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Black hole conserved charges in Generalized Minimal Massive Gravity

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ABSTRACT

In this paper we construct mass, angular momentum and entropy of black hole solution of Generalized Minimal Massive Gravity (GMMG) in asymptotically Anti-de Sitter (AdS) spacetimes. The Generalized Minimal Massive Gravity theory is realized by adding the CS deformation term, the higher derivative deformation term, and an extra term to pure Einstein gravity with a negative cosmological constant. We apply our result for conserved charge $Q^{\mu}(\bar{\xi})$ to the rotating BTZ black hole solution of GMMG, and find energy, angular momentum and entropy. Then we show that our results for these quantities are consistent with the first law of black hole thermodynamics.

1. Introduction

We know that the pure Einstein–Hilbert gravity in three dimensions exhibits no propagating physical degrees of freedom [1,2]. But adding the gravitational Chern–Simons term produces a propagating massive graviton [3]. The resulting theory is called Topologically Massive Gravity (TMG). Including a negative cosmological constant yields Cosmological Topologically Massive Gravity (CTMG). In this case the theory exhibits both gravitons and black holes. Unfortunately there is a problem in this model: with the usual sign for the gravitational constant, the massive excitations of CTMG carry negative energy. In the absence of a cosmological constant, one can change the sign of the gravitational constant, but if $\Lambda < 0$, this will give a negative mass to the BTZ black hole, so the existence of a stable ground state is in doubt in this model [4]. Recently an interesting three dimensional massive gravity has been introduced by Bergshoeff et al. [5], dubbed Minimal Massive Gravity (MMG), which has the same minimal local structure as Topologically Massive Gravity (TMG) [3]. The MMG model has the same gravitational degree of freedom as the TMG has and the linearization of the metric field equations for MMG yields a single propagating massive spin-2 field. So both models have the same spectrum [6]. However, in contrast to TMG, there is no bulk vs boundary clash in the framework of this new model. During last months some interesting works have been done on MMG model [6]. More recently, this model has been extended to Generalized Minimal Massive Gravity (GMMG) theory [7]. The GMMG is a unification of MMG with New Massive Gravity (NMG) [8], so this model is realized by adding the higher derivative deformation term to the Lagrangian of MMG.

In this paper we want to construct mass, angular momentum and entropy of black hole solution of GMMG in asymptotically Anti-de Sitter (AdS) spacetimes. There are several approaches to obtain mass and angular momentum of black holes for higher curvature theories [9–25]. The Arnowitt–Deser–Misner (ADM) method [10] uses a linearization of metric around asymptotically flat spacetime, so this approach fails here because we consider the solution which is not asymptotically flat. A method to calculate the energy of asymptotically AdS solution was given by Abbott and Deser [9]. Deser and Tekin have extended this approach to the calculation of the energy of asymptotically dS or AdS solutions in higher curvature gravity models and also to TMG [12]. In contrast to the ADM method, this ADT formalism is covariant. Another method is the Brown–York formalism [13] which is based on quasi-local concept, but this approach also is not covariant. The authors of [14] have obtained the quasi-local conserved charges for black holes in any diffeomorphically invariant theory of gravity. By considering an appropriate variation of the metric, they have established a one-to-one correspondence between the ADT approach and the linear Noether expressions. They have extended this work to a theory of gravity containing a gravitational Chern–Simons term in [15], and have computed the off-shell potential and quasi-local conserved charges of some black holes in TMG. We should mention that before these works, the authors of [16] have computed the ADT charges for a solution of TMG linearized about an arbitrary background and have applied the result to evaluate the mass and angular momentum of the non-asymptotically flat, non-asymptotically AdS black hole solution (ACL black hole) of

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TMG. Another way to obtain mass and angular momentum of black holes for higher curvature models in the case of asymptotically AdS space is application of AdS/CFT correspondence, because the definition of conserved charges in the dual field theory is clear and there are no ambiguities in their construction [20,22–24]. This method is covariant and takes into account the non-linear effects. However this formalism is applicable only to asymptotically (warped) AdS space. Moreover to obtain holographic conserved charges, one needs the boundary stress tensor which depends on the explicit form of Gibbons–Hawking terms [26] and counter term which are not known in general. So this approach becomes complicated for a higher derivative model of gravity [17]. Another way is the super-angular momentum approach [18,19]; by this method one can compute the conserved charges with non-linear effects. In this formalism the non-linear conserved charge is obtained by the first integral of equation of motion. However this method is not completely covariant, moreover it is inconsistent with the first law of black hole thermodynamics for warped AdS_3 black hole solution of TMG. Here we follow the method given by Abbott, Deser, and Tekin in [9,11,12], which needs to obtain the field equations and linearize them about the (A)dS vacuum of the model. By this method we obtain conserved charges which are consistent with the first law of black hole thermodynamics.

