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A bulk effect to SUSY effective potential in a 5D super-Yang–Mills model

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Abstract

Supersymmetric effective potential of a 5D super-Yang–Mills model compactified on S^1/Z_2, i.e., on an interval l of extra dimension, is estimated at the 1-loop level by the auxiliary field tadpole method. For the sake of infinite towers of Kaluza–Klein excitation modes of bulk fields involved in the tadpoles, there arises a definite bulk effect of linear growth of the effective potential along with the cutoff Λ which is greatly suppressed by l to produce a finite contribution. Incorporating the tree potential and a Fayet–Iliopoulos D-term, the effective potential is minimized at a specific value of l, corresponding to an intermediate mass scale $10^{11\text{–}14}$ GeV, where the supersymmetry is restored.

1. Introduction

Recently theories of extra-dimensions have attracted attention. Among them a 5-dimensional (5D) super-Yang–Mills (super-YM) theory with mirror-plane boundaries is very interesting since it has a possibility to lead to a realistic model of particle theory. In a previous Letter [1], we have analyzed the background configuration based on the Mirabelli–Peskin–Hebecker [2,3] model and obtained the 1-loop effective potential for some special cases in a 5D bulk-boundary theory compactified on S^1/Z_2 orbifold, i.e., on an interval of length l. One of virtues of the model is that the coupling of a 5D super-YM multiplet to a 4D orientifold boundary is explicitly given in an off-shell formulation.

In this Letter we try to evaluate a full 1-loop effective potential of 4D boundary in the same framework as [1] except adding a superpotential in the boundary. We use the auxiliary field tadpole method (AFTM) by Miller [4] based on the tadpole method by Weinberg [5], without eliminating auxiliary fields by their equation of motion. An advantage of this method is that only F and D auxiliary field tadpoles are sufficient to reconstruct the effective

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potential since the spin-0 tadpole contributions are generated automatically by the use of a supersymmetric (SUSY) boundary condition.

The model is generically non-renormalizable and should be viewed as an effective theory valid up to some high mass scale associated with an ultraviolet cutoff Λ. However, we require that it should be renormalizable in the limit of $l \to 0$.

Although the effective potential to be evaluated is 4D, we have a definite *bulk effect* which comes from the contribution of whole Kaluza–Klein (KK) excitation modes of the bulk fields involved in the tadpole diagrams. Such a bulk effect is very interesting since it might implement new aspects of breakings of gauge symmetry and/or supersymmetry through the minimization of the effective potential. In particular, by minimizing the effective potential which contains the tree level potential together with an additional contribution of Fayet–Iliopoulos (FI) D-term, we find a case that SUSY is restored at a specific value of the radius l of extra dimension corresponding to an intermediate mass scale $\approx 3 \times 10^{11}$ or 7×10^{13} GeV for the ultraviolet cutoff $\Lambda \approx M_{\mathrm{GUT}}$ or M_{Pl}.

2. 5D super-Yang–Mills model

Let us consider the 5D flat space–time with the signature $(+----)$. The space of the fifth component is taken to be S^1 with the periodicity $2l$ and the Z_2-orbifold condition $x^5 \sim -x^5$. We take a 5D SUSY action such as

$$S = \int d^5X \left\{\mathcal{L}_{\mathrm{blk}} + \delta\left(x^5\right)\mathcal{L}_{\mathrm{bnd}} + \delta\left(x^5 - l\right)\mathcal{L}'_{\mathrm{bnd}}\right\}, \tag{1}$$

where $X \equiv (x^0, x^1, x^2, x^3, x^5)$, $\int dX^5 \equiv \int d^4x \int_{-l}^{l} dx^5$, $\mathcal{L}_{\mathrm{blk}}$ is a 5D bulk Lagrangian and $\mathcal{L}_{\mathrm{bnd}}$ and $\mathcal{L}'_{\mathrm{bnd}}$ denote a 4D boundary Lagrangian on a "wall" at $x^5 = 0$ and a hidden sector Lagrangian on the other "wall" at $x^5 = l$, respectively.

The bulk dynamics is given by the 5D super-YM theory which is made of a vector field A^M $(M = 0, 1, 2, 3, 5)$, a scalar field Φ, a doublet of symplectic Majorana fields λ^i $(i = 1, 2)$, and a triplet of auxiliary scalar fields X^a $(a = 1, 2, 3)$:

$$\mathcal{L}_{\mathrm{blk}} = -\frac{1}{2}\,\mathrm{tr}(F_{MN})^2 + \mathrm{tr}(\nabla_M\Phi)^2 + \mathrm{tr}\left(i\bar{\lambda}^i\gamma^M\nabla_M\lambda^i\right) + \mathrm{tr}\left(X^a\right)^2 - \mathrm{tr}\left(\bar{\lambda}^i\left[\Phi, \lambda^i\right]\right), \tag{2}$$

where all bulk fields are of the adjoint representation of the gauge group G: $A^M = A^{M\alpha}T^\alpha$, etc., $\mathrm{tr}[T^\alpha T^\beta] = \delta^{\alpha\beta}/2$ and $\nabla_M\Phi = \partial_M\Phi - ig[A_M, \Phi]$. This system has the symmetry of 8 real supercharges.

We can project out $\mathcal{N} = 1$ SUSY multiplet, which has 4 real super charges, by assigning Z_2-parity to all fields in accordance with the 5D SUSY. A consistent choice is given as: $P = +1$ for A_m $(m = 0, 1, 2, 3)$, λ_L, X^3; $P = -1$ for A_5, Φ, λ_R, X^1, X^2. (The fields of $P = -1$ vanish on the boundaries $x^5 = 0, l$.) Then, $V \equiv (A_m, \lambda_L, X^3 - \nabla_5\Phi)$ and $\Sigma \equiv (\Phi + iA_5, -i\sqrt{2}\lambda_R, X^1 + iX^2)$ constitute an $\mathcal{N} = 1$ vector supermultiplet in Wess–Zumino gauge and a chiral scalar supermultiplet, respectively. Especially $X^3 - \nabla_5\Phi \equiv D^{(5)}$ plays the role of D-field on the wall, namely $D^{(5)}|_{x^5=0,l} = X^3 - \partial_5\Phi \equiv (2l)^{-1/2}D$.[1]

We introduce a 4D chiral supermultiplet[2] $S \equiv (\phi, \psi, F)$ of the fundamental representation which is localized on the wall, where ϕ, ψ and F stand for a complex scalar field, a Weyl spinor and an auxiliary field of complex scalar, respectively. This is the simplest matter content on the wall. Using the $\mathcal{N} = 1$ SUSY property, we can find

[1] It looks that $D \to 0$ as $l \to 0$ at first sight. However, if we introduce a dimensionless effective 4D gauge coupling, $\hat{g}^2 \equiv g^2/(2l)$ which is fixed for $l \to 0$, we have $gD^{(5)}|_{x^5=0} = \hat{g}D$ irrespective of l.

