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The virial relation for the Q-balls in the thermal logarithmic potential revisited analytically

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ABSTRACT

We study the properties of Q-balls dominated by the thermal logarithmic potential analytically instead of estimating the characters with only some specific values of model variables numerically. In particular, the analytical expressions for radius and energy of this kind of Q-ball are obtained. According to these explicit expressions we demonstrate strictly that the large Q-balls enlarge and the small ones become smaller in the background with lower temperature. The energy per unit charge will not be divergent if the charge is enormous. We find that the lower temperature will lead the energy per unit charge of Q-ball smaller. We also prove rigorously the necessary conditions that the model parameters should satisfy to keep the stability of the Q-balls. When one of model parameters of Q-balls, K, is positive, the Q-balls will not form or survive unless the temperature is high enough. In the case of negative K, the Q-balls are stable no matter the temperature is high or low.

1. Introduction

The Q-balls as nontopological solitons have attracted a lot of attention. This kind of nontopological solitons possess a conserved Noether charge because of a symmetry of their Lagrangian and appear in extended localized solutions of models with certain self-interacting complex scalar field [1,2]. The charge of these nontopological solitons and that their mass is smaller than the mass of a collection of scalar fields keep them stable. In general the Q-balls have self-interaction potential with absolute minima and the shape of potential determines the properties of the Q-ball. The Q-balls have been studied in many areas of physics in order to explain the origin of dark matter and the baryon asymmetry which cannot be explained by the standard model of elementary particle physics. The Affleck–Dine mechanism produces a scalar field condensating with baryon number while generating the baryon asymmetry [3]. These kinds of nontopological solitons may be considered as candidates of dark matter [4]. Recently the scalar field configuration of the Q-ball with a step function was considered to calculate the ratio of the Q-ball decay into the candidates for dark matter [5]. More attention from cosmology was paid to the Q-balls [6–9]. In the process of expanding universe with the sufficiently low temperature the Q-balls build up quickly with absorbing charged particles from the outside to result in a new kind of first-order phase transition [10]. In the cosmological context, the existence of the Q-balls was formulated and these kinds of Q-balls were further discussed to estimate the net baryon number of the universe, its dark matter and the ratio of the baryon to cold dark matter [11]. The solitosynthesis will lead the formation of large Q-balls in the process of graduate charge accretion if some primordial charge asymmetry and initial seed-like Q-balls exist [12]. It was also discussed that the phase transitions induced by solitosynthesis are possible [13]. We also probed the nontopological solitons in de Sitter and anti de Sitter spacetimes respectively to show the constrains from background on the models [14]. The Q-balls can become Boson stars as flat spacetime limits [15]. The compact Q-balls in the complex Signum–Gordon model were also discussed [16]. The Affleck–Dine field fragments into Q-balls which formed in the early universe and change the scenario of Affleck–Dine baryogenesis significantly [17].

A lot of efforts certainly have been contributed to the formation of Q-balls. The Q-ball generates naturally in the context of supersymmetry, in particular, in Affleck–Dine mechanism for baryogenesis [3,18–23]. During the process the homogeneous field as Q-ball solution begins to fluctuate and transforms into lumps. With calculations to the full non-linear dynamics of the complex scalar field, it was shown that the some flat directions consisting of combination of squarks and sleptons carry the baryonic charge in MSSM in the frame of the gravity-mediated supersymmetry breaking scenario [22]. According to the Affleck–Dine baryogenesis in the minimal supersymmetric standard model with gauge-mediated

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supersymmetry breaking, it was found that the Affleck–Dine field is naturally deformed into the form of the Q-ball when the temperature is high [23]. The Q-ball formation in the expanding universe was studied by means of 1D, 2D and 3D lattice simulations respectively [24]. The evolution of universe is associated with change of temperature. The thermal effects may control the potential for Q-balls. It is necessary to research on the Q-ball at finite temperature. The Q-ball formation in the thermal logarithmic potential was investigated in virtue of the lattice simulation [24]. The logarithmic potential appears during the reheating epoch. The evolution of the universe certainly provides the potential with the thermal correction [25]. It is found that the Q-ball subject to the gravity-mediation potential will transform from the thick-wall type to the thin-wall ones as the temperature decreases and will be destroyed at last when the temperature drops sufficiently.

