



37th National Conference on Theoretical and Applied Mechanics (37th NCTAM 2013) & The 1st International Conference on Mechanics (1st ICM)

## Reliability Analysis of Wind Turbine Towers

Yao Hsu<sup>a\*</sup>, Wen-Fang Wu<sup>b</sup>, Yung-Chang Chang<sup>b</sup>

<sup>a</sup>Department of Business and Entrepreneurial Management, Kainan University, No. 1, Kainan Road, Luchu, Taoyuan 33857, Taiwan, R.O.C.

<sup>b</sup>Department of Mechanical Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan, R.O.C.

---

### Abstract

To increase energy density in order to meet increasing electricity demands, increasingly large-scale designs for wind turbines have been developed. Wind turbine towers of larger sizes can generate more electricity, but such large sizes also create higher costs in terms of development and maintenance. The present research sets up a wind turbine tower model, wherein the loads of towers are calculated by their relation to wind speed. The finite element method is used to analyze the stress distribution of towers under these loads. Impacts from different loads are compared as well. The wind speed distribution is derived from data collected in Penghu, Taiwan using statistical methods. Fatigue analysis of towers is then conducted using fatigue loads and wind speed distribution, and the mean time to failure (MTTF) of towers is calculated with quantitative reliability theory. The results show that the main loads of towers are the wind force acting on the rotation area of wind turbine blades and the moment caused by non-uniform wind speed. After comparing this research finding with loads calculated by a wind turbine design software, it is concluded that it is a feasible and conservative method to analyze a wind turbine tower structure with the loads calculated by its relation to wind speed. In addition, it is shown that both the average and maximum hourly wind speeds in Penghu can be fitted into Weibull distribution. In conclusion, the fatigue analysis shows that the probability is greater than 99.8% for the tower model's failure time to be above 331,416 cycles, and it further shows that the tower model in this research possesses appropriate fatigue durability and is considered a safe tower design for Penghu.

© 2014 Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

Selection and peer-review under responsibility of the National Tsing Hua University, Department of Power Mechanical Engineering

*Keywords:* Wind Turbine; Reliability; Fatigue Analysis

---

---

\* Corresponding author. Tel.: +882-3-3412500 ext 6086; fax: +886-3-3412176.  
E-mail address: [yhsu@mail.knu.edu.tw](mailto:yhsu@mail.knu.edu.tw)

## 1. Introduction

Owing to growing environmental concerns over the global warming caused by the greenhouse effect, the development of renewable energy to reduce undesired gas emissions has become an important issue worldwide. Among the various forms of renewable energy, one promising type is wind power, which is widely considered as the most feasible form of renewable energy in Taiwan because of the island's windy environment. To increase energy density in order to meet the demand of electricity use, wind turbines have been developed toward large-scale designs. Under this trend, the structural safety of wind turbine towers will be an important issue to solve. To this end, the present study investigated the structural responses and the reliability of a 5 MW wind turbine tower subjected to static and fatigue loads. The finite element method was used to analyze the stress and displacement distributions of the tower under the loads. In addition, with quantitative reliability theory, a fatigue analysis of the tower was conducted, and the mean time to failure (MTTF) of the tower was evaluated.

## 2. Load analysis of wind turbine tower

The specifications for the wind turbine considered in the present study were described by Xie et al. [1]. The geometric dimensions and material properties of the tower are shown in Fig. 1 and listed in Table 1, respectively. The boundary condition between tower base and ground was set to be fixed to avoid separation and slippage. The loads applying to the top of tower come from the weight itself as well as the forces and moments resulting from the wind acting on the surfaces of the turbine blades. The loads can be resolved into x-y-z directions as shown in Fig. 2. All six of the force and moment values can be expressed in the following equations [2].

