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Ultraslow Muonium for a Muon beam of ultra high quality

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Abstract

Efficient conversion of a standard muon beam into a high quality slow muon beam by using the muonium ionization method is shown to be achievable with a new kind of converter. It consists of a superfluid helium layer from which muonium atoms escape because of their negative affinity. Application to pulsed muon beams in an optimized target configuration leads to a reduction of the necessary VUV laser energy by a factor of 100. The resulting muon beam has more than 4 orders of magnitude better phase space quality than what is obtained today with the silica powder converter. When using the superfluid helium converter in association with high quality DC muon beams in development at the Paul Scherrer Institute, it should be possible to use intense continuous lasers for ionization and obtain ultra-high quality continuous muon beams with precise timing of the muon arrival.

Keywords: muonium, muon beam

1. Introduction

The interest in beams of slow positive muon for applications in solid state physics and fundamental interactions have resulted in the proposal of various concepts for the conversion of common high energy muon beams into high quality slow beams [1, 2, 3]. Presently only one method [3] has been developed to the level of a full fledged beam available to a wide user community [6]. It uses the re-emission of muons of a few tens of eV from surface layer of solidified gases.

Another promising method which results in a better beam quality has been made possible by the discovery of the thermal muonium emission from either hot tungsten [4] or fine silica powder [5]. By inducing the ionization of the muonium atoms with laser beams, a thermal muon source is obtained that can be accelerated and focused onto small spots. Such a beam has been under development for two decades and the results are still far below the expectations. The main limitation is the laser fluence, especially when using the far ultraviolet wavelength of $0.122\,\mu$m in the first step transition from the 1S to the 2P level of muonium. Only very low ($1\,\mu$J) and far insufficient (by a factor of 100) laser radiation at this difficult wavelength could be achieved [7]. Although the excitation cross-section is high, the large Doppler width due to the velocity spread of the muonium atom requires a large laser bandwidth that increases the required laser energy.

In this paper we present a new converter concept that is operated at very low temperatures. It provides muonium atoms with very low transverse momentum that results in a much lower Doppler width and requires much less laser energy to achieve the 1S–2P excitation. Also a much higher quality of the outgoing muon beam is obtained.

The converter used is a thin layer of superfluid ^{4}He at a temperature near 0.3 K. What makes the emission into vacuum possible is that hydrogen atoms in superfluid helium have a negative affinity or a positive chemical potential. The hydrogen atom is an impurity atom for which the helium-hydrogen wave function overlap results in a repulsive

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interaction that raises the potential of the impurity atom. The lighter the hydrogen isotope, the greater its chemical potential. As calculated by various authors, its value for hydrogen, deuterium and tritium amounts to 37 K, 14 K and 7 K respectively [9]. The value for deuterium has been confirmed experimentally [10]. The calculations have recently been extended to muonium[12] giving a chemical potential of 270 K. As a result, a muonium atom that reaches the surface will be ejected vertically with a kinetic energy of 23 meV. The very small transverse momentum it had before the ejection remains conserved, so that the muonium atoms exit the liquid with a very small divergency. A nearly mono–energetic muonium beam is provided, which by itself has interesting experimental applications.

We first discuss in this paper how such a converter could be optimally used to replace the silicon powder converter for the application to an incoming standard high energy pulsed muon beam. In the next section we extend the technique to the case where the incoming muon beam is already a high quality slow beam and show that the very small muonium emission volume should allow continuous lasers to do the operation.