It is significant to investigate the Q-ball in the thermal logarithmic potential by means of virial theorem. This kind of Q-balls evolving in the expanding universe could become candidates for baryon asymmetry and dark matter. The formation and properties of this kind of Q-ball has been estimated numerically [25,26]. The conclusions from numerical estimation are certainly important. It is also fundamental to find the analytical expressions for Q-ball's charge, radius and energy to show how the Q-balls evolve with the change of temperature in detail because the field equations for Q-balls are nonlinear and the reliable and explicit relation among the more model parameters as the existence of Q-balls is difficult to be revealed by performing the burden numerical calculation repeatedly. To our knowledge little contribution was paid. A generalized virial relation for Q-balls with general potential in the spacetime with arbitrary dimensionality was obtained [27]. The analytical description instead of a series of curves for Q-balls was derived [27]. The analytical expressions can show clearly how the model variables influence on the Q-balls and the fate of these kinds of nontopological solitons. Here we will follow the procedure of Ref. [27] to study the Q-balls controlled by the thermal logarithmic potential. We hope to understand how this kind of Q-balls evolve with the decreasing temperature.

In this paper we investigate the Q-ball in the thermal logarithmic potential with virial theorem. It is difficult to solve the field equation of Q-balls with thermal log-type potential. A virial relation for this kind of Q-balls is found. We look for the analytical descriptions of the radius and energy of these Q-balls to estimate their properties in the case of large radius and small ones respectively and to show the relation between their existence and the expansion of the universe. We emphasize the results in the end.

2. The virial relation for Q-balls in the thermal logarithmic potential

We start to consider the Lagrangian density of this system as follows:

$$\mathcal{L} = \partial_\mu \Phi^+ \partial^\mu \Phi - V(\Phi\Phi^+) \tag{1}$$

where $\Phi = \Phi(x)$ is a complex scalar field. The index $\mu = 0, 1, 2, \cdots, d$ and the signature is $(+, -, -, \cdots)$. In the Affleck–Dine scenario the homogeneous field begins rotation with large amplitude in order to fluctuate and transform into lumps, so the two-loop thermal effects on the potential are crucial [28,29]. In the case of large field values, the potential is assumed to be

$$V(\Phi) = V_T(\Phi) + V_m(\Phi) \tag{2}$$

where

$$V_T(\Phi) = T^4 \ln(1 + \frac{|\Phi|^2}{T^2}) \tag{3}$$

$$V_m(\Phi) = m_{\frac{3}{2}}^2 |\Phi|^2 [1 + K \ln(\frac{|\Phi|^2}{M^2})] \tag{4}$$

with a global minimum at $\Phi = 0$ and T is temperature. $m_{\frac{3}{2}}$ is the gravitino mass. The parameter K is negative [19,20,29]. Here M is a normalization scale. This potential admits the formation of Q-balls. Under this potential Q-balls are nonperturbative excitation about this global vacuum state carrying a net particle number called Q which is conserved. Here the energy of the Q-ball E_Q is smaller than Qm_Φ with $m_\Phi^2 = V''(0)$ because the condition will keep the Q-ball to be stable although that is energetically preferred. The Lagrangian of the Q-ball constrained by the thermal logarithmic potential has a conserved $U(1)$ symmetry under the global transformation $\Phi(x) \longrightarrow e^{i\alpha}\Phi(x)$ where α is a constant. The associated conserved current density is defined as $j^\mu \equiv -i(\Phi^+ \partial^\mu \Phi - \Phi \partial^\mu \Phi^+)$ and the corresponding conserved charge can be given by $Q = \int d^d x j^0$. We introduce the ansatz for field configuration with lowest energy,