$$F_{XT} = C_p U_{hub}^2 \pi r^2 \quad (1)$$

$$F_{YT} = F_{XT} \cos \delta \sin \delta \quad (2)$$

$$F_{ZT} = mg \quad (3)$$

$$M_{XT} = 9550 \frac{P\eta}{\omega} \quad (4)$$

$$M_{YT} = \frac{4\rho}{27B} \pi r^3 (U_2^2 - U_1^2) \quad (5)$$

$$M_{ZT} \leq \frac{4}{9} \rho U_{hub}^2 \pi r^2 e \sin \delta \cos \delta \quad (6)$$

where  $C_p$  is the coefficient of wind energy,  $r$  is the radius of the blade,  $\delta$  is the angle between the wind direction and the normal line of blades,  $\eta$  is the mechanical efficiency,  $\omega$  is the angular velocity of the blade,  $\rho$  is the air density,  $U$  is the wind speed,  $e$  is the eccentricity;  $F_{XT}$ ,  $F_{YT}$ , and  $F_{ZT}$  are the forces coming from the action of wind on the blades, namely, the turning forces due to changes of wind direction and in the weights of the nacelle and blades, respectively;  $M_{XT}$ ,  $M_{YT}$ , and  $M_{ZT}$  are the moment of rotation, moment being caused by non-uniform wind speed, and the turning moment, respectively. Under the assumptions that the center of gravity of the nacelle and blades falls on the central line of the tower, and that the normal line direction of the blades coincides with the wind direction,  $F_{YT}$  and  $M_{ZT}$  can be neglected.

With appropriate value settings to the parameters above provided by Vorpahl et al. [3], and through the above equations,  $F_{XT}$ ,  $F_{ZT}$ ,  $M_{XT}$ , and  $M_{YT}$  were calculated as 2,733 kN, 1,962 kN, 3,552 kN·m, and 9,587 kN·m, respectively, when  $U_{hub}$  is 25 m/s. These calculated results are reasonable according to the aforementioned study by Xie et al. [1]. Furthermore, with these results as input loadings to finite element (FE) software ANSYS and performing the FE analysis, the maximum total displacement occurring in the x direction was calculated to be 2.401 m and the maximum von-Mises stress which occurs near the tower root was 300 MPa. These two values are both higher than those proposed by Xie et al. [1], indicating that our analysis approach provides a conservative solution, however, which can still be valuable for reference as high safety in wind turbine construction is required.

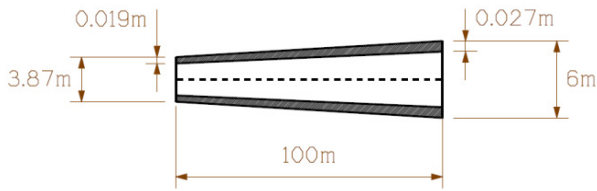


Fig. 1. Schematic diagram of wind turbine tower [1]

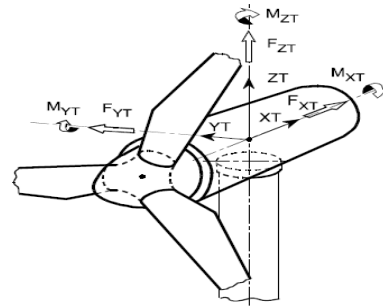


Fig. 2. Coordinate system of wind turbine tower

Table 1 Material property of wind turbine tower

Material	Steel
Modulus of Elasticity	210 GPa
Density	7.7 g/cm <sup>3</sup>
Poisson's Ratio	0.29
Ultimate Tensile Strength	460 MPa

### 3. Probability model of wind speed

Besides the static load analysis on the wind turbine tower, the present study also evaluated the behavior of the tower and its fatigue life when subjected to repetitive loads caused by wind. Therefore, we had to have the probability model of the wind speed first. In this research, the model was established by statistical methods and by reference to the wind speed data collected in Penghu County in Taiwan.