2. The superfluid ^{4}He converter

A special configuration is considered that optimizes the number of muonium atoms reaching the surface. It is based on pulling the stopped muons to the helium surface with an electric field and inducing their neutralization near the surface. μSR experiments have shown that muons stopping in superfluid helium initially thermalize as positive ions but are prone to neutralization via the attraction towards the close-by electrons from the ionization path. This can be hindered by applying an electric field acting in the direction of the incoming muon beam [13]. In our case this is the direction that brings the muons to the surface. At a field of 100 V/cm, 80% of the muons drift away from the attracting electrons and make their way towards the surface. The drift velocity of a positive muon is practically the same as that of He ions because in both cases they are in the snowball state with an effective mass of 200 atomic mass units. We can therefore rely on existing measurements of the drift velocity of He$^+$ ions [14]. Near 0.3 K the drift velocity depends sensitively on the ^{3}He concentration which acts as an impurity "gas" on which the positive ion loses its acceleration energy. The drift velocity increases with the electric field up to a critical value of the field over which it falls down rapidly, because of the build up of vortex rings that swallow the acceleration energy. Measurements done near 0.3 K show that at a fixed temperature the critical field increases linearly with the ^{3}He concentration. In all cases the maximum drift velocity is 42 m/sec. These measurements have been done up to a concentration of 500 ppm ^{3}He where the maximum drift velocity is reached at 45 V/cm. These results can be extrapolated to obtain the maximum drift velocity of 42 cm/sec at a field of 100 V/cm and a ^{3}He concentration near 1200 ppm. Under these conditions, a muon stopped 100 μ below the surface reaches the surface in 2.3 μs. This gives the maximum ^{4}He layer thickness useful for the extraction of the stopped muons.

Near the surface, neutralization will be induced by preparing, in the time between the muon pulses from the muon beam, a layer of electrons (electron pool) trapped below the surface. This is a well developed technique [15] where under the action of an external electric field that attracts electrons from the liquid to the surface (in opposite direction to the field considered above!) the electrons remain trapped in a potential maximum just below the surface. The electrons that fill the pool are produced within the liquid by field emission from a tungsten tip. The external electric field is set to be somewhat higher than 100 V/cm . This will allow to collect a surface density of 10^8 e/cm^2 which is a typical density used in the experiments.

After this preparation phase, just before the incoming of a muon pulse from the beam, the external field will be reduced to a very small value that will still be sufficient to hold the electron pool in place. But now the electric field in the liquid has reversed its direction, as the electric field is now mainly due to the negative charge of the electron pool resulting in a field of 100 V/cm pointing in the upwards direction. It attracts the positively charged muons towards the electron subsurface layer.

With such a dense electron layer any μ^+ approaching the surface is rapidly attracted by the closest electron, and neutralization takes place within a micron distance from the surface. The formed muonium atom diffuses by collisions with the dense ^{3}He impurity "gas" and reaches the surface in a short time to be emitted with an energy of about 270 K, a small energy spread and a small divergence.

For homogeneous muon stopping in a helium layer of a thickness slightly less than 100 μm at a field of 100 V/cm, it takes one muon lifetime for all the drifting muons(80%) to reach the surface and get re–emitted. The conversion efficiency for a thick helium layer can be compared with what have been obtained in an optimized experimental configuration where the incoming muon beam had a 21 MeV/c momentum and a 6 % momentum spread [11] . Such a beam has approximatively 800 μm range width in superfluid helium. The total muonium emission will be near 80 % of 100/800 or near 10 % of the incoming beam to be compared with the 5.7 % efficiency measured with the silica powder converter.

If nothing hinders the flow of the muonium swarm in vacuum, it has a vertical velocity of 0.63 cm/μsec which results in a muonium swarm of 1.4 cm width after one muon lifetime. Besides the fact that such a muonium "beam" can be used for various experimental applications, it provides a significant improvement in terms of the production of a μ^+ beam via laser ionization. Because of the small muonium beam divergence, the Doppler width may be 10 times smaller than in the case of the silica powder converter leading to a reduction of the required laser pulse energy by a factor of about 10 . Also, the resulting muon beam will have a much smaller divergence and energy spread giving an increase by more than two orders of magnitude in overall phase space density. The beam divergence which determines the reduction in the needed laser power should be the subject of an experimental investigation as it depends on various properties of superfluid helium.

Up to this level, the realization of the converter is based on standard superfluid helium technology and the yield is reliably predictable. Technically the incoming high energy muon beam has to be rotated vertically by inserting a vertically mounted 90 % dipole magnet.