Our paper is organized as follows. In Section 2 we review the GMMG briefly. In Section 3 we will obtain the formula for the calculation of conserved charges in this model in asymptotically AdS_3 spacetime. Then we apply our result for conserved charge $Q^{\mu}(\bar{\xi})$ to the rotating BTZ black hole solution of GMMG, and find energy, angular momentum and entropy. Section 4 is devoted to conclusions and discussions.

2. The Generalized Minimal Massive Gravity

We introduce the Lagrangian of GMMG model as [7]

$$L_{GMMG} = L_{GMG} + \frac{\alpha}{2} e.h \times h \tag{1}$$

where

$$L_{GMG} = L_{TMG} - \frac{1}{m^2}(f.R + \frac{1}{2} e.f \times f) \tag{2}$$

here m is a mass parameter of NMG term and f is an auxiliary one-form field. L_{TMG} is the Lagrangian of TMG,

$$L_{TMG} = -\sigma e.R + \frac{\Lambda_0}{6} e.e \times e + h.T(\omega) + \frac{1}{2\mu}(\omega.d\omega + \frac{1}{3}\omega.\omega \times \omega) \tag{3}$$

where Λ_0 is a cosmological parameter with dimension of mass squared, and σ a sign. μ is a mass parameter of Lorentz Chern–Simons term. α is a dimensionless parameter, e is a dreibein, h is the auxiliary field, ω is a dualized spin-connection, $T(\omega)$ and $R(\omega)$ are a Lorentz covariant torsion and a curvature 2-form respectively. So by adding extra term $\frac{\alpha}{2} e.h \times h$ to the Lagrangian of generalized massive gravity we obtain the Lagrangian of GMMG model. The equation for the metric can be obtained by generalizing field equation of MMG. Due to this we introduce the GMMG field equation as follows [7]

$$\Phi^{\mu\nu} = \Lambda_0 g^{\mu\nu} + \sigma G^{\mu\nu} + \frac{1}{\mu} C^{\mu\nu} + \frac{\gamma}{\mu^2} J^{\mu\nu} + \frac{s}{2m^2} K^{\mu\nu} = 0, \tag{4}$$

where $G^{\mu\nu}$ is Einstein's tensor, the Cotton tensor is

$$C^{\mu\nu} = \frac{1}{\sqrt{-g}} \varepsilon^{\mu\alpha\beta} \nabla_\alpha S^{\nu}_{\beta}, \tag{5}$$

where $S^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{4}\delta^{\mu}_{\nu} R$ is the Schouten tensor in 3 dimensions,

$$J^{\mu\nu} = \frac{1}{2g} \varepsilon^{\mu\rho\sigma} \varepsilon^{\nu\alpha\beta} S_{\rho\alpha} S_{\sigma\beta}, \tag{6}$$

and

$$K^{\mu\nu} = 2\Box R^{\mu\nu} - \frac{1}{2}\nabla^{\mu}\nabla^{\nu} R - \frac{1}{2} g^{\mu\nu}\Box R - 8R^{\mu\alpha}R^{\nu}_{\alpha} + \frac{9}{2} R R^{\mu\nu} + 3g^{\mu\nu} R^{\alpha\beta} R_{\alpha\beta} - \frac{13}{8} g^{\mu\nu} R^2, \tag{7}$$

s is sign, γ, σ, Λ_0 are the parameters defined in terms of cosmological constant $\Lambda = \frac{-1}{l^2}$, m, μ, and the sign of Einstein–Hilbert term. Here G_{mn} and C_{mn} denote the Einstein tensor and the Cotton tensor respectively. Symmetric tensors J_{mn} and K_{mn} are coming from MMG and NMG parts respectively [8,5].

