XIIIth International Scientific and Engineering Conference “HERVICON-2011”

Calculation Method of the Improved Design of Automatic Balancing Device with Hydraulically Unloaded Resiliently Mounted Ring

Nataliya Zuyeva\textsuperscript{a, a*}

\textsuperscript{a}Sumy State University, General Mechanics and Machine Dynamics Department, 2, Rimsky-Korsakov Street, Sumy 40007, Ukraine

Abstract

The mathematical model of static and dynamic calculation of the improved design of balancing device with the hydraulically unloaded resiliently mounted ring is represented in work. For the dynamic analysis of system it is necessary to consider the connected oscillations of resiliently mounted ring and disk of balancing device. For the improved design device rigidity of the disk is much more than rigidity of the resiliently mounted ring. Therefore at calculation of the balancing device special attention is to be paid to dynamic analysis of axially moving ring. The forced angular oscillations of ring, which are caused by a possible misalignment of the rotating disk, are also considered.

© 2011 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of Sumy State University Open access under CC BY-NC-ND license.

Keywords: Hydrodynamic characteristics; face throttle; automatic balancing device.

In most constructions of high-pressure multistage pumps for unloading of rotor from unstable axial force influencing on its impellers, the special automatic balancing devices are used. Such device along with the function of the thrust bearing with self-adjusting carrying capacity operates as end seal reducing fluid pressure from the discharge pressure to the inlet one [1,2].

Usually the balancing device (fig. 1) includes a stationary disk with a cylindrical bush 2 and the rotating disk 5 rigidly installed on a shaft, which being assembled form sequentially located cylindrical throttle 1 and face throttle 3 separated with chamber 4. The total pressure differential \( \Delta p = p_1 - p_3 \) is a

* Corresponding author. Tel.: +380-0504070983.
E-mail address: zueva@ukr.net.
difference of the discharge pressure $p_1$ and pressure in the chamber behind the balancing device $p_3$. This chamber is mostly connected with the inlet fitting of pump, thus $p_1$ is pressure at inlet. Part of the total pressure differential $\Delta p_2 = p_2 - p_1$ reduces on a face throttle 3, conductivity of which depends on the gap width. When rotor due to the action of excess axial force is displaced to the left, the gap reduces and pressure $p_2$ in the chamber increases restoring equality of force $T$ influencing on the rotor with balancing force $F$. Thus the face throttle 3 is a basic element providing efficiency of balancing device.

![Fig. 1. Scheme of automatic balancing device](image)

For the face throttle of balancing device the pressure differential is generally equal to 10 MPa at that the face gap of balancing device changes from 20 to 200 mkm [1-3]. Rather large pressures lead to deformations of thrust disk of balancing device, and these deformations conduce to appearance of diffusion gap, that reduces hydrostatical axial force acting in face throttle and reduces flow rate. Errors at manufacturing and assembling of balancing device lead to nonflatness of working surfaces. During pump mounting, the disk of balancing device and stationary pad are installed with some misalignment with respect to the shaft rotational axis, that result in increase of average face gap and resistance of the face throttle drops. Bending vibrations of rotor cause the periodic misalignment of unloading disk that conduces to the change of pressure in the chamber of balancing device and the change of balancing force. As a result of this, angular and axial vibrations of the balancing device disk rigidly connected with the rotor may occur, and additional hydrodynamic forces and moments can result in face motion variation and in loss of dynamic stability of balancing device. An initial misalignments and deformations of the rotating and stationary disks influence significantly on static and dynamic characteristics of balancing device. Therefore at designing of the balancing device special attention is to be paid to assurance of face gap flatness [1-4].

The task of balancing device reliability and hermiticity increasing leads to creation of wide range of various modifications. One of them is balancing device with hydraulically unloaded resiliently mounted ring (fig. 2) that gives opportunity to follow the possible misalignment of bearing pad and to save flatness of face gap [3,4]. There appears a possibility for rings’ self-alignment during work of machine. In such modification of the device, the hydrodynamic force acting on the disk of automatic balancing device is created almost fully due to pressure of fluid in the chamber of the balancing device, and force deformations of disk become less significant for regulation of axial force. The increasing of reliability
thanks to the hydrodynamic unloading allows working with less face gaps and, consequently, decreasing flow rate and increasing pump efficiency [3].

Calculation of balancing device as automatic device, which regulates bearing capacity depending on the size of face gap, represents construction of static characteristic - dependence of face gap size on the balanced force \( T \), which acts on the rotor [1]. The desired dependence is determined from the condition of axial equilibrium of balancing device disk (fig. 3):

\[
T = F + F_s,
\]

where: \( F \) - is resulting axial force of pressure, which acts on disk. It includes force of pressure in the balancing device chamber and force of pressure in a face gap, \( F_s = k(\Delta - h_{w0}) \), \( k \) - is resulted constant of squeezer springs, \( \Delta \) - is their pre-compression, \( h_{w0} \) - is a size of face gap.