The next step which expands significantly the potential of the converter is now presented and discussed. A detailed treatment needs theoretical calculations on the interaction of muonium with ^{4}He surfaces that are in preparation. Once they will be available a full simulation of the process will be feasible.

The idea is to slow down the "energetic" muonium atoms prior to their final emission. The basic way to achieve this is to place on the top of the helium surface a thin layer of a very porous material that hinders the free flow out of the muonium atoms. Because superfluid helium will coat all surfaces, the diffusion of the muonium atoms takes place only via collisions with superfluid helium surfaces. The muonium atom cannot re-enter into bulk helium as soon as it has lost some energy or hit the surface with an oblique angle. The collisions with the helium surface will either be inelastic (with momentum and energy transfer to superfluid helium excitations) or elastic (specular reflection).

The inelastic collisions depend on the relative shapes of the well known dispersion curve for helium excitations and the dispersion curve (parabola) of the free muonium. At its starting energy, the muonium momentum is 11 nm^{-1}. At this momentum, maxon excitation is predominent. A single maxon emission gives an energy loss of ΔE=13 K or a relative energy loss of near 5 % initially. As the muonium slows down, E decreases but ΔE stays almost constant, and therefore ΔE/E increases. At cryogenic muonium temperatures the excitations will be phonons, ΔE falls down but ΔE/E still increases reaching values approaching 1 (strong energy loss) at energies just above the intersection point of the two dispersion curves. Below the intersection point at 1.5 K, phonon emission is no more possible. Further energy loss takes place via ripplon excitation, however the shallow shape of the ripplon dispersion curve reduces ΔE/E back to a very small value.

Elastic collisions have a probability that is small at high momentum and increases with the reduction of the perpendicular momentum as shown for the case of hydrogen collisions on the surface of superfluid helium [16]. Assuming a similar momentum dependence for hydrogen and muonium which have the same long range near surface potential, 20 times more elastic collisions than inelastic collisions should take place at muonium energies below a few Kelvins.

The overall outcome is a relatively fast slowing down to below 1.5 K which nevertheless will need 100 collisions or more to take place. Further slowing down is strongly reduced because it can only result from ripplon excitation and also the specular reflection fraction is high. Slowing down to below 0.1 K will therefore require many 100 collisions. The average number of collisions that takes place before the muonium atoms escape can be controlled by the constitution and the thickness of the porous membrane, so that it should be possible to obtain most of the muonium atoms escaping between 0.1 K and 1 K.

In the search of an optimum porous membrane we found the so called self-ordered nanochannel array materials whose most perfect realization is the self-organized anodic alumina [17]. It is produced by anodizing an annealed and polished aluminum surface under high voltage to form an alumina layer full of pores hexagonally distributed. The substrate is then etched away and the pores are etched to increase their diameter. The pores are perfectly aligned as cylindrical holes perpendicular to the membrane and have an open ratio that can reach 65 %. Any pore diameter between 40 nm and 300 nm can be produced.

For a given pore diameter, the thickness of the membrane defines the length to diameter ratio which should be