[2] We do not introduce extra 5D matter multiplets (the hypermultiplets) differently from [3].

the following boundary Lagrangian with a definite supersymmetric coupling between bulk and boundary fields:

$$\begin{aligned}\mathcal{L}_{\text{bnd}} &= S^\dagger e^{gV} S\big|_{\theta^2\bar\theta^2} + W(S)\big|_{\theta^2} \\ &= \nabla_m\phi^\dagger\nabla^m\phi + \psi^\dagger i\bar\sigma^m\nabla_m\psi + F^\dagger F - \sqrt{2}\,g\big(\phi^\dagger\lambda_L^t\sigma^2\psi + \psi^\dagger\sigma^2\lambda_L^*\phi\big) + g\phi^\dagger D^{(5)}\phi \\ &\quad - \left[m_{\alpha'\beta'}\left(\phi_{\alpha'}F_{\beta'} - \frac{1}{2}\psi_{\alpha'}\psi_{\beta'}\right) + \frac{1}{2}\lambda_{\alpha'\beta'\gamma'}\left(\phi_{\alpha'}\phi_{\beta'}F_{\gamma'} - \psi_{\alpha'}\psi_{\beta'}\phi_{\gamma'}\right) + \text{h.c.}\right],\end{aligned} \tag{3}$$

where $\nabla_m \equiv \partial_m - igA_m$, α', β' and $\gamma'$are the suffices of the fundamental representation and we have taken the following superpotential:

$$W(S) = \frac{1}{2}m_{\alpha'\beta'}S_{\alpha'}S_{\beta'} + \frac{\lambda_{\alpha'\beta'\gamma'}}{3!}S_{\alpha'}S_{\beta'}S_{\gamma'}, \tag{4}$$

with the coefficients $m_{\alpha'\beta'}$ and $\lambda_{\alpha'\beta'\gamma'}$ being such that the gauge symmetry is respected.

Since the hidden sector is irrelevant to the present purpose, we do not specify its Lagrangian $\mathcal{L}'_{\text{bnd}}$.

3. Effective Lagrangian for AFTM

The 1-loop SUSY effective potential $V_{\text{1-loop}}$ can be calculated only by the scalar loop (tadpole) up to the F- and D-independent terms in the off-shell treatment in which the auxiliary fields F and D are not eliminated by their equations of motion. This is because the auxiliary fields cannot have the Yukawa coupling with fermions and vectors. This method is called "auxiliary field tadpole method (AFTM)" [4].

The evaluation of the 1-loop effective potential $V_{\text{1-loop}}$ according to AFTM is by the following recipe:

(1) Find an effective Lagrangian by translating auxiliary and spin-0 fields such that "original field" $\to$ "classical part (VEV)" + "quantum part".
(2) Write an effective action of the translated theory with the effective Lagrangian plus the source terms, set aside all terms quadratic in the quantum fields to get $\mathcal{L}^{(2)}$ and calculate from the generating functional *full* propagators of those which couple with the auxiliary fields.
(3) Evaluate a 1PI 1-point vertex function $\Gamma^{(1)}$ for the relevant tadpole diagrams and its momentum space representation, up to the delta function for the momentum conservation, which is nothing but the 1-loop auxiliary field tadpole amplitude $\hat\Gamma^{(1)}_{p_{\text{ext}}=0}$ in the momentum space at zero external momentum.
(4) Integrate the equation

$$\frac{\partial V_{\text{1-loop}}}{\partial\langle\text{auxiliary field}\rangle} = -\hat\Gamma^{(1)}_{p_{\text{ext}}=0}, \tag{5}$$

to obtain $V_{\text{1-loop}}$, where $\langle\cdots\rangle$ means the VEV.
(5) Determine the final form of $V_{\text{1-loop}}$ by making use of SUSY boundary condition, i.e., $V_{\text{1-loop}}(\langle\text{auxiliary field}\rangle = 0) = 0$.

To begin with, we put the following conditions:

$$A_m = 0 \quad (m = 0, 1, 2, 3), \quad \lambda^i = \bar\lambda^i = 0, \quad \psi = 0 \tag{6}$$

to secure the scalar property of the vacuum. The extra (fifth) component of the bulk vector A_5 is not taken to be zero because it is regarded as a 4D scalar on the wall.

Then, we split all the scalar fields (Φ, X^3, A_5; ϕ, F) into the *quantum field* (which is denoted again by the same symbol) and the *classical field* (VEV) ($\varphi \equiv \langle\Phi\rangle$, $\chi^3 \equiv \langle X^3\rangle$, $a_5 \equiv \langle A_5\rangle$; $\eta \equiv \langle\phi\rangle$, $f \equiv \langle F\rangle$) as follows:

$$\Phi \to \varphi + \Phi, \qquad X^3 \to \chi^3 + X^3, \qquad A_5 \to a_5 + A_5, \qquad \phi \to \eta + \phi, \qquad F \to f + F. \tag{7}$$

We allow the classical part of bulk fields φ, χ^3, a_5 to depend in general on the extra coordinate x^5. These VEVs do not violate the Z_2 symmetry as far as they obey the boundary condition.