$$\Phi(x) = \frac{1}{\sqrt{2}} F(\mathbf{r}) e^{i\omega t} \tag{5}$$

Here the field $F(\mathbf{r})$ can be taken to be spherically symmetry meaning $F(\mathbf{r}) = F(r)$ and $\{\mathbf{r}\}$ represents the spatial components of coordinates of coordinates and certainly $r = |\mathbf{r}|$. The field equation for this Q-ball can read,

$$(\nabla_d^2 + \omega^2)F - m_{\frac{3}{2}}^2 K F - \frac{2T^4}{F^2 + 2T^2} F - m_{\frac{3}{2}}^2 (1 + K \ln \frac{F^2}{2M^2}) F = 0 \tag{6}$$

From Lagrangian (1), the total energy of the system is,

$$E[F] = \int d^d x [\frac{1}{2}(\nabla_d F)^2 + \frac{1}{2}\omega^2 F^2 + V(F^2)] \tag{7}$$

According to Ref. [27], the virial relation is a generalization of Derrick's theorem for Q-balls in the $(d+1)$-dimensional spacetime can be expressed as,

$$d\langle V \rangle = (2-d)\langle \frac{1}{2}(\nabla_d F)^2 \rangle + \frac{d}{2}\frac{Q^2}{\langle F^2 \rangle} \tag{8}$$

Since $\langle V \rangle \geq 0$, the absolute lower bound for Q-balls to be a preferred energy state is shown as,

$$Q^2 \geq \frac{2(d-2)}{d} \langle F^2 \rangle \langle \frac{1}{2}(\nabla_d F) \rangle \tag{9}$$

leading to

$$\frac{E}{Q} = \omega(1 + \frac{1}{d - 2 + d\frac{\langle V \rangle}{\langle \frac{1}{2}(\nabla_d F)^2 \rangle}}) \leq m_\phi \tag{10}$$

In the four-dimensional spacetimes, the energy per unit charge can be written as,

$$\frac{E}{Q} = \omega(1 + \frac{1}{1 + \frac{3\langle V \rangle}{\langle \frac{1}{2}(\nabla F)^2 \rangle}}) \tag{11}$$

where $\langle \cdots \rangle = \int \cdots d^3x$ and

$$\langle V \rangle = \int [T^4 \ln(1 + \frac{F^2}{2T^2}) + m_{\frac{3}{2}}(1 + K \ln \frac{F^2}{2M^2}) F^2] d^3x \tag{12}$$

The total charge of the Q-balls is,

$$Q = \omega \int F^2 d^3x \tag{13}$$

The condition (11) is fundamental to Q-balls' stability. Because the potential has terms associated with the temperature, the cosmic temperature decreases and so does the thermal logarithmic potential during the process of the Universe expansion. According to Eqs. (11) and (12) the properties including the structure and stability of thermal log-type Q-ball will change with time.

3. The variational approach for large Q-balls with the thermal logarithmic potential

Here we focus on the Q-balls dominated by the thermal log-type potential in the case of large charge and radius. First of all we make use of the Coleman issue [2] to probe this kind of large Q-balls. We choose the scalar field composing the Q-balls to be a step function which is equal to be a constant denoted as F_c within the model and vanishes outside the ball's volume v. The system energy is,

$$E = \frac{1}{2}\frac{Q^2}{F_c^2 v} + vV(F_c) \tag{14}$$

and $V(F_c) = \frac{1}{2}m_{\frac{3}{2}}{}^2(1 + K\ln\frac{F_c^2}{2M^2})F_c^2 + T^4\ln(1 + \frac{F_c^2}{2T^2})$. Having extremized the expression (14) with respect to the volume v, we obtain the minimum energy per unit charge and the condition for thermal log-type Q-ball's stability as follows:

$$\frac{E_{\min}}{Q} = \frac{\sqrt{2}}{F_c}\sqrt{V(F_c)} < m_\phi \tag{15}$$

It is interesting that the minimum energy per unit charge will be lower with decreasing temperature, which keeps the energy density lower than the kinetic energy which is necessary for the Q-ball to disperse. This kind of Q-balls can survive.