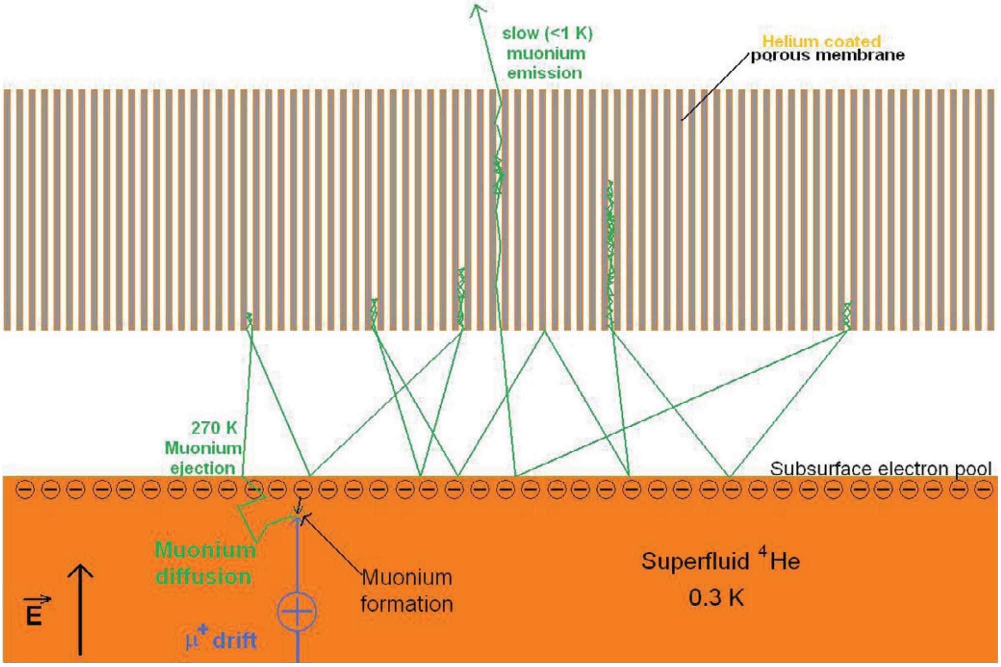

Figure 1: Sketch of ultra-slow muonium formation. The energetic muon beam comes from beneath and the stopped muons drift to the surface where they neutralize with a subsurface electron. The muonium atom is formed by neutralization with a subsurface electron, diffuses below the surface and then in vacuum above the liquid helium and within the pores of the membrane. The trajectory of one muon shown in the figure is just an illustration of the large number of surface collisions taking place (a few 100) before the slowed-down muonium atoms are finally emitted from the membrane. Careful stretching of the membrane should allow to hold it at a distance of 5 μm above the liquid helium surface.

selected according to the optimal number of collisions required. For the optimization, a full simulation of the slowing down process for the given geometry is necessary. This will be done when the calculation of the relevant scattering process for muonium will be available. Ideally, a small pore diameter and a thin membrane lead to the fastest slowing down, but there is a limitation set by the fragility of the membrane and how straight it can be stretched to keep it at a small distance from the liquid helium surface. For a 300 nm pore diameter and a 5 μm distance between the membrane and the helium surface the average time delay between ejection from the liquid and the final emission at an average energy below 1 K should be less than a microsecond. The helium film thickness inside the pores can be controlled via the height relative to the level of a superfluid helium reservoir. It has been shown that a thickness of 15 nm is sufficient to attribute to the film the same scattering properties as bulk helium [16]. Care has to be taken that the membrane has no contact with the liquid helium target and that no superfluid flow take place between the target and the reservoir.

A schematic drawing of the arrangement together with a sketch of a muonium trajectory is shown in Fig. 1.

The kind of porous membrane selected here has an important further property. At low muonium energies where specular reflection dominates, the vertical path of the muonium atom between two inelastic collisions increase strongly for a motion with the velocity vector having a direction approaching the vertical. The consequence, similar to the well known property of capillary molecular beam sources, is that the outgoing muonium atoms are predominantly directed in the vertical direction. The average divergence of the outgoing muonium beam will therefore be quite small and its effective value depends on the detailed properties of the inelastic muonium - surface scattering. We assume here that it will reach a value below 300 mrad. Altogether, the outgoing muonium beam should have more than half its atoms with an average energy below 1 K (corresponding to a velocity of 0.4 mm/μs) and an average divergence below 300 mrad. By shooting the lasers about 2 μs after the incoming muon beam pulse, a laser of less than 1 mm width is sufficient to interact with most of the emitted muonium atoms. This is about 5 times less than what has to be used in existing configurations using the silica powder converter.

The very low energy and the small divergence result in a Doppler width more that 20 times less than that of the room temperature silica converter so that a overall reduction of the required Lyman α laser intensity by a factor of 100 is achieved. This is the intensity level achieved in the latest experiments [7].

