Abstract

To clarify the navigation error of strapdown inertial navigation system (SINS) with low navigation accuracy, short working period, and fickle operating environment that installed in tactical missiles, a new navigation error model of SINS was established. Founded by a six dimensional kinematic model of tactical missile and a three dimensional target model, the paper rebuilt the whole course of tactical missile attacking the moving target, where correlative output parameters were used to calculate the navigation error model of SINS simultaneously. Experiment result revealed the influence of different error sources to navigation errors of SINS, which is constructive to the improvement of navigation accuracy of SINS.

1. Introduction

With the development of fabrication technology and computational capability, SINS has been applied to tactical missiles with short and medium ranges (SMR) progressively, which takes on some new traits:

- Low navigation accuracy. Limited by cost, volume and weight of the tactical missile, the navigation error of missile-borne strapdown inertial navigation system (MSINS) may far above inertial navigation system with stationary base or higher navigation accuracy.

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Short working period. The effective range of tactical missiles with SMR is usually within $400 km$. Suppose the velocity of missile is $250 m/s$, hence its flight time will not exceed $1600 s$, that’s the maximum working period of MSINS.

Fickle operating environment. For tactical missile with path planning capabilities, it performs maneuver frequently in vertical and horizontal direction during the attacking. This results in rapid changes in kinetic parameters of the missile, and compels MSINS to work in a fickle environment.

The above new traits indicate that routine solution may not model the navigation error of MSINS effectively. For instance, predigest error sources[1], setting the carrier trajectory beforehand[2, 3], or using analytic method to obtain the approximate solution[4]. Consequently, a new navigation error model of SINS with dynamic base was established. Due to the new traits mentioned above, all error sources were handled properly with no predigestion for purpose of exposing their influence to the navigation errors.

2. SINS Error Model

2.1. Coordinate Frames and Error Sources

In the development of SINS navigation Error equations, it is necessary to use five coordinate frames: the body($b$), earth($e$), inertial($i$), geographical($g$) and navigation($n$) frames. Their definitions can be found in [5].

As a complex system, the MSINS has many different error sources[6]. Ignore errors of principles and methods, the NEM takes into account the following error sources:

- Initial errors. Such as initial navigation parameters error, accelerometer angular misalignment error, installation angle error and calibration factor error of accelerometer and gyro.
- Accelerometer bias error and gyro drifts error.
- Dynamic error caused by angular rate of missile.
- Random error of accelerometer bias and gyro drifts.

2.2. SINS Error Propagation Model

1) Velocity Error Equation

Adopt local east-north-upside frame $g$ as navigation frame $n$, the velocity error equation projected on $n$-frame is given by

$$\delta V^n = -\phi^a \times f^n + C^n_b([\delta K_2] + [\delta A]) f^b + \delta V^n \times (2\omega_{we}^n + \omega_{ew}^n) + V^n \times (2\delta \omega_{we}^n + \delta \omega_{ew}^n) + \nabla^n + r_A^n$$

(1)

Where the superscripts denote the coordinate frame in which the vectors are coordinated, and $\phi$ denotes the attitude error matrix of accelerometer, $C^n_b$ denotes the direction cosine matrix[5] from $b$-frame to
n-frame, \([\delta A]\) and \([\delta K_A]\) denote installation angle error and calibration factor error matrix\(\footnote{7}\) of accelerometer separately, and \(\omega_{e_{\text{m}}}\) denotes rotation angular rate vector from \(n\)-frame to \(e\)-frame.

Write \(C^a_b\) and \([[\delta A]+[\delta A]]\) as \((c_{mk})_{3\times3}\) and \((\delta \alpha_{mk})_{3\times3}\) separately, where \(m = 1, 2, 3\) and \(k = 1, 2, 3\). Since the vertical axis is divergence, vertical direction movement can be ignored. Then the east and north components in \(n\)-frame of (1) yield

\[
\begin{align*}
\delta V_E &= \delta V_N (2\omega_{e_{\text{m}}} \sin L + L + V_E \tan L) + \delta \omega_L (2\omega_{e_{\text{m}}} (V_E \sin L + V_N \cos L) + \frac{V_N V_E}{R_N + h}) + \delta V_N \left(\frac{V_N \tan L - V_E}{R_N + h}\right) \\
&= \phi_X f_N - \phi_N f_U + c_{\text{ia}} \delta \alpha_{mk} f_b + \nabla + r_{\text{ae}} \\
\delta V_N &= -\delta V_E (2\omega_{e_{\text{m}}} \sin L + \frac{2V_E \tan L}{R_N + h}) - \delta \omega_N \frac{V_E}{R_M + h} - \delta \omega_L (2\omega_{e_{\text{m}}} \cos L + \frac{V_E \sec^2 L}{R_N + h}) + \delta \omega_N \left(\frac{V_N \tan L - V_E}{R_N + h}\right) \\
&= c_{\text{ia}} \delta \alpha_{mk} f_b + \nabla + r_{\text{ne}}
\end{align*}
\]

(2)

2) Attitude Angle Error Equation

Attitude error equation projected on \(n\)-frame is given by

\[
\phi = \delta \omega_{e_{\text{m}}} + \phi \times \omega_{e_{\text{m}}} - C^a_b [[\delta K_G]] + \delta K_G \omega_{e_{\text{m}}} + \delta \omega_{e_{\text{m}}} + r_{\text{G}}
\]

(3)

Where the superscripts denote the coordinate frame in which the vectors are coordinated, \([\delta K_G]\) and \([\delta \alpha_{mk}\]) denote installation angle error and calibration factor error matrix\(\footnote{7}\) of gyro separately.