In order to describe the true large Q-balls with the potential possessing the thermal logarithmic terms, we introduce the field profile,

$$F(r) = \begin{cases} F_c & r < R \\ F_c e^{-\alpha(r-R)} & r \geq R \end{cases} \tag{16}$$

where α is a variational parameter and R represents a region where the field configuration keeps constant instead of diminishing quickly. The scalar field of Q-balls can distribute a little widely. According to the large-Q-ball ansatz (16), the energy of model reads,

$$\begin{aligned}
E &= \frac{1}{2}\frac{Q^2}{\langle F^2\rangle} + \frac{1}{2}\langle(\nabla F)^2\rangle + \langle V(F)\rangle \\
&= \frac{1}{2}Q^2[\frac{4\pi}{3}F_c^2R^3 + 8\pi F_c^2(\frac{1}{8\alpha^3} + \frac{R}{4\alpha^2} + \frac{R^2}{4\alpha})]^{-1} \\
&\quad + 4\pi\alpha^2F_c^2(\frac{1}{8\alpha^3} + \frac{R}{4\alpha^2} + \frac{R^2}{4\alpha}) \\
&\quad + \frac{2\pi}{3}m_{\frac{3}{2}}^2F_c^2(1 + K\ln\frac{F_c^2}{2M^2})R^3 \\
&\quad + 4\pi m_{\frac{3}{2}}^2F_c^2[1 + K(\ln\frac{F_c^2}{2M^2} + 2\alpha R)][\frac{1}{(2\alpha)^3} + \frac{R}{(2\alpha)^2} + \frac{R^2}{4\alpha}] \\
&\quad - 12\pi\pi m_{\frac{3}{2}}^2KF_c^2[\frac{1}{(2\alpha)^3} + \frac{R}{(2\alpha)^2} + \frac{R^2}{4\alpha} + \frac{R^3}{6}] \\
&\quad + \frac{4\pi}{3}R^3T^4\ln(1 + \frac{F_c^2}{2T^2}) \\
&\quad + 4\pi T^4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(\frac{F_c^2}{2T^2})^n[\frac{2}{(2n\alpha)^3} + \frac{2R}{(2n\alpha)^2} + \frac{R^2}{2n\alpha}]
\end{aligned} \tag{17}$$

where the conserved charge is,

$$\begin{aligned}
Q &= \omega\int F^2d^3x \\
&= \frac{4\pi}{3}\omega F_c^2R^3 + 8\pi\omega F_c^2(\frac{1}{8\alpha^3} + \frac{R}{4\alpha^2} + \frac{R^2}{4\alpha})
\end{aligned} \tag{18}$$

The above expression means that the topological charge can take the place of the frequency ω. In order to further discuss the properties such as the stability of this kind of Q-ball controlled by the cosmic temperature, we just leave several dominant terms in the expressions of the energy and this approximation is acceptable for large Q-balls. Combining the conserved charge and reduced energy, we have,

$$\begin{aligned}
E_Q &\leq \frac{3Q^2}{8\pi F_c^2}R^{-3} + [\frac{2\pi}{3}m_{\frac{3}{2}}^2F_c^2(1 + K\ln\frac{F_c^2}{2M^2}) \\
&\quad + \frac{4\pi}{3}T^4\ln(1 + \frac{F_c^2}{2T^2})]R^3 \\
&\quad + [\pi\alpha F_c^2 + \frac{\pi}{\alpha}m_{\frac{3}{2}}^2F_c^2(1 + K\ln\frac{F_c^2}{2M^2} - K) \\
&\quad + \frac{2\pi}{\alpha}T^4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}(\frac{F_c^2}{2T^2})^n]R^2
\end{aligned} \tag{19}$$