In terms of the phase space properties of the muon source after ionization, the transverse emittance is reduced by the same factor as the Doppler width or by more than 20 and the longitudinal emittance is reduced by the velocity ratio between room temperature and 1 K or in total by a factor greater than 50. Altogether this corresponds to a phase space improvement by a factor greater than 40,000.

The muon beam is extracted without loss in phase space by applying a short pulsed electric field in the vertical direction after the ionization: a precisely positioned fine grid placed 2 mm above the converter is pulsed for a short time to impart to the muons an acceleration (to less than 0.01 eV energy) so that they cross the grid to a region where they get further accelerated.

Altogether it is necessary to check thoroughly the possible sources of phase space blow-up. The muon recoil momenta from photon absorbtion (and re-emission), it is still far below the transverse momentum spread; but from the ionization, a recoil energy of 1/200 of the electron energy is induced. In order to make it negligible, the wavelength of the laser inducing ionization has to be precisely set to induce ionization within 0.1 meV above threshold. Electrons with such a low energy will be subjected to the attraction of the free μ^+ layer. At high pulsed muon beam intensities, the very low thermal energy of both muons and electrons make them sensitive to space charge effects. These have to be taken into account in the extraction at high muon pulse intensity to insure a minimum increase in the longitudinal energy spread.

Concluding the first part of this paper, a novel kind of converter for the production of slow muons via the ionization method has been proposed. Based on cold muonium atoms emitted from superfluid helium, it should lead to a great improvement in ionization efficiency and in final muon beam quality. A dedicated experimental program will determine the feasibility and the achievable performances. The option with the added porous membrane may be technically challenging, but the effort invested may be more rewarding than those needed to increase the Lyman α laser intensity by a factor of 100. Also the prospect of a much higher beam quality should be an incentive to implement this kind of converter, as it extends significantly the potential of the slow muon beam for both μSR and fundamental experiments.

3. Application to an incoming slow DC muon beam

In this section we discuss the extension of the above considerations to an incoming muon beam which is itself already a high quality slow muon beam. If the muonium emission can be limited to a small enough area, it may be possible to induce ionization with strongly focused CW laser beams and apply the muonium ionization method to beams produced by DC accelerators as the one available at PSI. The requirement of most μSR experiments to know the time when the muon enter the specimen will be fulfilled by detecting the electron ejected during ionization. The electron signal would also be used to trigger the application of a pulsed electric field at the μ^+ site to impart to the ultra-slow μ^+ a fixed longitudinal momentum and obtain an ultra-quality muon beam of variable energy.

The incoming muon beam will be taken here to be the outcome of the intense slow muon production technique recently proposed [8] and under development at PSI. It is based on stopping a standard muon beam in a long helium gas target in a longitudinal high magnetic field and compressing the stop volume via adequate electric fields until it can exit a hole of 1 mm diameter in a side wall of the target. Once in vacuum, the muons are extracted by a pulsed electric field and accelerated to form a high quality beam that can be focused into a small spot. For its use in μSR experiments or a novel concept of the g–2 experiment [19], the timing information can be provided either by the extraction time [8] or by a subsequent window–free gas detector which provides a precise muon timing with limited loss in phase space [18]. For the present application this timing is not required, and a fast pulsed extraction can be used that results in a quasi-continuous slow muon beam of the highest quality.

The muon source has, before it is extracted from the magnetic field, a transverse size of 0.5 mm x 1 mm and an average energy of 1 eV [8] [18]. After extraction with phase space conservation and acceleration, it is rotated into the vertical direction and focused into a thin superfluid helium target so that it stops in the upper few micrometer near the surface. Optimal focusing is provided by a superconducting magnetic lens which should be shielded in order to hinder its magnetic field from reaching the target. Under these conditions the beam will be focused onto a spot of 0.05 mm x 0.1 mm. With no electric field applied, the stopped μ^+ is attracted by a nearby track electron and