3. Charges of GMMG

In this section we would like to obtain the conserved charges of GMMG for asymptotically (A)dS space-times. Here we follow the method given in [9,11,12] (see also [27]), which needs to obtain the field equations and linearize them about the (A)dS vacuum of the model.[1]

The field equations of the model can be written as

$$\Phi_{\mu\nu}(g, R, \nabla(Riemann), R^2, \ldots) = 8\pi T_{\mu\nu}. \tag{8}$$

We assume that (A)dS is the background solution $\Phi_{\mu\nu}(\bar{g}) = 0$. The linearized form of the above equation can be written symbolically as

$$\mathcal{O}(\bar{g})_{\mu\nu\alpha\beta} h^{\alpha\beta} = 8\pi T_{\mu\nu}, \tag{9}$$

where the deviation of background is $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$. If the field equation (8) comes from a diffeomorphism invariant action, then we have

$$\nabla_\mu \Phi^{\mu\nu} = 0. \tag{10}$$

From (9) and (10) we have

$$\bar{\nabla}_\mu \mathcal{O}^{\mu}{}_{\nu\alpha\beta} h^{\alpha\beta} = 0. \tag{11}$$

In order to define globally conserved charges, we use the Killing vector $\bar{\xi}$, and energy–momentum tensor $T^{\mu\nu}$. Then we can define conserved current as

$$\sqrt{-\bar{g}}\bar{\nabla}_\mu(\bar{\xi}_\nu T^{\mu\nu}) = \partial_\mu(\sqrt{-\bar{g}}\bar{\xi}_\nu T^{\mu\nu}) = 0. \tag{12}$$

So we obtain conserved charge by

$$Q^{\mu}(\bar{\xi}) = c \int_{\mathcal{M}} d^{D-1}x \sqrt{-\bar{g}} \bar{\xi}_\nu T^{\mu\nu} \equiv c \int_{\Sigma} dl_i \mathcal{F}^{\mu i}, \tag{13}$$

where we have used Stoke's theorem, c is an arbitrary constant, and $\mathcal{M}$ is a $(D-1)$-dimensional spatial manifold with boundary Σ. We have assumed that $\bar{\xi}_\nu T^{\mu\nu} = \bar{\nabla}_\nu \mathcal{F}^{\mu\nu}$, where $\mathcal{F}^{\mu\nu}$ is an anti-symmetric tensor. For the background metric we have:

[1] Here we should mention that although we obtain the conserved charges from the linearization of the field equations around a background, the method is general. There is no dependence on background (except that there must be asymptotic Killing vectors and spatial infinity of course) in the method of [11]. Multiple vacua are universal features of all $R + R^2$ etc. models, since they clearly allow both flat and (A)dS vacua, but energy is still definable around each branch, though one is unstable (see for example [28]).

$$\bar{R}_{\mu\alpha\nu\beta} = \Lambda\left(\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} - \bar{g}_{\mu\beta}\bar{g}_{\nu\alpha}\right),$$
$$\bar{R}_{\mu\nu} = 2\Lambda\bar{g}_{\mu\nu}, \qquad \bar{R} = 6\Lambda. \tag{14}$$

Now we obtain the linearized form of field equation (4) around the AdS_3 space-time. So at the first order the field equation can be written as

$$\Phi^{\mu\nu}(\bar{g}) + \Phi^{\mu\nu}_L = 8\pi T^{\mu\nu}, \tag{15}$$

where

$$\Phi^{\mu\nu}_L = -\Lambda_0 h^{\mu\nu} + \sigma G^{\mu\nu}_L + \frac{1}{\mu}C^{\mu\nu}_L + \frac{\gamma}{\mu^2}J^{\mu\nu}_L + \frac{s}{2m^2}K^{\mu\nu}_L, \tag{16}$$

$$\mathcal{G}^L_{\mu\nu} = R^L_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}R^L - 2\Lambda h_{\mu\nu}, \tag{17}$$

$$C^{\mu\nu}_L = \frac{1}{\sqrt{-\bar{g}}}\varepsilon^{\mu\alpha\beta}\bar{g}_{\beta\sigma}\nabla_\alpha\left(R^{\sigma\nu}_L - \frac{1}{4}\bar{g}^{\sigma\nu}R_L + 2\Lambda h^{\sigma\nu}\right), \tag{18}$$