In \(n\)-frame, write \([[\delta K_G]] + \delta K_G\) as \((\delta \alpha_{mk})_{3\times3}\), then the east, north and upside components of equation (3) yield

\[
\begin{align*}
\dot{\phi}_E &= \phi_X (\omega_{e_{\text{m}}} \sin L + \frac{V_E \tan L}{R_N + h} - \phi_U (\omega_{e_{\text{m}}} \cos L + \frac{V_E}{R_N + h}) - \frac{\delta \omega_N}{R_M + h} - c_{\text{ia}} \delta \alpha_{mk} \omega_{e_{\text{m}}} - r_{\text{GE}} \\
\dot{\phi}_N &= -\phi_E (\omega_{e_{\text{m}}} \sin L + \frac{V_E \tan L}{R_M + h} - \phi_U (\omega_{e_{\text{m}}} \cos L + \frac{V_E}{R_M + h}) - \frac{\delta \omega_N}{R_N + h} - c_{\text{ia}} \delta \alpha_{mk} \omega_{e_{\text{m}}} - r_{\text{GN}} \\
\dot{\phi}_U &= \phi_X \omega_{e_{\text{m}}} \cos L + \phi_U V_E + \phi_N V_N + \delta \omega_N \cos L + \frac{V_E \sec^2 L}{R_N + h} + \frac{\delta V_E}{R_N + h} - c_{\text{ia}} \delta \alpha_{mk} \omega_{e_{\text{m}}} - r_{\text{GU}}
\end{align*}
\]

(4)

3) Position Error Equation

Position error equation projected on \(n\)-frame is given by

\[
\begin{align*}
\delta L &= \frac{1}{R_M + h} \frac{\delta V_N}{R_M} \\
\delta \lambda &= \frac{\sec L}{R_N + h} \frac{\delta V_E + V_E \tan L \sec L}{R_N + h} \delta L
\end{align*}
\]

(5)

Equation (2), (4) and (5) are the whole navigation error equations of NEM.

3. Simulation and Analysis of NEM

Take anti-ship missile with high subsonic speed for instance. In \(g\)-frame, suppose the missile was launched straight and level in direction of north by east 24°, initial velocity 300m/s, launching position: north latitude 30° and east longitude 125°. The target keeps uniform linear motion on the sea in direction of north by east 305°, initial position: north latitude 31°40’ and east longitude 125°47’53”.
Hence, the relative distance of missile and ship is about 200km, or 185km in north direction and 76km in east direction.

Adopt Krasovsky ellipsoid as the earth model. On the assumption that the three initial attitude angle errors are 0.01°, the three installation angle errors and calibration factor errors of accelerometer and gyro are 10^{-4}, the three-axis gyro drift errors are 1°/h, the three-axis accelerometer bias errors are 10^{-4}g, and random error $r_d \sim N(0, \sigma_d^2)$ and $r_g \sim N(0, \sigma_g^2)$, where $\sigma_d = 0.5 \times 10^{-4}$ and $\sigma_g = 0.5 \times 10^{-4}g$.

The experiment data of navigation errors caused by different error sources were showed in table 1.

Table 1. Navigation error of different error sources

<table>
<thead>
<tr>
<th>Navigation error</th>
<th>$S_0(m)$</th>
<th>$S_1(m)$</th>
<th>$S_2(m)$</th>
<th>$S_3(m)$</th>
<th>$S_4(m)$</th>
<th>$S_5(m)$</th>
<th>$S_6(m)$</th>
<th>$S_7(m)$</th>
<th>$S_8(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>East position</td>
<td>1735.3</td>
<td>2333.9</td>
<td>-215.9</td>
<td>1.7</td>
<td>1.5</td>
<td>536.9</td>
<td>13</td>
<td>351.2</td>
<td>19.6</td>
</tr>
<tr>
<td>North position</td>
<td>-422.5</td>
<td>2344.5</td>
<td>-208.1</td>
<td>58.2</td>
<td>4.3</td>
<td>-1407.5</td>
<td>1.2</td>
<td>-367.4</td>
<td>-17.8</td>
</tr>
</tbody>
</table>

where, $S_0$ denotes the total navigation error of MSINS, and $S_1$–$S_8$ denotes gyro drifts error, accelerometer bias error, latitude error, calibration factor error, installation angle error, dynamic error, accelerometer angular misalignment error, and random error in turn.

From table 1, the navigation error of SINS is larger than that of inertial navigation system with higher navigation accuracy[8], and navigation error caused by a single error source may larger than the total error due to the counteraction of different error sources. Besides, MSINS is a nonlinear system judging from table 1. Fig 1–3 showed the simulation result of some parameters of the missile and MSINS.

![Fig. 1. True and desired trajectory in east-north-upside frame](image)

![Fig. 2. (a) East-north-upside velocity of missile; (b) East-north-upside attitude angle error of MSINS](image)
In Fig 1, “true trajectory” means the actual trajectory that contains navigation error of SINS and “desired trajectory” means trajectory without navigation error. Figure 2(a) displays the three missile velocity in $g$-frame, and Fig 2(b) is the attitude angle error of MSINS. Due to the short working time of SINS, the periodicity of the attitude angle error is not obvious. Fig 3(a) and (b) are the navigation error projected in east and north direction, and Fig 3(c) and (d) displays the “true trajectory” and “desired trajectory” of missile in the corresponding direction.

4. Conclusion

The paper established a new navigation error model of MSINS based on the 6 dimensional kinematic model of tactical missile, and the influence of difference error sources to the navigation error was revealed by simulation.

MSINS is a nonlinear system, and the navigation error of a single error source may larger than the total. Experiment result in table 1 indicated that gyro drifts, accelerometer bias, accelerometer angular misalignment, and installation angle of accelerometer and gyro are the main error sources of MSINS. Therefore, if higher navigation accuracy of MSINS were required, the improvement of these error sources will be beneficial and substantial.

References