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Invited Paper

Foreword

Disjunctive programming extends traditional linear programming by considering feasible sets which are unions of convex polyhedra. Integer programming is a special case. Egon Balas established the foundations of disjunctive programming in 1974, in a research report written at Carnegie Mellon University. Unfortunately, this often cited paper was never published. Looking back on it from a quarter century perspective, we can see how advanced Balas's ideas were, and, indeed, how many of the ideas he introduced have developed into significant areas of activity in integer programming. Here are four examples.

(i) A key idea of disjunctive programming is that a disjunctive program is equivalent to a linear program in a higher-dimensional space (Theorem 2.1). The insight that reformulation of a problem can lead to a new problem with a much smaller number of constraints in a higher-dimensional space is very powerful and has been successfully applied to many combinatorial optimization problems. Pursuing this theme, the insight is that, in order to tighten a relaxation, it can be beneficial to reformulate it in a higher-dimensional space. This approach is now common. The important area of semi-definite relaxations is of this flavor.

(ii) Another key idea of disjunctive programming is that, when projecting back into the original space of variables, cutting planes can be obtained by simply solving a linear program (Theorem 3.1). These disjunctive cuts have been used very successfully in recent computational work.

(iii) Section 4 describes the facets of the convex hull of feasible solutions. It pioneers an approach repeated over the last twenty years for various combinatorial optimization problems. This sort of endeavor is central to modern polyhedral combinatorics.

(iv) Theorem 5.3 is related to the notion of rank of a valid inequality in integer programming. The question is, how difficult is it to establish the validity of an inequality satisfied by all points in the integer hull? The well-known Chvátal–Gomory rank measures the number of necessary iterations of a procedure, based on rounding inequalities obtained as nonnegative linear combinations of valid inequalities. Similarly, the "disjunctive" rank introduced by Balas can be defined as follows. Starting from a polytope P_0, any inequality which is not valid for P_0, but is valid when imposing one of the disjunctions, is said to have disjunctive rank 1. Let P_1 be the polytope obtained from P_0 by adding all such rank 1 inequalities. Now, repeating the process starting from P_1, one obtains rank 2 inequalities, and so on. A key question in integer programming is to give a bound on the rank of valid inequalities. Theorem 5.3 provides such a bound for

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facial disjunctive programs. In particular, for 0, 1 programs, the disjunctive rank never exceeds the number of 0, 1 variables. This is in contrast with the Chvátal–Gomory rank for which no such bound is known.

We hope the readers enjoy the seminal 1974 Balas paper on disjunctive programming, published here for the first time.

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