It should be pointed out that the variables such as radius R and coefficient α do not belong to the model described by Lagrangian (1) although the energy of model depends on these variables. We extremize the reduced energy expression (19) with respect to R and α respectively. We proceed performance $\frac{\partial E}{\partial R}|_{R=R_{cl}} = 0$ to find the equation that the critical radius R_{cl} of Q-balls satisfies,

$$3CR_{cl}^6 + 2BR_{cl}^5 - 3A = 0 \tag{20}$$

where

$$A = \frac{3}{8\pi}\frac{Q^2}{F_c^2} \tag{21}$$

$$\begin{aligned}
B &= \pi\alpha F_c^2 + \frac{\pi}{\alpha}m_{\frac{3}{2}}^2F_c^2[1 + K(\ln\frac{F_c^2}{2M^2}) - K] \\
&\quad + \frac{2\pi T^4}{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}(\frac{F_c^2}{2T^2})^n
\end{aligned} \tag{22}$$

$$C = \frac{2\pi}{3}m_{\frac{3}{2}}^2F_c^2(1 + K\ln\frac{F_c^2}{2M^2}) + \frac{4\pi}{3}T^4\ln(1 + \frac{F_c^2}{2T^2}) \tag{23}$$

The approximate solution to Eq. (20) is,

$$\begin{aligned}
R_{cl} &\approx (\frac{3}{8\pi}\frac{Q^2}{F_c^2})^{\frac{1}{6}}[\frac{2\pi}{3}m_{\frac{3}{2}}^2F_c^2(1 + K\ln\frac{F_c^2}{2M^2}) \\
&\quad + \frac{4\pi}{3}T^4\ln(1 + \frac{F_c^2}{2T^2})]^{-\frac{1}{6}} \\
&\quad - \frac{1}{9}[\pi\alpha F_c^2 + \frac{\pi}{\alpha}m_{\frac{3}{2}}^2F_c^2[1 + K(\ln\frac{F_c^2}{2M^2}) - K] \\
&\quad + \frac{2\pi T^4}{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}(\frac{F_c^2}{2T^2})^n] \\
&\quad \times[\frac{2\pi}{3}m_{\frac{3}{2}}^2F_c^2(1 + K\ln\frac{F_c^2}{2M^2}) \\
&\quad + \frac{4\pi}{3}T^4\ln(1 + \frac{F_c^2}{2T^2})]^{-1}
\end{aligned} \tag{24}$$

and this solution is valid for large Q-balls at finite temperature. It is interesting that the radius of Q-ball becomes larger as the temperature decreases. Certainly the larger Q-balls have larger size. We can also impose the condition $\frac{\partial E_Q}{\partial\alpha}|_{\alpha=\alpha_c}=0$ into Eq. (19) to obtain,

$$\alpha_c^2=m_{\frac{3}{2}}^2(1+K\ln\frac{F_c^2}{2M^2}-K)+\frac{2T^4}{F_c^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}(\frac{F_c^2}{2T^2})^n \quad (25)$$

According to Eq. (25),

$$\frac{2T^4}{F_c^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}(\frac{F_c^2}{2T^2})^n>m_{\frac{3}{2}}^2(1-K\ln\frac{F_c^2}{2M^2}+K) \quad (26)$$

where $\ln\frac{F_c^2}{2M^2}<0$. If $K<0$, the inequality (26) can certainly be satisfied no matter whether the temperature is high or low. In the case of positive parameter K, the temperature cannot be so low that the α_c will be complex, then the field will oscillate outside the core to make the Q-ball to disperse. The large Q-balls with positive parameter K will not be stable when the temperature is lower than a critical magnitude. The inequality (26) can help us to estimate the critical temperature for any large Q-balls with a set of parameters. Here we prove analytically that the Q-ball should obey the necessary conditions from Refs. [17,24–26] instead of solving the nonlinear differential equation numerically a lot of times corresponding to various values of Q-ball model parameters. Our results for positive K are also consist with the relevant conclusions in Ref. [11]. Combining Eqs. (19), (24) and (25), we find the minimum energy of large Q-balls per unit charge,