The quadratic part of action which is relevant for the present purpose is given by

$$S^{(2)}[\Phi, A_5; \phi, F] = \int d^5X \left[\mathcal{L}^{(2)}_{\text{blk}} + \delta\left(x^5\right)\mathcal{L}^{(2)}_{\text{bnd}} + \text{source terms}\right], \tag{8}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\text{blk}} &= \frac{1}{2}\partial_M \Phi_\alpha \partial^M \Phi_\alpha + \frac{1}{2}\partial_M A_{5\alpha}\partial^M A_{5\alpha} - g f_{\alpha\beta\gamma}\left\{\partial_5\varphi_\alpha A_{5\beta}\Phi_\gamma + \partial_5\Phi_\alpha(a_{5\beta}\Phi_\gamma + A_{5\beta}\varphi_\gamma)\right\} \\ &\quad - g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} a_{5\alpha}\varphi_\beta A_{5\gamma}\Phi_\delta - \frac{g^2}{2}\left\{f_{\alpha\beta\tau}(a_{5\alpha}\Phi_\beta + A_{5\alpha}\varphi_\beta)\right\}^2,\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\text{bnd}} &= \partial_m\phi^\dagger\partial^m\phi + \hat{g}\left\{\hat{d}_\alpha\phi^\dagger T^\alpha\phi - \partial_5\Phi_\alpha\left(\eta^\dagger T^\alpha\phi + \phi^\dagger T^\alpha\eta\right)\right\} + F^\dagger F - \frac{\hat{g}^2}{2}\delta(0)\left(\eta^\dagger T^\alpha\phi + \phi^\dagger T^\alpha\eta\right)^2 \\ &\quad - \left[\phi_{\alpha'}(m_{\alpha'\beta'} + \lambda_{\alpha'\gamma'\beta'}\eta_{\gamma'})F_{\beta'} + \frac{1}{2}\lambda_{\alpha'\beta'\gamma'}\phi_{\alpha'}\phi_{\beta'} f_{\gamma'} + \text{h.c.}\right],\end{aligned} \tag{10}$$

where $\hat{d}_\alpha \equiv \langle D_\alpha\rangle = (2l)^{1/2}(\chi^3_\alpha - \partial_5\varphi_\alpha)$, $\phi^\dagger T^\alpha\phi \equiv \phi^\dagger_{\alpha'}(T^\alpha)_{\alpha'\beta'}\phi_{\beta'}$, etc. and the 5D auxiliary field X^3 has been integrated out at the price of giving rise to a singular term ($\propto \delta(0)$) [2].

4. Mass-matrix and the 1PI vertex function

We are now ready for the calculation of the 1-loop effective potential.

The effective action (8) can be expressed as

$$S^{(2)} = \int d^5X \left[\frac{1}{2}\Psi^\dagger M\Psi + \Psi^\dagger J\right], \tag{11}$$

where

$$\Psi^\dagger_A = \left(\phi^\dagger_{\alpha'},\quad \phi^t_{\alpha'},\quad F^\dagger_{\alpha'},\quad F^t_{\alpha'},\quad \Phi^t_\alpha,\quad A^t_{5\alpha}\right), \tag{12}$$

$$J^t_A = \left(J^t_{\phi^\dagger_{\alpha'}},\quad J^t_{\phi_{\alpha'}},\quad J^t_{F^\dagger_{\alpha'}},\quad J^t_{F_{\alpha'}},\quad J^t_{\Phi_\alpha},\quad J^t_{A_{5\alpha}}\right), \tag{13}$$

with $A = (\alpha', \alpha)$, $B = (\beta', \beta)$ and J's denote sources.

We can perform the integration of (11) w.r.t. x^5 by KK-expanding Φ and A_5 as follows;

$$\Phi_\alpha\left(x, x^5\right) = \frac{1}{\sqrt{l}}\sum_{n=1}^{\infty}\Phi_{n\alpha}(x)\sin\left(\frac{n\pi}{l}x^5\right), \tag{14}$$

$$A_{5\alpha}\left(x, x^5\right) = \frac{1}{\sqrt{l}}\sum_{n=1}^{\infty}A_{n\alpha}(x)\sin\left(\frac{n\pi}{l}x^5\right). \tag{15}$$

We obtain

$$S^{(2)} = \int d^4x \left[\frac{1}{2}\hat{\Psi}^\dagger\mathcal{M}\hat{\Psi} + \hat{\Psi}^\dagger\hat{J}\right], \tag{16}$$

where

$$\hat{\Psi}^\dagger = \left(\phi^\dagger_{\alpha'},\quad \phi^t_{\alpha'},\quad F^\dagger_{\alpha'},\quad F^t_{\alpha'},\quad \hat{\Phi}^t_\alpha,\quad \hat{A}^t_{5\alpha}\right), \tag{17}$$

$$\hat{J}^t_A = \left(J^t_{\phi^\dagger_{\alpha'}},\quad J^t_{\phi_{\alpha'}},\quad J^t_{F^\dagger_{\alpha'}},\quad J^t_{F_{\alpha'}},\quad \hat{J}^t_{\Phi_\alpha},\quad \hat{J}^t_{A_{5\alpha}}\right), \tag{18}$$

with

$$\hat{\Phi}^t_\alpha = (\Phi_{1\alpha}, \quad \Phi_{2\alpha}, \quad \dots), \tag{19}$$

$$\hat{A}^t_\alpha = (A_{1\alpha}, \quad A_{2\alpha}, \quad \dots), \tag{20}$$

$$\hat{J}^t_{\Phi_\alpha} = (J_{\Phi_{1\alpha}}, \quad J_{\Phi_{2\alpha}}, \quad \dots), \tag{21}$$

$$\hat{J}^t_{A_{5\alpha}} = (J_{A_{1\alpha}}, \quad J_{A_{2\alpha}}, \quad \dots), \tag{22}$$

and

$$(\mathcal{M}_{AB}) = \begin{pmatrix} \mathcal{A}_{\alpha'\beta'} & \mathcal{B}_{\alpha'\beta} & 0 \\ \mathcal{C}_{\alpha\beta'} & \mathcal{M}_{\hat{\Phi}_\alpha \hat{\Phi}_\beta} & \mathcal{M}_{\hat{\Phi}_\alpha \hat{A}_{5\beta}} \\ 0 & \mathcal{M}_{\hat{A}_{5\alpha}\hat{\Phi}_\beta} & \mathcal{M}_{\hat{A}_{5\alpha}\hat{A}_{5\beta}} \end{pmatrix}, \tag{23}$$

