I. Hinchliffe, A. Kwiatkowski, Annu. Rev. Nucl. Part. Sci. 46 (1996) 609, hep-ph/9604210.

[6] F.J. Yndurain, The Theory of Quark and Gluon Interactions, Springer, Berlin, 1993.

[7] D.A. Ross, C.T. Sachrajda, Nucl. Phys. B 149 (1979) 497.

[8] G. Curci, W. Furmanski, R. Petronzio, Nucl. Phys. B 175 (1980) 27.

[9] W.A. Bardeen, A.J. Buras, D.W. Duke, T. Muta, Phys. Rev. D 18 (1978) 3998.

[10] W.L. van Neerven, E.B. Zijlstra, Phys. Lett. B 272 (1991) 127.

[11] S. Moch, J.A. Vermaseren, Nucl. Phys. B 573 (2000) 853, hep-ph/9912355.

[12] S.A. Larin, T. van Ritbergen, J.A. Vermaseren, Nucl. Phys. B 427 (1994) 41;
S.A. Larin, P. Nogueira, T. van Ritbergen, J.A. Vermaseren, Nucl. Phys. B 492 (1997) 338, hep-ph/9605317;
A. Retey, J.A. Vermaseren, Nucl. Phys. B 604 (2001) 281, hep-ph/0007294.

[13] A.L. Kataev, G. Parente, A.V. Sidorov, Phys. Part. Nucl. 34 (2003) 20, Fiz. Elem. Chastits At. Yadra 34 (2003) 43, hep-ph/0106221.

[14] R.S. Towell, et al., FNAL E866/NuSea Collaboration, Phys. Rev. D 64 (2001) 052002, hep-ex/0103030.

[15] S. Catani, et al., hep-ph/0005025;
S. Catani, et al., in: Standard model physics (and more) at the LHC, CERN Report 2000-004, Geneva, 2000.

[16] A. Szczurek, V. Uleshchenko, Phys. Lett. B 475 (2000) 120, hep-ph/9911467.

[17] M. Karliner, H.J. Lipkin, Phys. Lett. B 533 (2002) 60, hep-ph/0202099.

[18] S. Moch, J.A. Vermaseren, A. Vogt, Nucl. Phys. B 646 (2002) 181, hep-ph/0209100.