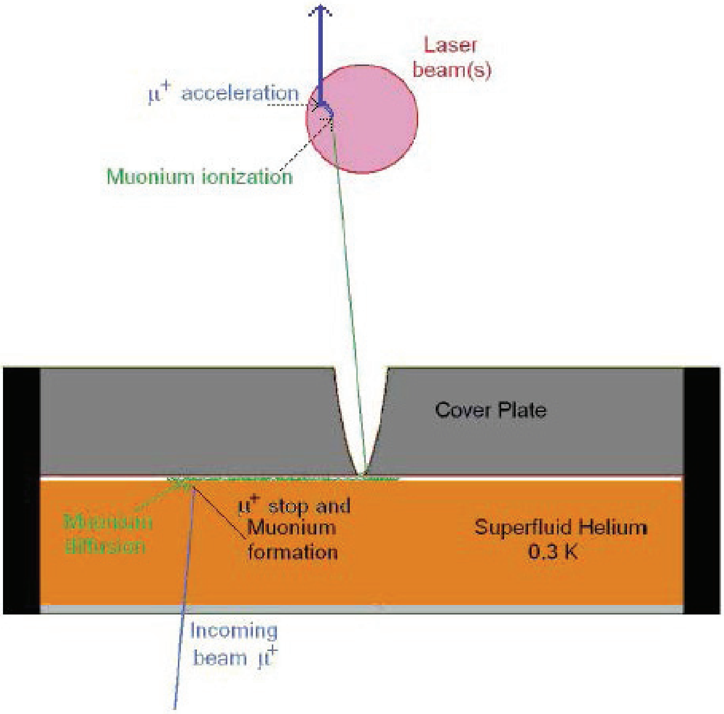

Figure 2: Sketch of ultra-slow muonium formation. View perpendicular to the laser beam of 20 μm waist. The muonium diffusion in vacuum above the liquid helium below the cover plate shown in the figure is just an illustration of the large number of surface collisions taking place (a few 100) before the slowed-down muonium atom is finally emitted through the parabolic shaped cover plate and directed towards the laser.

neutralizes [13]. The formed muonium diffuses by collisions with the ^{3}He impurity atoms whose concentration is selected so that the muonium atoms predominantly reach the close–by liquid–vacuum interface. As in the previous section the "energetic" muonium atoms emitted will be slowed down to thermal energies by collision with helium films. An adapted configuration will allow to reduce the emission area. It uses a flat horizontal thin plate placed on the top of the helium surface at a short distance. Being in contact with the target at its edges, it becomes covered with a superfluid helium layer. A thin gap (less that 1 μm) should separate the two opposite helium surfaces, so that the emitted muonium atom will undergo multiple surface collisions (inelastic or specular as explained above), slow down and diffuse in the horizontal direction. At the center of the plate, a thin slit opening is cut in the plate from which the muonium atoms can escape after an average large number of collisions. A reduction of the effective final emission area by a factor greater 100 can be achieved so that a slit width of less than 1 μm will reduce the 100 μm stopping width into a less than 1 μm wide ultra–slow muonium emission. With a of the slit opening having the shape of a parabola cut within the plate and the slit opening placed at its facal point the exiting ultra–slow muonium atoms are focused by specular reflection on the helium coated walls to reach the laser ionization region of 50 μm length and 15 μm width. A sketch of the process is shown in Fig. 2.

Because no CW Lyman α laser is available with sufficient intensity, a different ionization pathway from the one considered above will be used. There will first be a 1S–2S Doppler–free two–photon transition (induced by a laser with 5.1 eV photon energy in a resonant cavity) followed by a 2S-continuum laser induced transition. This pathway have been realized in the few completed 1S–2S spectroscopy experiments either with pulsed or continuous lasers [22, 23]. In these experiments, the second transition is induced by the laser inducing the 1S–2S transition itself in an average time that is half the average time needed by the first transition resulting in a very efficient ionization. As the binding energy of the 2S state is 3.4 eV the outcome is an electron emitted with a 1.7 eV energy and a muon with a recoil energy of 8.2 meV or about 100 K.