$$\mathcal{A}_{\alpha'\beta'} = \begin{pmatrix} \mathcal{M}_{\phi^\dagger\phi} & \mathcal{M}_{\phi^\dagger\phi^\dagger} & 0 & \mathcal{M}_{\phi^\dagger F^\dagger} \\ \mathcal{M}_{\phi\phi} & \mathcal{M}_{\phi\phi^\dagger} & \mathcal{M}_{\phi F} & 0 \\ 0 & \mathcal{M}_{F^\dagger\phi^\dagger} & I & 0 \\ \mathcal{M}_{F\phi} & 0 & 0 & I \end{pmatrix}_{\alpha'\beta'}, \tag{24}$$

$$\mathcal{B}_{\alpha'\beta} = \begin{pmatrix} \mathcal{M}_{\phi^\dagger\hat{\Phi}} \\ \mathcal{M}_{\phi\hat{\Phi}} \\ 0 \\ 0 \end{pmatrix}_{\alpha'\beta}, \tag{25}$$

$$\mathcal{C}_{\alpha\beta'} = \big(\mathcal{M}_{\hat{\Phi}\phi}, \quad \mathcal{M}_{\hat{\Phi}\phi^\dagger}, \quad 0, \quad 0\big)_{\alpha\beta'}, \tag{26}$$

with

$$\begin{aligned}
\mathcal{M}_{\phi^\dagger_{\alpha'}\phi_{\beta'}} &= -\Box\delta_{\alpha'\beta'} + \hat{g}\hat{d}_\gamma\big(T^\gamma\big)_{\alpha'\beta'} - g^2\delta(0)\big(T^\gamma\eta\big)_{\alpha'}\big(\eta^\dagger T^\gamma\big)_{\beta'}, \\
\mathcal{M}_{\phi_{\alpha'}\phi^\dagger_{\beta'}} &= -\Box\delta_{\alpha'\beta'} + \hat{g}\hat{d}_\gamma\big(T^\gamma\big)_{\beta'\alpha'} - g^2\delta(0)\big(\eta^\dagger T^\gamma\big)_{\alpha'}\big(T^\gamma\eta\big)_{\beta'}, \\
\mathcal{M}_{\phi^\dagger_{\alpha'}\phi^\dagger_{\beta'}} &= -\lambda^*_{\alpha'\beta'\gamma'} f^\dagger_{\gamma'} + g^2\delta(0)\big(T^\gamma\eta\big)_{\alpha'}\big(T^\gamma\eta\big)_{\beta'}, \\
\mathcal{M}_{\phi_{\alpha'}\phi_{\beta'}} &= -\lambda_{\alpha'\beta'\gamma'} f_{\gamma'} + g^2\delta(0)\big(\eta^\dagger T^\gamma\big)_{\alpha'}\big(\eta^\dagger T^\gamma\big)_{\beta'}, \\
\mathcal{M}_{F^\dagger_{\alpha'}\phi^\dagger_{\beta'}} &= (\mathcal{M}_{\phi^\dagger F^\dagger})_{\alpha'\beta'} = -\big(m^*_{\alpha'\beta'} + \lambda^*_{\alpha'\gamma'\beta'}\eta^\dagger_{\gamma'}\big) \equiv \chi^\dagger_{\alpha'\beta'}, \\
\mathcal{M}_{F_{\alpha'}\phi_{\beta'}} &= (\mathcal{M}_{\phi F})_{\alpha'\beta'} = -(m_{\alpha'\beta'} + \lambda_{\alpha'\gamma'\beta'}\eta_{\gamma'}) \equiv \chi_{\alpha'\beta'}, \\
\mathcal{M}_{\phi^\dagger_{\alpha'}\Phi_{n\beta}} &= -g\big(T^\beta\eta\big)_{\alpha'}\big(n\pi/l^{3/2}\big), \\
\mathcal{M}_{\Phi_{n\alpha}\phi^\dagger_{\beta'}} &= -g\big(T^\alpha\eta\big)_{\beta'}\big(n\pi/l^{3/2}\big), \\
\mathcal{M}_{\phi_{\alpha'}\Phi_{n\beta}} &= -g\big(\eta^\dagger T^\beta\big)_{\alpha'}\big(n\pi/l^{3/2}\big), \\
\mathcal{M}_{\Phi_{n\alpha}\phi_{\beta'}} &= -g\big(\eta^\dagger T^\alpha\big)_{\beta'}\big(n\pi/l^{3/2}\big), \\
\mathcal{M}_{\Phi_{m\alpha}\Phi_{n\beta}} &= -\big(\Box + (n\pi/l)^2\big)\delta_{mn}\delta_{\alpha\beta}.
\end{aligned} \tag{27}$$

The explicit form of sources $\hat{J}$'s is not required in the following computation. The matrix elements $(M_{\hat{\Phi}\hat{A}_5})_{\alpha\beta}$, $(M_{\hat{A}_5\hat{\Phi}})_{\alpha\beta}$ and $(M_{\hat{A}_5 A_5})_{\alpha\beta}$ do not depend on $f_{\alpha'}$, $f^\dagger_{\alpha'}$ and $\hat{d}_\alpha$ so that they are irrelevant to the effective potential to be estimated.

The generating functional $Z[\hat{J}]$ is given by

$$\ln Z[\hat{J}] = -\frac{1}{2}\int d^4x\, d^4y\, \hat{J}^{\dagger}(x) i\mathcal{M}^{-1}(x, y)\hat{J}(y), \tag{28}$$

from which we can extract a *full* propagator $\Delta_F(x-y)_{ij}$ through $\delta^2 \ln Z[\hat{J}]/\delta\hat{J}_j\delta\hat{J}_i^{\dagger}$, namely

$$\Delta_F(x-y)_{ij} = -\mathcal{M}_{ij}^{-1} = -\frac{m_{ji}}{\det\mathcal{M}}, \tag{29}$$

where m_{ji} denotes the jith minor of $\mathcal{M}$.