#### 3.1. Weibull probability density function

In the field of wind engineering, the two-parameter Weibull distribution is widely used by many researchers, such as Chang et al. [4], and is expressed as:

$$f(U) = \frac{\beta}{\theta} \left( \frac{U}{\theta} \right)^{\beta-1} \exp \left[ - \left( \frac{U}{\theta} \right)^{\beta} \right] \quad (7)$$

in which  $U$  is the wind speed,  $\beta$  is the shape parameter and  $\theta$  is the scale parameter.

#### 3.2. Vertical distribution of wind speed

In 1916, Hellman used the power law to describe the wind speed distribution with varying heights. The wind speed increases as height increases. When height reaches a threshold called the gradient height, the wind speed will remain constant and be uniformly distributed above the gradient height. The wind speed at the moment is named the gradient velocity. Below this velocity, the relation of wind speed and height is shown as:

$$U_{z_g} = U_z \left( \frac{z_g}{z} \right)^\alpha, \quad 0 \leq z \leq z_g \quad (8)$$

where  $U_{z_g}$  is the gradient velocity,  $U_z$  is the wind speed at height  $z$ ,  $z_g$  is the gradient height, and  $\alpha$  is a coefficient related to ground condition and was set to be 0.26 in this study.

### 3.3. Distribution of mean wind speed

In this study, the wind speed data came from the Data Bank Atmospheric Research (DBAR), which contains 43,824 hourly wind speeds observed between 2006 and 2010 in Penghu County in Taiwan [5]. The hourly mean wind speed  $U_{mean}$  is defined as the mean of wind speed in the first ten-minute interval at the time when measuring and recording the data. It should be mentioned that the data were observed and collected at the altitude of 10 m. The study used the Weibull++7 statistical analysis software to distribute the data into the two-parameter Weibull probability density function with goodness-of-fit test. Under the significance level of 0.05, the shape and scale parameter in the Weibull probability model are 1.6396 and 4.5598, respectively. The obtained function is expressed in Eq. (9) and its corresponding distribution is depicted in Fig. 3.

$$f(U_{mean}) = \frac{1.6396}{4.5598} \left( \frac{U_{mean}}{4.5598} \right)^{0.6396} \exp \left[ - \left( \frac{U_{mean}}{4.5598} \right)^{1.6396} \right] \quad (9)$$

## 4. Fatigue analysis of wind turbine tower

Fatigue is known as one of the important factors leading to structural damage, so it should be given extra attention when designing a structure for the sake of safety. Since winds always blow repetitively and their velocities change with time, wind loads can be regarded as fatigue loadings. This study sampled 1,000 wind speeds from the established probability model of wind speed and then transferred them into 1,000 loads acting on the wind turbine tower through Eqs. (1)~(6). Afterwards, with these loads as input data, the fatigue responses and fatigue lives of the tower were analyzed by using ANSYS.

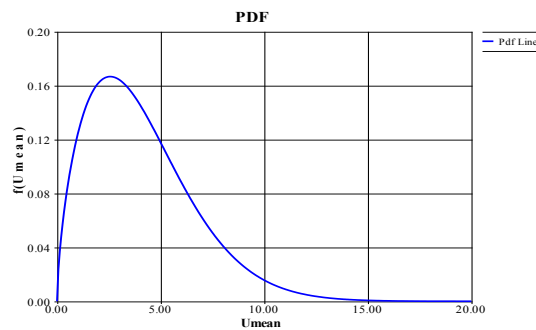


Fig. 3. Weibull probability density function for hourly mean wind speed

### 4.1. Fatigue load setting

Wind turbine towers are made of steel, and they were considered as isotropic structures in the present study. The material properties of steel in the FE analysis are provided by ANSYS. During the process of analysis, it should be noted that when the fatigue life  $N$  of the material was more than  $10^6$  cycles, the material was considered to be non-fracture in the present study. In our study case, the fatigue load stress was  $8.62 \times 10^7$  Pa, corresponding to the fatigue life of  $10^6$  cycles, which means that when the fatigue loading is less than  $8.62 \times 10^7$  Pa, the fatigue life will be longer than  $10^6$  cycles. In addition, the fatigue loading was imposed on the structure in the form of constant-amplitude with zero base, and Goodman theory was used to modify the fatigue life in this research.