The horizontal laser beam with a waist of 20 μm provides a cylindrical ionization volume that will be crossed by the emitted muonium beam. Sufficient distance from the emission slit insures minimum losses in the laser resonant cavity. The muonium atoms enter the laser beam within a few 100 ns after their emission. They spend an average time of 60 ns inside the laser covered volume. Within this time they have to be excited and ionized. A recent calculation has shown that at a laser fluence of 2.3 MW/m^2 for a time of 5 ms ionizes more than 90 % of the atoms. This translates into a fluence of 190 GW/m^2 for the 60 ns available time. Multiplying this fluence by the beam area (taken to be a circle whose diameter is the beam waist) gives 60 W of laser power to be contained in the resonant cavity. With high reflectivity UV laser mirrors available today, a laser resonant cavity with an enhancement factor exceeding 200 can be used. The required external laser power at 244 nm is less than 300 mW . Intensity of 50 mW have been demonstrated at the close-by wavelength of 252 nm by tripling the frequency of a 500 mW Ti:sapphire CW laser [20]. Also, stabilization to a linewidth as small as 10 kHz has been demonstrated at a Ti:Sa power as high as 2 W [21]. It appears therefore that available laser systems are not far from doing the job.

We will first discuss the case where the 2S–ionization transition is induced by the 5.1 eV photons of the laser that induces the two–photon 1S–2S transition. Besides the resulting increase by a factor of 100 of the muon kinetic energy due to muon recoil, the effective size of the muon source increases to a significantly higher value than the basic volume of 15 μmm x 20 μm x 50 μm. This is so because in order to use a pulsed electric field for the formation of the muon beam, and also in order to have a timed source, the ionization electron need to be used to trigger the extracting electric field. The electron is detected with high efficiency as, even in the case it hits the helium coated plate, it will be backscattered with a high probability. The electron is collected above the laser, and in an electric field slowly increasing with distance, it accelerates to a few keV energy before it hits a scintillator viewed by a high gain fast microchannel which gives an output signal within 5 ns. This signal triggers the extracting electric field. Assuming the whole operation takes place in a time as short as 10 ns, the muon, with its 100 K recoil energy has moved by 40 μm so that at extraction time the effective full volume of the source is 95 μm x 100 μm x 130 μm. The procedure caused therefore a phase space blow–up because of the increase in both momentum and spacial spread. Nevertheless the improvement factor in phase space density compared to the initial slow muon source is of the order of 10^5.

The following improved configuration will eliminate the above degradation. A second laser with 366 nm wavelength is introduced and made to nearly overlap the first laser. It induces the transition from the 2S state to a ionized level of energy just above threshold with an ionization cross-section that is more than 10 times greater than the laser at 244 nm. Therefore a laser fluence at 366 nm that is smaller than that at 244 nm is sufficient to force most of the ionizations to take place at threshold. The outcome is a very small electron energy and a negligible muon recoil. A very small DC electric field is applied to collect the electron without affecting much the muon, until the signal from the electron detector induces the pulsed electric field for the muon extraction.

The phase space compression factor resulting from the operation can be calculated to exceed 10^{10}. The timed muon source obtained this way can, after acceleration, be focused on tiny spots. Given the source properties, the beam transverse emittances will be asymmetric because of the slit shaped emission and the parabolic focusing in the direction perpendicular to the slit. However, emittance transfer techniques can be used. For example, in the application to a muon microscope operated at energies over 10 keV, the intrinsic longitudinal emittance of the beam need not to be so good, so that the larger transverse emittance in one direction can be reduced at the expense of the longitudinal emittance. This is done by letting the beam pass a 90° dipole magnet that converts beam width into time delay. The resulting beam can be focused in a microscope with superconducting lenses onto spots of 10 nm diameter. Clearly the realization of such an ultra-high quality beam has to wait for the completion of the slow beam used as the entrance beam which itself is still in its initial phase. Nevertheless, awareness of this possible extension to a beam quality far better than what has been dreamed about until now should significantly increase the interest in development of the project.

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