The 1PI vertex function $\Gamma^{(1)}$ corresponding to the auxiliary field tadpole is defined as the proper Green function with the propagator of external line amputated, i.e.,

$$\langle 0|\mathrm{T}\boldsymbol{B}_A(x)|0\rangle_{\mathrm{prop}} = \int d^4y\, \Delta_F(x-y)_{B_A}\Gamma^{(1)B_A}(y), \tag{30}$$

where $\boldsymbol{B}_A$ stands for the renormalized Heisenberg fields $\boldsymbol{F}_{\alpha'}$, $\boldsymbol{F}_{\alpha'}^{\dagger}$ or $\boldsymbol{D}_\alpha$ corresponding to $f_{\alpha'}$, $f_{\alpha'}^{\dagger}$ or $\hat{d}_\alpha$, respectively.

The momentum space representation $\hat{\Gamma}^{(1)B_A}$ of $\Gamma^{(1)B_A}(y)$ is defined in general by

$$\int d^4y\, e^{ipy}\Gamma^{(1)B_A}(y) \equiv (2\pi)^4\delta^4(p)\hat{\Gamma}^{(1)B_A}(p). \tag{31}$$

Then, as $\Gamma^{(1)B_A}(y)$ is written by the propagator $\Delta_F(y-y)_{ij} = -\mathcal{M}^{-1}(y-y)_{ij}$ and $\mathcal{M}$ depends linearly on $f_{\alpha'}$, $f_{\alpha'}^{\dagger}$ and $\hat{d}_\alpha$, we find [4]

$$\hat{\Gamma}^{(1)B_A} \equiv \hat{\Gamma}^{(1)B_A}_{p_{\mathrm{ext}}=0} = \hat{\Gamma}^{(1)B_A}(0) = -\frac{1}{2}\frac{\partial}{\partial\hat{b}_A}\int\frac{d^4k}{(2\pi)^4}\ln\det\mathcal{M}(k), \tag{32}$$

where $\hat{b}_A \equiv \langle B_A\rangle$ and $\mathcal{M}(k)$ is the momentum representation of $\mathcal{M}(x)$ (23):

$$\mathcal{M}(k) = \begin{pmatrix} \mathcal{A}(k) & \mathcal{B}(k) & 0 \\ \mathcal{C}(k) & \mathcal{M}_{\hat{\Phi}\hat{\Phi}}(k) & \mathcal{M}_{\hat{\Phi}\hat{A}_5}(k) \\ 0 & \mathcal{M}_{\hat{A}_5\hat{\Phi}}(k) & \mathcal{M}_{\hat{A}_5\hat{A}_5}(k) \end{pmatrix}. \tag{33}$$

The estimation proceeds as

$$\begin{aligned}
\hat{\Gamma}^{(1)B_A} &= -\frac{1}{2}\frac{\partial}{\partial\hat{b}_A}\int\frac{d^4k}{(2\pi)^4}\ln\Big[\det\mathcal{M}_{\hat{\Phi}\hat{\Phi}}(k)\det\big\{\mathcal{A}(k) - \mathcal{B}(k)\mathcal{M}_{\hat{\Phi}\hat{\Phi}}(k)^{-1}\mathcal{C}(k)\big\} \\
&\qquad \times\det\mathcal{M}_{\hat{A}_5\hat{A}_5}(k)\det\big\{\mathcal{M}_{\hat{A}_5\hat{A}_5}(k) - \mathcal{M}_{\hat{\Phi}\hat{A}_5}(k)\mathcal{D}^{-1}(k)\mathcal{M}_{\hat{A}_5\hat{\Phi}}(k)\big\}\Big] \\
&= -\frac{1}{2}\frac{\partial}{\partial\hat{b}_A}\int\frac{d^4k}{(2\pi)^4}\ln\det\big\{\mathcal{A}(k) - \mathcal{B}(k)\mathcal{M}_{\hat{\Phi}\hat{\Phi}}(k)^{-1}\mathcal{C}(k)\big\} + O\big(\hat{g}^5, \hat{g}^4\lambda\big),
\end{aligned} \tag{34}$$

where

$$\mathcal{D} = \begin{pmatrix} \mathcal{A}(k) & \mathcal{B}(k) \\ \mathcal{C}(k) & \mathcal{M}_{\hat{\Phi}\hat{\Phi}}(k) \end{pmatrix}. \tag{35}$$

Thus, we obtain

$$\hat{\Gamma}^{(1)B_A} = -\frac{1}{2}\frac{\partial}{\partial \hat{b}_A}$$
$$\times \int \frac{d^4k}{(2\pi)^4}\frac{1}{2l}\ln\det\left\{\begin{pmatrix} -k^2-\chi^\dagger\chi+\hat{g}\hat{d}T \\ \quad -g^2\delta(0)(T\eta)(\eta^\dagger T) & -\lambda^\dagger f^\dagger - g^2\delta(0)(T\eta)(T\eta)^t \\ -\lambda f - g^2\delta(0)(\eta^\dagger T)^t(\eta^\dagger T) & -k^2-\chi\chi^\dagger+\hat{g}\hat{d}T^t \\ & \quad -g^2\delta(0)(\eta^\dagger T)^t(T\eta)^t \end{pmatrix}\right.$$
$$\left. + g^2\left(\delta(0)-\frac{1}{2}k\coth(lk)\right)\begin{pmatrix} (T\eta)(\eta^\dagger T) & (T\eta)(T\eta)^t \\ (\eta^\dagger T)^t(\eta^\dagger T) & (\eta^\dagger T)^t(T\eta)^t \end{pmatrix} + O\left(g^4\right)\right\}$$
$$\approx -\frac{1}{2}\frac{\partial}{\partial \hat{b}_A}\int \frac{d^4k}{(2\pi)^4}\frac{1}{2l}\operatorname{tr}\ln \Delta(\hat{b}), \tag{36}$$