### 4.2. Modification of fatigue load

In consideration of wind turbine safety, when the wind speed in the hub center  $U_{hub}$  of the blades is bigger than the cut-out wind speed (which was 25 m/s for the wind turbine studied herein), the wind turbine will usually be forced to stop running. In the meantime, the loads on the wind turbine towers such as  $F_{XT}$  and  $M_{YT}$  would start to decrease because the blades stop rotating. Therefore, modification of these two loads is necessary. When  $U_{mean}$  is 13.93 m/s,  $U_{hub}$  will reach the cut-out wind speed in this study. Moreover, since  $M_{YT}$  is quite small in comparison with  $F_{XT}$ ,  $M_{YT}$  is directly modified to zero in the present study. The final relation diagrams of  $F_{XT}$  and  $U_{mean}$ , as well as of  $M_{YT}$  and  $U_{mean}$ , are shown in Fig. 4 and Fig. 5.

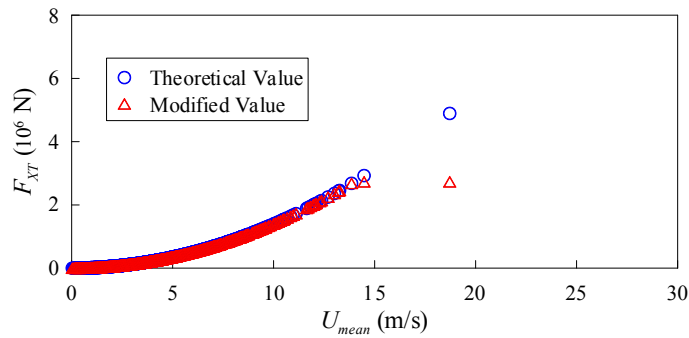


Fig. 4. Relation of  $F_{XT}$  and  $U_{mean}$

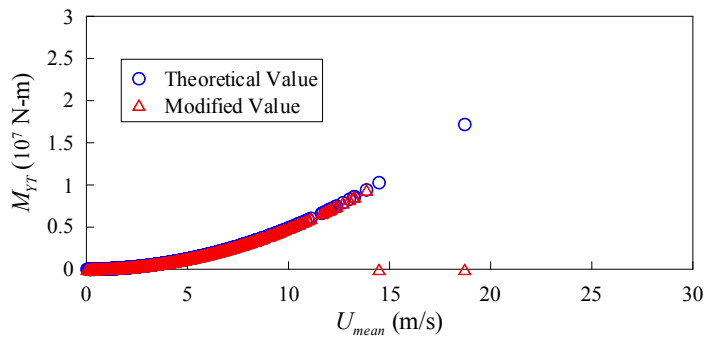


Fig. 5. Relation of  $M_{YT}$  and  $U_{mean}$

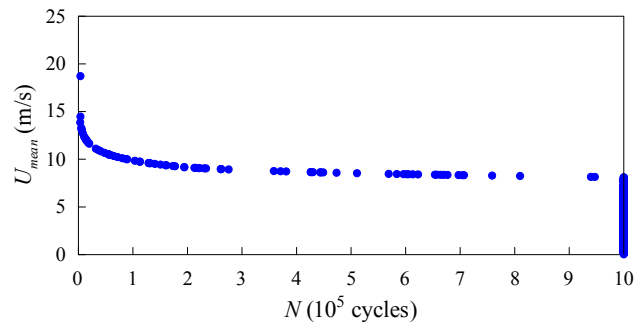


Fig. 6. Relation of fatigue life and hourly mean wind speed

#### 4.3. Fatigue life and mean time to failure

With an initial sample size of 1,000 wind speeds, 1,000 fatigue lives  $N$  for the wind turbine tower can eventually be obtained by FE simulation through the analysis procedure described above. The simulated fatigue lives related to corresponding  $U_{mean}$  are shown in Fig. 6. Among these 1,000 fatigue lives, data for only 82 fatigue lives were screened out to be used for subsequent analysis. The large majority of the other 918 fatigue lives were not used because their values were more than  $10^6$  cycles. After using Weibull++7 software to conduct the goodness-of-fit test, it was found that the data for the 82 fatigue lives were best fitted by the three-parameter Weibull probability model whose form is as shown in Eq. (10),

$$f(N) = \frac{\beta}{\theta} \left( \frac{N - N_0}{\theta} \right)^{\beta-1} \exp \left[ - \left( \frac{N - N_0}{\theta} \right)^{\beta} \right] \quad (10)$$

where  $N_0$  is the location parameter. MTTF can then be obtained from Eq. (11).

$$\text{MTTF} = N_0 + \theta \cdot \Gamma \left( 1 + \frac{1}{\beta} \right) \quad (11)$$

In this analysis,  $N_0$  is 2,899,  $\beta$  is 0.7405 and  $\theta$  is  $2.7307 \times 10^5$ . MTTF is therefore 331,416 cycles that is equivalent to 37.8 years.