$$\frac{E_Q[F_c]|_{R_c,\alpha_c}}{Q}=[m_{\frac{3}{2}}^2+m_{\frac{3}{2}}^2K\ln\frac{F_c^2}{2M^2}+\frac{2T^4}{F_c^2}\ln(1+\frac{F_c}{2T^2})]^{\frac{1}{2}}(1+\xi_cQ^{-\frac{1}{3}}) \quad (27)$$

where

$$\xi_c=(\frac{9\pi}{2}F_c^2)[m_{\frac{3}{2}}^2(1+K\ln\frac{F_c^2}{2M^2}-K)+\frac{2T^4}{F_c^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}(\frac{F_c^2}{2T^2})^n]\times[m_{\frac{3}{2}}^2+m_{\frac{3}{2}}^2K\ln\frac{F_c^2}{2M^2}+\frac{2T^4}{F_c^2}\ln(1+\frac{F_c}{2T^2})]^{-\frac{5}{6}} \quad (28)$$

The asymptotic behavior of the minimum energy of large Q-balls with huge charge Q per unit charge is

$$\lim_{Q\longrightarrow\infty}\frac{E_Q[F_c]|_{R_c,\alpha_c}}{Q}=\sqrt{m_{\frac{3}{2}}^2+m_{\frac{3}{2}}^2K\ln\frac{F_c^2}{2M^2}+\frac{2T^4}{F_c^2}\ln(1+\frac{F_c}{2T^2})} \quad (29)$$

Here we obtain the explicit expression for the minimum energy of large Q-balls. We can discuss the influence from the model variables and the temperature. Having compared Eq. (15) with Eq. (27), we discover that the lower bound on the energy per one particle in large Q-balls with description (16) is larger than that of Coleman's issue because of the ξ_c-term. The minimum energy over total charge is finite even the number of particles is extremely large. We show the dependence of the minimum energy per unit charge of Q-balls with thermal logarithmic potential on the cosmic temperature for a definite charge in Fig. 1. It is clear the minimum energy per unit charge is a decreasing function of temperature, which means that the decreasing temperature due to the expanding universe leads the thermal log-type Q-balls more stable. For various magnitudes of temperature T, the shapes of curves of minimum energy over our charge are similar.

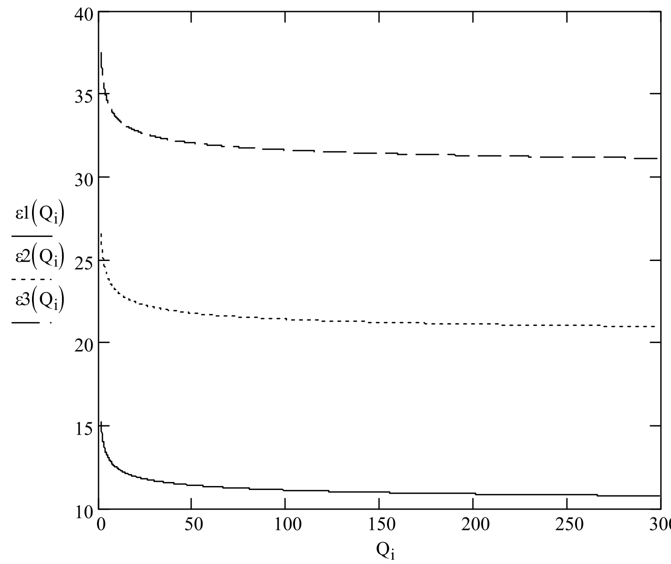

Fig. 1. The solid, dot and dashed curves of the minimum energy per unit charge of large Q-balls in the thermal logarithmic potential as functions of charge Q for temperature $T=10,20,30$.