$$\Delta(\hat{b}) = \left[k^2+\chi^\dagger\chi-\hat{g}\hat{d}T+\frac{1}{2}g^2k\coth(lk)(T\eta)\left(\eta^\dagger T\right)\right]\left[k^2+\chi\chi^\dagger-\hat{g}\hat{d}T^t+\frac{1}{2}g^2k\coth(lk)\left(\eta^\dagger T\right)^t(T\eta)^t\right]$$
$$-\left[\lambda^\dagger f^\dagger+\frac{1}{2}g^2k\coth(lk)(T\eta)(T\eta)^t\right]\left[\lambda f+\frac{1}{2}g^2k\coth(lk)\left(\eta^\dagger T\right)^t\left(\eta^\dagger T\right)\right], \tag{37}$$

where we have performed a Wick rotation and used the formula

$$\sum_{k^5}\frac{(k^5)^2}{k^2+(k^5)^2} = 2l\left(\delta(0)-\frac{1}{2}k\coth(lk)\right), \tag{38}$$

with k^5 being summed over the values $\pi n/l (n = \text{integer})$, i.e., over whole KK modes. An interesting observation here is that the $\delta(0)$-singularity coming from the KK mode summation has been neatly cancelled by that from the elimination of the 5D auxiliary field X^3.

5. 1-loop effective potential

The effective potential $V_{\text{1-loop}}$ is nothing but a generator of $\hat{\Gamma}^{(1)}(0)$, namely,

$$\frac{\partial V_{\text{1-loop}}}{\partial \hat{b}_A} = -2l\,\hat{\Gamma}^{(1)B_A}_{P_{\text{ext}}=0}, = \frac{1}{2}\frac{\partial}{\partial \hat{b}_A}\int \frac{d^4k}{(2\pi)^4}\ln\det\mathcal{M}(k), \tag{39}$$

where $\hat{\Gamma}^{(1)B_A}_{P_{\text{ext}}=0} = \hat{\Gamma}^{(1)B_A}(0)$ has been multiplied by $2l$ in order for $V_{\text{1-loop}}$ to be 4D. Eq. (39) is integrated to give

$$V_{\text{1-loop}} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\operatorname{tr}\ln \Delta\left(f, f^\dagger, \hat{d}\right) + K\left(\eta, \eta^\dagger\right), \tag{40}$$

where K is an integration constant.

Finally, we apply the SUSY boundary condition

$$V_{\text{1-loop}}\left(f = f^\dagger = \hat{d} = 0\right) = 0, \tag{41}$$

and obtain

$$\begin{aligned}V_{1\text{-loop}} &= \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\big[\ln\Delta\big(f,f^\dagger,\hat{d}\big)-\ln\Delta(0,0,0)\big]\\ &= \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\ln\bigg[1-\frac{2\hat{g}(\hat{d}_\alpha T^\alpha)}{k^2+\chi^\dagger\chi}+\frac{\mathcal{G}_- - lk\coth(lk)\mathcal{H}}{(k^2+\chi^\dagger\chi)^2}+O\big(\lambda^4,\hat{g}^4,\hat{g}^2\lambda^2,\hat{g}^3\lambda\big)\bigg],\end{aligned} \tag{42}$$

where

$$\mathcal{G}_\pm \equiv \hat{g}^2\big(\hat{d}_\alpha T^\alpha\big)^2 \pm \big(\lambda^{\dagger\alpha'} f^\dagger_{\alpha'}\big)\big(\lambda^\beta p f_{\beta'}\big), \tag{43}$$

$$\begin{aligned}\mathcal{H} \equiv{}& \hat{g}^2\big\{\big(\lambda^{\dagger\alpha'} f^\dagger_{\alpha'}\big)\big(\eta^\dagger T^\alpha\big)^t\big(\eta^\dagger T^\alpha\big)+\big(T^\alpha\eta\big)\big(T^\alpha\eta\big)^t\big(\lambda^\alpha f_{\alpha'}\big)\big\}/2\\ &+\hat{g}^3\big\{\big(\hat{d}_\alpha T^\alpha\big)\big(\eta^\dagger T^\beta\big)^t\big(T^\beta\eta\big)^t+\big(T^\beta\eta\big)\big(\eta^\dagger T^\beta\big)\big(\hat{d}_\alpha T^\alpha\big)\big\}/2.\end{aligned} \tag{44}$$

The resultant effective potential (42) is now ready for being integrated w.r.t. the four momentum. Before doing so, it is useful to comment on the renormalizability. Higher-dimensional field theories are generically non-renormalizable. The present 5D super-YM model is not exceptional and must be viewed as an effective theory valid up to some high mass scale associated with an ultraviolet cutoff Λ. However, it should be required that the present model is renormalizable in the limit of $l \to 0$.

If T has a component such as $\mathrm{tr}\,T \neq 0$, i.e., the gauge group has a U(1) factor, which we denote as $\mathrm{U}(1)_X$, the term proportional to $\hat{g}T$ in (42) provides a dominant contribution, namely

$$V_{1\text{-loop}} \approx \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\ln\bigg[1-\frac{2\hat{g}\hat{d}T}{k^2+\chi^\dagger\chi}\bigg], \tag{45}$$

the integral of which yields a quadratic divergence. As (45) is independent of l, the quadratic divergence remains in the limit of $l \to 0$ and will spoil the non-renormalization theorem. To get around it, we introduce additional chiral scalar supermultiplets with $\mathrm{tr}\,Q_X = 0$ in the boundary and require that only one of the chiral scalar supermultiplets, say ϕ, the $\mathrm{U}(1)_X$-charge of which is normalized to be 1, has a non-trivial VEV η.