If we assume that pressure in the balancing device chamber \( p_2 \) is constant, the balancing force will be [5]:

\[
F = p_2 \pi (r_1^2 - r_2^2) + F_m - p_2 \pi (r_1^2 - r_2^2),
\]

where: \( F_m = 2\pi b \left( \frac{p_2 + p_3}{2} + \frac{\Delta p_2}{2} \left[ \frac{3A + 3\bar{\beta}}{3} \frac{\zeta_m}{\zeta_{m0}} + \frac{\zeta_{m0} + \zeta_{m0}'}{\zeta_{m0}} \right] \right) \) - is hydrodynamic force of pressure in the face gap; \( \Delta p_2 = (p_2 - p_3) + 0.3 \rho \omega^2 r_n b \) - is fluid pressure differential on a face throttle when we take into account inertia head pressure; dimensions of face gap: \( r_n = (r_i + r_f)/2 \), \( b = r_f - r_i \), \( A = b^2/2r_n \), \( \bar{\beta} = b\beta/(2h_{w0}) \), \( \beta \) - is angle of taper of face throttle; resistance head coefficients: \( \zeta_{m0} = \zeta_{m0} = \lambda_n b/(2h_{w0}) \), for self-stimulating zone of turbulent flow \( \lambda_n = 0.06 \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \), \( \zeta_{m0} = \zeta_{m0} \).
Fig. 3. Design model of rotor unloading system

Pressure in the unloading chamber $p_2$ depends on the size of face gap $h_{w0}$ and is calculated from balance of flow rates through cylindrical $Q_c$ and face $Q_m$ throttles:

$$Q_c = Q_m,$$  \hspace{1cm} (3)

where: $Q_c = g_c \sqrt{p_1 - p_2}$ and $Q_m = g_m \sqrt{p_2 - p_3}$; $g_c$ and $g_m$ - are conductivities of cylindrical and face throttles [1].

Conductivity of cylindrical throttle $g_c$ depends on the size of radial gap and length of throttle, which are constant. Conductivity of face throttle $g_m$ depends on the size of face gap and sets conditions for dependence of pressure in the unloading chamber $p_2$ on the size of face gap. Conductivities of cylindrical and face throttles for the turbulent flow are the following:

$$g_c = 2 \pi r_1 h_1 \left[0.5 \rho \zeta_{c0}\right]^{0.5} \text{ and } g_m = 2 \pi r_2 h_{w0} \left[0.5 \rho \zeta_{m0}\right]^{0.5},$$  \hspace{1cm} (4)

where: $\zeta_{c0} = \zeta_{c1} + \zeta_{c2} - \zeta_{c12}$ - is coefficient of total losses on a cylindrical throttle; $\zeta_{c1}$ and $\zeta_{c2}$ - are coefficients of losses on the inlet and outlet of cylindrical throttle, and $\zeta_{c2} = \lambda l_1 / (2h_1)$ - is coefficient of losses along the length of channel. For the turbulent flow $\lambda = 0.4$ [1]; $\zeta_{m0}$ - is coefficient of total losses on a face throttle.

To obtain the force deformations, which are conditioned by pressure of working environment, as a first approximation, we assume that pressure in a face throttle changes under the linear law. Then it is possible to determine the dependence of angle of taper $\beta$ from pressure in the balancing device chamber $p_3$. To define the force deformations it is possible to use calculation method of face seal ring deformations [1] or the numerical solution of the given problem. The angle of taper can be calculated on the maximum deformations:

$$\beta = \frac{w_2 - w_3}{b},$$  \hspace{1cm} (5)

where: $w_2$ and $w_3$ - are axial deformations of a face surface on radiiuses $r_2$ and $r_3$ respectively.
Deformations depending on pressure in the balancing device chamber for traditional design device and the device with hydraulically unloaded resiliently mounted ring are represented in figure 4a. For the improved design device the value of taper angle of face gap is less than for the traditional design device. Moreover, selecting radius of installation of a rubber sealing ring it is possible to reduce taper to a minimum and to achieve more flatness of the channel.

Static characteristics are represented in figure 4b. As we can see at any certain value of axial force, the size of a face gap is less for the improved design device. It is connected with the increase of pressure force in a face gap at smaller diffusion channel angle. Thus the device of improved design is more reliable in work at smaller face gap and provides lower flow rate and higher efficiency of the pump.

For the dynamic analysis of system it is necessary to consider the connected oscillations of resiliently mounted ring and disk of balancing device. Dynamic behavior of system for improved design is described by seven connected equations, namely: the equations of axial and angular oscillation of a disk and a ring, and also the equation of balance of the flow rate of a fluid with taking into consideration change of pressure in the balancing device chamber. Last equation defines dependence of pressure in the chamber on the size of a face throttle gap. At small oscillations hydrodynamic axial force depends only on axial displacement, and the moment depends on angular displacement. Small change of pressure in the chamber of balancing device defines change of hydro-dynamic force and the flow rate through a face throttle. Two independent systems are considered: the connected axial oscillations with accounting change of pressure in the balancing device chamber, and angular oscillations of resiliently mounted ring at steady-state pressure in the chamber. The values of steady-state pressure and face gap are defined at static calculation.

\[
Q_c = Q_n + Q_m + Q_p, \quad (6)
\]