4. The variational approach for small Q-balls with the thermal logarithmic potential

Now we pay attention to the small thermal log-type Q-balls. The small Q-balls with radii $R\geq m_\Phi^{-1}$ cannot be described well with the help of thin-wall approximation. We bring about a Gaussian ansatz in order to consider the small Q-balls subject to thermal logarithmic potential,

$$F(\mathbf{r})=F(r)=F_ce^{-\frac{r^2}{R^2}} \quad (30)$$

Substituting the ansatz (30) into the expression (7), we write the total energy in the case of small Q-balls at finite temperature as follows:

$$E=\int d^3x[\frac{1}{2}\omega^2F^2+\frac{1}{2}(\nabla F)^2+V(F^2)]$$
$$=\frac{\sqrt{2}Q^2}{\pi^{\frac{3}{2}}F_c^2R^3}+\frac{3\pi^{\frac{3}{2}}}{2^{\frac{5}{2}}}F_c^2R+\frac{1}{2}(\frac{\pi}{2})^{\frac{3}{2}}m_{\frac{3}{2}}^2F_c^2R^3[1+K\ln F_c^2-(\frac{3}{2}+\ln 2M^2)K]+[(\frac{\pi}{2})^{\frac{3}{2}}T^4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{\frac{5}{2}}}(\frac{F_c^2}{2T^2})^n]R^3 \quad (31)$$

According to Gaussian ansatz, the charge is,

$$Q=\omega(\frac{\pi}{2})^{\frac{3}{2}}F_c^2R^3 \quad (32)$$

replacing the frequency ω. The energy for small Q-balls controlled by thermal log-type potential just involve the dominant terms. In order to establish the equation for the critical radius R_c, we extremize the expression of the energy with respect to R like $\frac{\partial E_Q}{\partial R}|_{R=R_{cs}}=0$, then

$$3cR_{cs}^6+bR_{cs}^4-3a=0 \quad (33)$$

where

$$a=\frac{1}{2}\frac{Q^2}{(\frac{\pi}{2})^{\frac{3}{2}}F_c^2} \quad (34)$$

$$b = \frac{3}{2}(\frac{\pi}{2})^{\frac{3}{2}} F_c^2 \quad (35)$$

$$c = \frac{1}{2}(\frac{\pi}{2})^{\frac{3}{2}} m_{\frac{3}{2}}^2 F_c^2 [1 + \ln F_c^2 - (\frac{3}{2} + \ln 2M^2)K] + (\frac{\pi}{2})^{\frac{3}{2}} T^4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\frac{5}{2}}} (\frac{F_c^2}{2T^2})^n \quad (36)$$

Similarly the acceptable approximate solution reads,

$$R_{cs} = [1 - \frac{b}{2(9cR_0^2 + 2b)}]R_0 \quad (37)$$

where

$$R_0 = [\frac{1}{2}\frac{Q^2}{(\frac{\pi}{2})^{\frac{3}{2}} F_c^2}]^{\frac{1}{6}} \{\frac{1}{2}(\frac{\pi}{2})^{\frac{3}{2}} m_{\frac{3}{2}}^2 F_c^2 [1 + \ln F_c^2 - (\frac{3}{2} + \ln 2M^2)K] + (\frac{\pi}{2})^{\frac{3}{2}} T^4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\frac{5}{2}}} (\frac{F_c^2}{2T^2})^n\}^{-\frac{1}{6}} \quad (38)$$

The critical radius of small thermal-logarithmic-potential controlled Q-ball is shown as a function of temperature with a definite charge Q. The lower temperature will lead the small Q-balls to shrink. The minimum energy per unit charge for small Q-balls can be expressed in terms of the critical radius as,

$$\frac{E[F_c]|_{R=R_{cs}}}{Q} = (\frac{\tilde{a}}{c})^{\frac{1}{2}} [2c + b(\frac{\tilde{a}}{c})^{-\frac{1}{3}} Q^{-\frac{2}{3}} - \frac{b^2}{18} (\tilde{a}^2 c)^{-\frac{1}{3}} Q^{-\frac{4}{3}}] \quad (39)$$

where

$$\tilde{a} = \frac{1}{2(\frac{\pi}{2})^{\frac{3}{2}} F_c^2} \quad (40)$$

In the limit of too much charge, the minimum energy per unit charge for small Q-balls becomes,

$$\lim_{Q \longrightarrow \infty} \frac{E[F_c]|_{R=R_{cs}}}{Q} = 2\sqrt{\tilde{a}c} \quad (41)$$