The factor $lk\coth(lk)$ in (42) alters the high k behaviour of the integrand and yields a term which is linear in Λ but suppressed by l. In fact, we obtain

$$\begin{aligned}V_{1\text{-loop}} &\approx -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{tr}\,\mathcal{G}_+ + lk\coth(lk)\,\mathrm{tr}\,\mathcal{H}}{(k^2+\chi^\dagger\chi)^2}\\ &\approx \frac{1}{16\pi^2}\Bigg[\int_0^\Lambda dk\frac{k^3\,\mathrm{tr}\,\mathcal{G}_+}{(k^2+\chi^\dagger\chi)^2}+\Bigg\{\int_0^{\tilde{k}} dk\frac{k^3}{(k^2+\chi^\dagger\chi)^2}+\int_{\tilde{k}}^\Lambda dk\,l\coth(lk)\Bigg\}\mathrm{tr}\,\mathcal{H}\Bigg]\\ &\approx \frac{1}{32\pi^2}\bigg[\ln\bigg(\frac{\Lambda^2}{\chi^\dagger\chi}\bigg)\mathrm{tr}\,\mathcal{G}_+ - l\Lambda\,\mathrm{tr}\,\mathcal{H}\bigg]+\text{finite part},\end{aligned} \tag{46}$$

where terms that vanish as Λ goes to infinity have been neglected and we have split the integral of the term proportional to $\mathrm{tr}\,\mathcal{H}$ into two regions, $0 \leqslant k \leqslant \tilde{k}$ and $\tilde{k} \leqslant k \leqslant \Lambda$, assuming $|\chi| \ll \tilde{k} \ll l^{-1}$.

The leading term in r.h.s. of (46) is apparently linealy divergent. However, the cutoff Λ is multiplied by the length l of the extra dimension which may suppress the growth of Λ so as to give a finite and significant contribution to the effective potential. This is nothing but a bulk effect and plays an important role in the minimization of the effective potential as will be described in the next section.

6. Minimization of the effective potential

The utility of effective potential is to search a true vacuum by its minimization. Our effective potential is, however, SUSY so that it vanishes trivially at the minimum point $f = \hat{d} = 0$. In order to examine its physical property, therefore, it is appropriate to introduce a term such as FI D-term[3] into the boundary Lagrangian $\mathcal{L}_{\mathrm{bnd}}$ and observe how the spontaneous breaking of SUSY as well as that of gauge symmetry is realized. For this purpose, we choose the gauge group to be $\mathrm{U}(1)_X$ discussed in the previous section.

The effective potential to be minimized is then as follows:

$$V^{\mathrm{eff}} = V_{\mathrm{tree}} + V_{1\text{-loop}} + V_{\mathrm{FI}}, \tag{47}$$

$$V_{\mathrm{tree}} = -f^\dagger f + \eta^t m f + \eta^\dagger m f^* + \frac{1}{2}\eta^t \lambda f \eta + \frac{1}{2}\eta^\dagger \lambda^\dagger f^\dagger \eta^* - \frac{1}{2}\hat{d}^2 - \hat{g}\eta^\dagger \hat{d}\eta, \tag{48}$$

$$V_{1\text{-loop}} = -\alpha\,\mathrm{tr}\big\{\big(\lambda^\dagger f^\dagger \eta^{\dagger 2} + \eta^2 \lambda f\big)/2 + \hat{g}\eta^\dagger \hat{d}\eta\big\}, \tag{49}$$

$$V_{\mathrm{FI}} = -\xi\hat{d}, \tag{50}$$

where V_{tree} is a tree level potential which is directly read from (2) and (3), $V_{1\text{-loop}}$ is the dominant part of (46) with

$$\alpha \equiv \frac{l\Lambda\hat{g}^2}{32\pi^2}, \tag{51}$$

and V_{FI} comes from the FI D-term $\mathcal{L}_D = \xi D^{(5)}$.

In order to trace essential features of our analysis, we assume that the components of each classical scalar field vanish except for a certain real component which we denote by the same symbol. Then, we have

$$V^{\mathrm{eff}}(f, \hat{d}, \eta) = -f^2 + 2mf\eta + \lambda f(1-\alpha)\eta^2 - \frac{1}{2}\hat{d}^2 - (1+\alpha)\hat{g}\hat{d}\eta^2 - \xi\hat{d}. \tag{52}$$

The auxiliary fields f, $\hat{d}$ are written as functions of η through the conditions $\partial V^{\mathrm{eff}}/\partial f = \partial V^{\mathrm{eff}}/\partial\hat{d} = 0$ as follows;

$$f = m\eta + \frac{\lambda}{2}(1-\alpha)\eta^2 \equiv \tilde{f}, \tag{53}$$

$$\hat{d} = -\xi - (1+\alpha)\hat{g}\eta^2 \equiv \tilde{d}, \tag{54}$$

by which we eliminate f, $\hat{d}$ from V^{eff}:

$$V^{\mathrm{eff}}(\tilde{f}, \tilde{d}, \eta) = V^{\mathrm{eff}}(\eta) = \left\{\frac{\lambda^2(1-\alpha)^2}{4} + \frac{(1+\alpha)^2\hat{g}^2}{2}\right\}\eta^4 + \lambda m(1-\alpha)\eta^3 + \big\{m^2 + (1+\alpha)\hat{g}\xi\big\}\eta^2 + \frac{\xi^2}{2}. \tag{55}$$

From now on, we assume that ϕ is massless ($m = 0$) for simplicity. Then, if $\hat{g}\xi > 0$, $V^{\mathrm{eff}}(\eta)$ has a minimum at $\eta = 0$ with a minimum value $\xi^2/\{2(1+2\alpha)\}$, which measures the SUSY breaking scale M_{SUSY}. The gauge symmetry is not broken. If $\hat{g}\xi < 0$, on the other hand, $V^{\mathrm{eff}}(\eta)$ is minimized at

$$\eta^2 = \frac{-2(1+\alpha)\hat{g}\xi}{\lambda^2(1-\alpha)^2 + 2(1+\alpha)^2\hat{g}^2} \equiv \tilde{\eta}^2, \tag{56}$$

with the minimum value

$$V^{\mathrm{eff}}(\eta = \tilde{\eta}) = \tilde{V}^{\mathrm{eff}} = \frac{(1-\alpha)^2\lambda^2}{(1-\alpha)^2\lambda^2 + 2(1+\alpha)^2\hat{g}^2}\,\frac{\xi^2}{2}, \tag{57}$$

[3] Such a D term has been introduced into the hidden sector in [2].