Fig. 4. Dependence of deformations (a), axial force and the flow rate (b) on size of a face gap
where: \( Q_z = Q_{z_{01}} \left( 1 - \frac{P_{z_{01}}}{P_1 - P_{z_{01}}} \frac{\xi_{z_{01}}}{\xi_{z_{01}} - \xi_{z_{01}}} \psi_{z_{01}} \right) \), \( Q_{z_{01}} = Q_{z_{01}} \left( 1 + \frac{P_{z_{01}}}{\Delta P_{z_{01}}} \frac{\xi_{z_{01}}}{\xi_{z_{01}} - \xi_{z_{01}}} \psi_{z_{01}} + v_{z_{01}} \mu \right) +
+ 2\pi [v_{z_{01}} - (1 - \lambda)^2] \frac{r_{z_{01}} h_{z_{01}}}{2} \dot{\psi}_{z_{01}} - 2\pi \frac{r_{z_{01}} h_{z_{01}}}{4} \frac{\xi_{z_{01}}}{\xi_{z_{01}} - \xi_{z_{01}}} \psi_{z_{01}} \right),
\( Q_\psi = S_\psi \dot{\psi}_d \).
\[ Q_p = \frac{V_1}{E} \dot{p}_2 = \frac{V_1}{E} \frac{P_{z_{01}}}{\psi_{z_{01}}}; \quad \psi_{z_{01}} = \Delta p_z / P_{z_{01}} \] is relative change of fluid pressure in the chamber; \( \Delta p_{z_{01}} = (P_{z_{01}} - p_1) + \rho \sigma \omega^2 r_n b \) is steady-state differential of fluid pressure on the face throttle with taking into account the inertia head pressure; \( u = \Delta h_{z_{01}} / h_{z_{01}} \) is relative change of size of face gap; \( z_d \) is axial displacement of disk.

Motion equations for disk and ring mounted on balancing device disk (fig.2b) are:
\[ m \dot{z} + c \dot{z} + k \ddot{z} = F_z, \]
where: \( F_z = F_{z_{01}} - A_c \ddot{u} - B_c \dot{u} - C_c u - B_p \dot{\psi}_z - C_p \psi_z \) is hydrodynamic characteristic of face throttle; \( F_{z_{01}} \) is force of steady-state pressure in a face gap; \( m, c, k \) are coefficients which characterize inertial, damping and rigid properties of parts of device; \( A_c, B_c, C_c, B_p, C_p \) are force coefficients which characterize inertial, damping and rigid properties of a fluid layer in face gap; \( u = \xi_z \) is relative change of a face gap value [5].

Amplitude characteristics of axial oscillations of a disk and resiliently mounted ring are represented in figure 5.

Fig. 5. Amplitude-frequency characteristic of axial oscillation of a disk (a) and resiliently mounted ring (b)
Angular rigidity of the disk is much more than angular rigidity of the ring for the design of balancing device with resiliently mounted ring. Therefore it is necessary to pay attention to the forced angular oscillations of ring, which are caused by a possible misalignment of the rotating disk. This problem is completely similar to the problem of the forced oscillations of axially moving ring of face seal. Angular oscillations of the ring are expressed by the following way:

\[
J \ddot{\theta}_r + J_\omega \dot{\theta}_r + c_i \frac{r_m^2}{2} (\theta_r + \omega \theta_r) + k_i \frac{r_m^2}{2} \theta_r = M_s + M_{\alpha},
\]

\[
J \ddot{\theta}_r - J_\omega \dot{\theta}_r + c_i \frac{r_m^2}{2} (\theta_r - \omega \theta_r) + k_i \frac{r_m^2}{2} \theta_r = M_s + M_{\gamma},
\]

where:

\[ M_s = -A_1 \left( \frac{\dot{\theta}_r + \frac{\omega}{2} \dot{\theta}_r - \left( \frac{\omega}{2} \right)^2 \theta_r}{} \right) - B_1 \left( \dot{\theta}_r + \frac{\omega}{2} \dot{\theta}_r \right) - C_1 \theta_r, \]

\[ M_{\alpha} = -A_2 \left( \dot{\theta}_r - 2 \frac{\omega}{2} \dot{\theta}_r - \left( \frac{\omega}{2} \right)^2 \theta_r \right) \]

\[ M_{\gamma} = -B_2 \left( \frac{\dot{\theta}_r - \frac{\omega}{2} \dot{\theta}_r}{} \right) - C_2 \theta_r \]

- are hydrodynamic characteristics of face throttle;
- \( M_s \) is forcing moment caused by misalignment of the rotating disk \( \gamma_s \), which is transferred by rigid elements;
- \( J, c, k \) are coefficients which characterize inertial, damping and rigid properties of parts of device;
- \( A_i, B_i, C_i \) are coefficients which characterize inertial, damping and rigid properties of fluid layer in face gap.

Amplitude characteristic of the forced angular oscillations of the resiliently mounted ring caused by a possible misalignment of rotating disk rigidly connected with rotor is represented in figure 6.

![Amplitude-frequency characteristic of angular oscillation of a resiliently mounted ring](image)
References