$$J^{\mu\nu}_L = -\frac{1}{2}\Lambda\mathcal{G}^{\mu\nu}_L - \frac{1}{4}\Lambda^2 h^{\mu\nu}, \tag{19}$$

$$K^{\mu\nu}_L = 2\bar{\Box}\mathcal{G}^{\mu\nu}_L + \frac{1}{2}\bar{g}^{\mu\nu}\bar{\Box}R_L - \frac{1}{2}\bar{\nabla}^\mu\bar{\nabla}^\nu R_L$$
$$- 5\Lambda\mathcal{G}^{\mu\nu}_L - \Lambda\bar{g}^{\mu\nu}R_L + \frac{1}{2}\Lambda^2 h^{\mu\nu}. \tag{20}$$

Here we have defined that $\mathcal{G}_{\mu\nu} = G_{\mu\nu} + \Lambda g_{\mu\nu}$. The linear forms of Ricci tensor and Ricci scalar are given by the following equations respectively

$$R^L_{\mu\nu} = \frac{1}{2}\left(-\bar{\Box}h_{\mu\nu} - \bar{\nabla}_\mu\bar{\nabla}_\nu h + \bar{\nabla}^\lambda\bar{\nabla}_\mu h_{\lambda\nu} + \bar{\nabla}^\lambda\bar{\nabla}_\nu h_{\lambda\mu}\right), \tag{21}$$

$$R^L = -\bar{\Box}h + \bar{\nabla}_\mu\bar{\nabla}_\nu h^{\mu\nu} - 2\Lambda h. \tag{22}$$

Using the Ricci tensor and the Ricci scalar of AdS_3 background in (14), it is easy to see that

$$\bar{G}^{\mu\nu} = -\Lambda\bar{g}^{\mu\nu}, \qquad \bar{C}^{\mu\nu} = 0,$$
$$\bar{J}^{\mu\nu} = \frac{1}{4}\Lambda^2\bar{g}^{\mu\nu}, \qquad \bar{K}^{\mu\nu} = -\frac{1}{2}\Lambda^2\bar{g}^{\mu\nu}. \tag{23}$$

Then field equation for AdS_3 reduces to a quadratic equation for

$$\Lambda_0 - \sigma\Lambda + \frac{\gamma\Lambda^2}{4\mu^2} - \frac{s\Lambda^2}{4m^2} = 0, \tag{24}$$

so

$$\Lambda = \frac{\left(\sigma \pm \sqrt{\sigma^2 - \Lambda_0(\frac{\gamma}{\mu^2} - \frac{s}{m^2})}\right)}{\frac{1}{2}(\frac{\gamma}{\mu^2} - \frac{s}{m^2})}. \tag{25}$$

Since $\Phi_{\mu\nu}(\bar{g}) = 0$, (15) takes the following form

$$\Phi^{\mu\nu}_L = 8\pi T_{\mu\nu}. \tag{26}$$

Substituting (17)–(20) and (24) in (26), we have

$$\left(\sigma\Lambda - \frac{\gamma\Lambda}{2\mu^2}\right)\mathcal{G}^{\mu\nu}_L + \frac{1}{\mu}C^{\mu\nu}_L + \frac{s}{2m^2}\left(K^{\mu\nu}_L - \frac{1}{2}\Lambda^2 h^{\mu\nu}\right)$$
$$= 8\pi T^{\mu\nu}. \tag{27}$$

One can show that $\bar{\nabla}_\nu\mathcal{G}^{\mu\nu}_L = \bar{\nabla}_\nu C^{\mu\nu}_L = 0$, then by the following identities [12]

$$\bar{\nabla}_\nu\left[(\bar{g}^{\mu\nu}\bar{\Box} - \bar{\nabla}^\mu\bar{\nabla}^\nu + 2\Lambda\bar{g}^{\mu\nu})R_L\right] = 0,$$
$$\bar{\nabla}_\nu\left[\bar{\Box}\mathcal{G}^{\mu\nu}_L - \Lambda\bar{g}^{\mu\nu}R_L\right] = 0, \tag{28}$$

we conclude that

$$\bar{\nabla}_\nu\left(K^{\mu\nu}_L - \frac{1}{2}\Lambda^2 h^{\mu\nu}\right) = 0. \tag{29}$$