#### 5. Concluding remarks

The present study constructed a FE model for a 5 MW wind turbine tower and investigated its structural responses under static and fatigue loads. In the static case, the numerical results from our FE analysis were shown to be correct by comparison with those calculated by specifically-designed wind turbine software GH Bladed. In the fatigue analysis, with the wind speed distribution and statistical method, MTTF of the wind turbine tower can be obtained. It should be noted that in this paper, for simplicity, it was assumed that the wind turbine tower was subjected to constant-amplitude wind loadings resulting from a generated wind speed. A sample of 1,000 was thus collected. The sample values of these wind speeds, loadings, and fatigue lives thus calculated may reflect those that can be found in more rigorous manners based on more advanced random theories. From the results of this study, several conclusions can be drawn:

1. For static loads, when comparing our results, such as maximum displacement and maximum stress, with those calculated by the wind turbine software, it can be indicated that the analysis approach based on the relation of loads and wind speeds proposed in the study is viable and conservative for analyzing the tower structure.
2. The majority of loads existing on the tower are the wind forces acting on the rotational area of the wind turbine blades, and the moments resulting from the non-uniform wind speed.
3. Weibull distribution is found to best fit both the average and maximum hourly wind speeds in Penghu County in Taiwan.
4. The probability can be up to 99.8% when the tower's failure time is more than 331,416 cycles. It suggests that the tower model in this research has adequate fatigue durability and is considered a safe tower design in the present study.

## References

- [1] K.R. Xie, J.T. Tseng, Y.Y. Chang, Load analysis of tower for wind turbine, Taiwan Wind Energy Association, 2010.
- [2] W.T. Zhao, P.Z. Cao, J.F. Chen, The research of load calculation method and loads combination about wind turbine tower, *Special Structures*. 27 (2010).
- [3] F.R. Vorpahl, M. Strobel, H.G. Busmann, S. Keliñhansl, Superelement Approach in Full-coupled Offshore Wind Turbine Simulation: Influence of the Detailed Support Structure Modelling on Simulation Results for a 5-MW Turbine on a Tripod Substructure, *Journal of System Simulation*, ISOPE (2010) 286-293.
- [4] T.J. Chang, Y.T. Wu, H.Y. Hsu, C.R. Chu, C.M. Liao, 2003. Assessment of wind characteristics and wind turbine characteristics in Taiwan, *Renewable Energy*. 28 (2003) 851-871.
- [5] Atmospheric Research Data Bank, <http://stdank.as.ntu.edu.tw/>