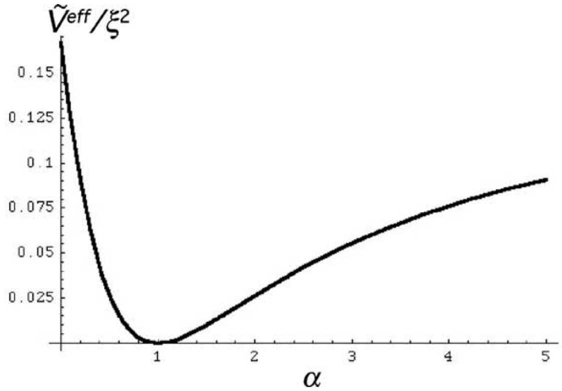

Fig. 1. α-dependence of $\tilde{V}^{\text{eff}}$ for $\lambda = \hat{g}$.

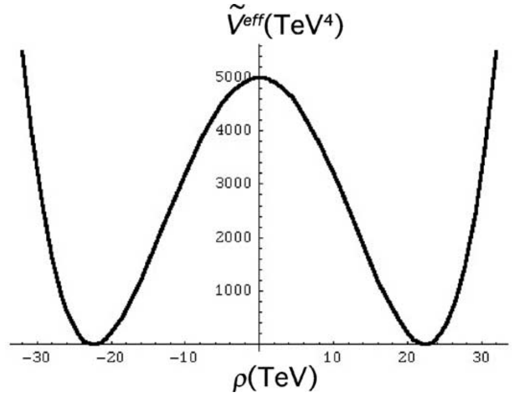

Fig. 2. η-dependence of V^{eff} for $\hat{g} = 0.1$ and $\xi = 100\ \text{TeV}^2$.

which is a function of α for given λ, $\hat{g}$ and ξ and has an absolute minimum at $\alpha = 1$ where $\tilde{V}^{\text{eff}} = 0$ as shown in Fig. 1. Therefore, the size of the extra dimension is settled at $l^{-1} = \Lambda\hat{g}^2/32\pi^2$ making SUSY restored in the true vacuum. For example, $l^{-1} \approx 3.0 \times 10^{11}$ GeV ($l^{-1} = 7.2 \times 10^{13}$ GeV) for $\Lambda = M_{\text{GUT}} \approx 10^{16}$ GeV ($\Lambda = M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV) and $\hat{g} \approx 0.1$.

At $\alpha = 1$, (55) becomes

$$V^{\text{eff}}(\eta) = 2\left(\hat{g}\eta^2 + \frac{\xi}{2}\right)^2, \tag{58}$$

for any λ, which has minima at $\eta = \pm\sqrt{|\xi/2\hat{g}|} \equiv \tilde{\eta}$ (hence $\tilde{f} = \tilde{d} = 0$) as shown in Fig. 2, provided $\hat{g}\xi < 0$. Namely ϕ plays the role of Higgs field which breaks the gauge symmetry with the breaking scale $\langle\phi\rangle = \tilde{\eta}$, while SUSY is restored in spite of the presence of FI D-term.[4] It is not M_{SUSY} but the gauge symmetry breaking scale $\tilde{\eta}$ that ξ affects.

7. Concluding remarks

We have estimated a SUSY effective potential of the 5D super-YM model with the extra dimension compactified on S^1/Z_2 at the 1-loop level. Under such assumptions that the quadratic divergence does not arise, its dominant part is apparently linealy divergent and proportional to $l\Lambda$, i.e., a product of the size of extra dimension and the cutoff scale. If l is small but much bigger than the "cutoff size" Λ^{-1} corresponding to the Planck, string or GUT scale, the divergence is suppressed and the term proportional to $l\Lambda$ of 1-loop effective potential proves to be finite and not negligible. This is just the bulk effect which originates from taking in all the KK excitation modes of the bulk field Φ and reveals an interesting situation. In fact, taking the tree level contributions and the FI D-term into account, we find that the effective potential is minimized at a specific value of l, where SUSY is restored but the gauge symmetry is broken. It is remarkable that the value of l corresponds to an intermediate energy scale where new ingredients of gauge theory are expected to be disclosed.

As an approach to regard the extra-space radius as a dynamical variable, the radion model is, at present, most promising. There, the radius parameter l is regarded as a vacuum expectation value of the field "radion" [6,7]. It would be more complete to treat the result of Section 6 in the framework of radion model. In order to incorporate

[4] Such a phenomenon is known to occur for $\alpha = 0$, i.e., at the tree level, too, only if $\lambda = 0$ as far as $\hat{g}\xi < 0$. In our case ($\alpha = 1$), (55) is valid irrespectively of λ.

the radion and the dilaton in a multiplet, we are naturally led to consider 5D supergravity (SUGRA). Consistency of FI-terms in the 5D SUGRA has been investigated in [8]. It remains as a future work to examine the conclusion of our model in connection with the radius stabilization in this context.

Such a phenomenon that the radiative correction appears to be proportional to $l\Lambda$ seems inherent in gauge theories with extra dimensions compactified on flat space. Indeed, it has been observed that the renormalization group running of the gauge coupling constants changes from "logarithmic" to "linear" in a 5D version of minimal SUSY Standard Model with the flat extra dimension compactified on S^1/Z_2 [9]. This fact is due to the presence of infinite towers of KK states and causes an accelerated unification of strong, electromagnetic and weak couplings only a little above $\mu_0 \equiv l^{-1}$. However, if the extra 5th dimension is warped as in the case of Randall–Sundrum (RS) [6], the gauge coupling running can be logarithmic [10,11]. It is, therefore, worth to try to compute the SUSY effective potential in the 5D super-YM model with the RS background and examine whether it is minimized at a non-trivial value of the radius of extra dimension or not. The bulk effect to our SUSY effective potential is principally due to the contribution from the bulk propagator of Φ. Since the interaction of Φ with ϕ's takes place only in the 4D boundary, the relevant loop amplitude including the Φ-propagator might not be affected by x^5-dependent cutoff [11] in the RS background, so that the "linear" growth of effective potential along with the cutoff would have a popssibility to be retained even if the extra dimension is warped. The details will be discussed in the forthcoming paper [12].

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