It should also be pointed out that the variable c must keep positive according to Eq. (36) and Eq. (39). When the model parameter K is negative, the variable c is certainly positive. For the Q-balls with positive K, only the sufficiently high temperature can keep the variable c positive. The weaker thermal corrections must result in the nonexistence of the small Q-balls containing positive K. The powers of charge Q in the terms are negative in Eq. (39), so the energy over charge will not be divergent if the charge becomes huge. In Fig. 2, for simplicity we also choose $F_c = 1$ without losing generality and show that the minimum energy per unit charge $\frac{E[\Phi_c]|_{R=R_{cs}}}{Q}$ also decreases in the process of universe expanding. Certainly the energy of one particle remains smaller than the kinetic energy of a free particle as the temperature becomes lower, which keeps the stability of small Q-balls in the colder universe.

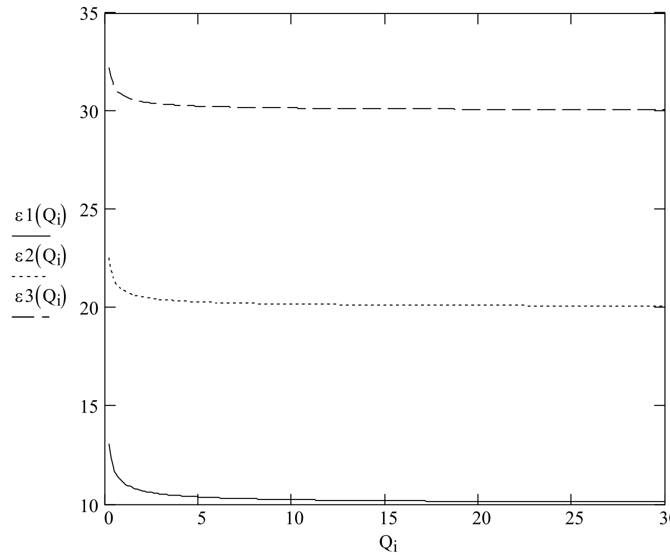

Fig. 2. The solid, dot and dashed curves of the minimum energy per unit charge of small Q-balls in the thermal logarithmic potential as functions of charge Q for temperature $T = 10, 20, 30$.

5. Summary

In this work we research on the Q-balls with the thermal logarithmic potential by means of variational estimation instead of lattice simulation. Some numerical solutions to the nonlinear field equation with respect to several given values of the model parameters cannot be reliable and only these numerical solutions cannot reveal the relations between the characters of the Q-balls and their construction completely. Here we obtain the analytical results on the Q-balls properties such as radii, energies related to the temperature and their stability without solving the nonlinear field equation numerically. Our analytical estimations help us to discuss the thermal log-type Q-balls in detail and their accuracy is acceptable. We also declare that the Q-balls minimum energy per unit charge will not be divergent if the charge is extremely large and the minimum energy over charge decrease to a quantity depending on the model parameters excluding the charge Q during which the universe expands. In the colder and colder background the energy per unit charge of Q-balls subject to the thermal logarithmic potential becomes lower, so the Q-balls remain stable. We also prove rigorously the necessary conditions that the model parameters should obey for the formation and stability of Q-balls under the influence from background temperature. The Q-balls involving negative parameter K will survive no matter whether the temperature is high or low. The sufficiently low temperature makes the Q-balls with positive parameter K to disappear. The expanding universe with decreasing temperature leads the large Q-balls enlarge and small ones contract. We can further study the related topics.

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