So (27) obeys the Bianchi identities, and we can use (27) for definition of conserved charges. One can check that [12,29]

$$\sqrt{-\bar{g}}\bar{\xi}_\nu\mathcal{G}^{\mu\nu}_L = \frac{1}{2}\partial_\nu q^{\mu\nu}_E(\bar{\xi}),$$
$$\sqrt{-\bar{g}}\bar{\xi}_\nu C^{\mu\nu}_L = \frac{1}{2}\partial_\nu\left(\frac{1}{2\mu}q^{\mu\nu}_E(\bar{\Xi}) + \frac{1}{2\mu}q^{\mu\nu}_C(\bar{\xi})\right),$$
$$\sqrt{-\bar{g}}\bar{\xi}_\nu\left(K^{\mu\nu}_L - \frac{1}{2}\Lambda^2 h^{\mu\nu}\right) = \frac{1}{2}\partial_\nu\left(q^{\mu\nu}_N(\bar{\xi}) - \Lambda q^{\mu\nu}_E(\bar{\xi})\right), \tag{30}$$

where

$$q^{\mu\nu}_E(\bar{\xi}) = 2\sqrt{-\bar{g}}\Big(\bar{\xi}_\lambda\bar{\nabla}^{[\mu}h^{\nu]\lambda} + \bar{\xi}^{[\mu}\bar{\nabla}^{\nu]}h + h^{\lambda[\mu}\bar{\nabla}^{\nu]}\bar{\xi}_\lambda$$
$$+ \bar{\xi}^{[\nu}\bar{\nabla}_\lambda h^{\mu]\lambda} + \frac{1}{2}h\bar{\nabla}^\mu\bar{\xi}^\nu\Big),$$
$$q^{\mu\nu}_C(\bar{\xi}) = \varepsilon^{\mu\nu}{}_\alpha\mathcal{G}^{\alpha\beta}_L\bar{\xi}_\beta + \varepsilon^{\beta\nu}{}_\alpha\mathcal{G}^{\mu\alpha}_L\bar{\xi}_\beta + \varepsilon^{\mu\beta}{}_\alpha\mathcal{G}^{\alpha\nu}_L\bar{\xi}_\beta,$$
$$q^{\mu\nu}_N(\bar{\xi}) = \sqrt{-\bar{g}}\Big[4\left(\bar{\xi}_\lambda\bar{\nabla}^{[\nu}\mathcal{G}^{\mu]\lambda}_L + \mathcal{G}^{\lambda[\nu}_L\bar{\nabla}^{\mu]}\bar{\xi}_\lambda\right)$$
$$+ \bar{\xi}^{[\mu}\bar{\nabla}^{\nu]}R_L + \frac{1}{2}R_L\bar{\nabla}^\mu\bar{\xi}^\nu\Big], \tag{31}$$

also $\bar{\Xi}^\beta = \frac{1}{\sqrt{-\bar{g}}}\varepsilon^{\alpha\lambda\beta}\bar{\nabla}_\alpha\bar{\xi}_\lambda$. Using (30), we can rewrite (27) as

$$16\pi\sqrt{-\bar{g}}\bar{\xi}_\nu T^{\mu\nu} = \partial_\nu\Big[\left(\sigma - \frac{\gamma\Lambda}{2\mu^2} - \frac{s\Lambda}{2m^2}\right)q^{\mu\nu}_E(\bar{\xi})$$
$$+ \frac{1}{2\mu}q^{\mu\nu}_E(\bar{\Xi}) + \frac{1}{2\mu}q^{\mu\nu}_C(\bar{\xi}) + \frac{s}{2m^2}q^{\mu\nu}_N(\bar{\xi})\Big], \tag{32}$$

substituting this result in (13), we obtain

$$Q^\mu(\bar{\xi}) = \frac{c}{16\pi}\int_\Sigma dl_i\Big[\left(\sigma - \frac{\gamma\Lambda}{2\mu^2} - \frac{s\Lambda}{2m^2}\right)q^{\mu i}_E(\bar{\xi}) + \frac{1}{2\mu}q^{\mu i}_E(\bar{\Xi})$$
$$+ \frac{1}{2\mu}q^{\mu i}_C(\bar{\xi}) + \frac{s}{2m^2}q^{\mu i}_N(\bar{\xi})\Big], \tag{33}$$

where i denotes the space direction orthogonal to the boundary Σ.

In the limiting case $\frac{1}{m^2} \to 0$, where GMMG reduces to the MMG, our result for conserved charge $Q^\mu(\bar{\xi})$ in the above equation reduces to the result of [29] for MMG. Now we apply Eq. (33) to the rotating BTZ black hole solution of GMMG, in order to obtain the energy, angular momentum and entropy of this black hole. The BTZ line-element is

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{l^2r^2}dt^2 + \frac{l^2r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2$$
$$+ r^2\left(d\phi - \frac{r_+r_-}{lr^2}dt\right)^2, \tag{34}$$

where $\Lambda = -\frac{1}{l^2}$, also r_+ and r_- are the outer and inner horizons respectively. The case of $r_+ = r_- = 0$ corresponds to the background. Then one can read that

$$h_{tt} = \frac{r_+^2 + r_-^2}{l^2}, \qquad h_{t\phi} = -\frac{r_+r_-}{l}, \qquad h_{rr} = \frac{l^2(r_+^2 + r_-^2)}{r^4}, \tag{35}$$

Σ is a circle and $dl_i = (d\phi, 0)$.

Energy corresponds to the Killing vector $\bar{\xi} = \partial_t$ and $c = -8$, then

$$E = \left(\sigma + \frac{\gamma}{2\mu^2 l^2} + \frac{s}{2m^2 l^2}\right)\frac{r_+^2 + r_-^2}{l^2} - \frac{2r_+ r_-}{\mu l^3}. \tag{36}$$

On the other hand angular momentum of BTZ black hole corresponds to the Killing vector $\bar{\xi} = \partial_\phi$ and $c = 8$, so we have

$$J = \left(\sigma + \frac{\gamma}{2\mu^2 l^2} + \frac{s}{2m^2 l^2}\right)\frac{2r_+ r_-}{l} - \frac{r_+^2 + r_-^2}{\mu l^2}. \tag{37}$$

If one writes the metric of rotating BTZ black hole in terms of mass M and angular momentum parameter a, the above expression for energy E and angular momentum J can be rewritten as

$$E = \left[(\sigma - \frac{\gamma\Lambda}{2\mu^2} - \frac{s\Lambda}{2m^2})M + \frac{\Lambda a}{\mu}\right], \tag{38}$$

$$J = \left[(\sigma - \frac{\gamma\Lambda}{2\mu^2} - \frac{s\Lambda}{2m^2})a - \frac{M}{\mu}\right]. \tag{39}$$

The above equations reduce to the corresponding results for MMG in the limit $\frac{1}{m^2} \to 0$ [29]. If we take $\bar{\xi} = \partial_t + \frac{r_-}{l r_+}\partial_\phi$ and $c = -\frac{32\pi}{\kappa}$, where $\kappa = \frac{r_+^2 - r_-^2}{l^2 r_+}$ is a surface gravity, then

$$S = 4\pi\left[\left(\sigma + \frac{\gamma}{2\mu^2 l^2} + \frac{s}{2m^2 l^2}\right)r_+ - \frac{r_-}{\mu l}\right]. \tag{40}$$

One can check that these results satisfy the first law of thermodynamics, that is

$$dE = T_H dS + \Omega_H dJ, \tag{41}$$

where $T_H = \frac{\kappa}{2\pi}$ and $\Omega_H = \frac{r_-}{l r_+}$.

4. Conclusion

In this paper we have investigated the Abbott–Deser–Tekin charge construction in the framework of Generalized Minimal Massive Gravity in asymptotically AdS space-time. We have applied our result for conserved charge $Q^\mu(\bar{\xi})$ in Eq. (33) to the rotating BTZ black hole solution of GMMG. In the limit $\frac{1}{m^2} \to 0$, where GMMG reduces to the MMG model, our conserved charge $Q^\mu(\bar{\xi})$ reduces to the corresponding result for MMG, which has been obtained in [29]. By this method and correspondence to the Killing vector fields $\bar{\xi} = \partial_t$ and $\bar{\xi} = \partial_\phi$, we have obtained energy and angular momentum of rotating BTZ black hole respectively. After that by considering the Killing vector field $\bar{\xi} = \partial_t + \frac{r_-}{l r_+}\partial_\phi$ we have obtained the entropy of BTZ black hole. Then we have shown that our result for entropy is consistent with the first law of black hole thermodynamics.

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