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Natural Convection of Magnetic Fluid in a Rectangular Hele-Shaw Cell of Different Aspect Ratios

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Abstract

The nature convection of a magnetic fluid in a rectangular Hele-Shaw cell with an imposition of an even vertical magnetic field is studied experimentally. Heat transfer measurements with thermocouples and flow visualization with liquid crystal thermography are conducted in Hele-Shaw cells of aspect ratios 1.0, 1.5, and 2.0. Both results show that the vertically imposed magnetic field has destabilizing influence. The flow instability modes become different from that in two-dimensional cavity cases, with and without the magnetic field. A pair of symmetric counter-rotating vortices is observed for the first instability mode. Increasing the aspect ratio destabilizes the flow. Second instability mode is observed in the cases of aspect ratios 1.0 and 1.5, but not in 2.0.

Keywords: Natural convection; Magnetic fluid; Hele-Shaw cell; Liquid crystal thermography

1. Introduction

The heat transfer characteristics associated with the hydrodynamic instabilities of a magnetic fluid have attracted a great interest of engineers and scientists for decades [1-6]. Yamaguchi et al. [3-4] and Wen et al. [5-6] have investigated the flow behaviors of the Rayleigh-Bénard instabilities of magnetic fluids in two-dimensional rectangular cavities of different aspect ratios ($AR$) and in a square Hele-Shaw cell ($AR=1.0$), respectively, with an imposition of an even vertical magnetic field. The corresponding heat transfer characteristics were also studied. The measured heat transfer rates [3-6] and images of flow visualization with shadowgraph [5] and liquid crystal thermograph techniques [6] showed that the vertically imposed magnetic field has a destabilizing influence, and at the super critical state the flow mode becomes substantially different from that with no magnetic field.

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In this study, the work of Wen and Su [6] is further extended to consider the effect of aspect ratios on the heat transfer characteristics and flow behaviors in a rectangular Hele-Shaw cell, heated from below. Experiments of two other aspect ratios, \( AR=1.5 \) and 2.0 were conducted and compared with that of \( AR=1.0 \), with and without the externally imposed vertical magnetic field.

2. Experimental Setup

Fig. 1 depicts a schematic diagram of the present experimental apparatus. The experimental apparatus of local heat transfer measurements, flow visualization and magnetic field imposition is similar to that of Wen and Su [6]. Three aspect ratios \( AR=1.0, 1.5, \) and 2.0 are used while keeping the 15.0-mm cell height and 1.5-mm cell depth constant. The magnetic fluid used in the current experiment is the temperature-sensitive magnetic fluid (Ferrotech Corp. APG-E26, viscosity=300cp@ 27°C, Ms = 275 gauss, oil based), whose magnetization for the temperature \( T \) and magnetic field can be approximated by a linear relation. The magnetic field was imposed to the cell by the electromagnet vertically. The bottom of the cell was heated through a copper sheet while the upper wall of the cell was cooled through another cooper sheet. Two accurate temperature-controlling systems were used to assure the constant-temperature boundary conditions of the top (\( T_H \)) and bottom walls (\( T_C \)). An automatic data acquisition system and seven carefully calibrated K-type thermocouples were adopted for heat transfer measurements. The liquid crystal thermography was used to visualize the flow fields with the wide-band cholesterol liquid crystal of 5 °C dynamic range (Hallerest, BM/R25C5W/C17-10). The liquid crystal was sprayed on a glass substrate as one of the cell surfaces. Four fluorescent tubes were used as the light source to avoid significant heating of the substrate and irreversible influence on the calibration of liquid crystals[7].

![Fig. 1. Experimental setup of heat transfer measurements and liquid crystal thermography](image)

Note that the continuity equation for incompressible fluid, the energy equation, and the Maxwell equations for electrically nonconductive media for the natural convection of the magnetic fluid in a Hele-Shaw cell with an imposition of an even vertical magnetic field are the same as its counterpart in a two-dimensional rectangular cavity [5]. Only is the momentum equation replaced by the Darcy’s law. Performing dimensional analyses of this Hele-Shaw cell flow and its governing equations yields four important dimensionless parameters to correlate the experimental heat transfer data:

\[
Dh = \frac{d}{h}, \quad Ra = \frac{\rho c \theta g \beta d^4}{\kappa \eta}, \quad Ram = \frac{\mu_s H_0 M_s d^2}{\kappa \eta}, \quad Nu = \frac{d}{\Delta T} \left( \frac{\partial T}{\partial z} \right)_{z=0},
\]

where \( Dh \) is the height-thickness ratio, \( Ra \) is the Rayleigh number, \( Ram \) the magnetic Rayleigh number and \( Nu \) the Nusselt number with \( \Delta T=T_H-T_C \), \( \beta=(T_H-T_C)/d \), \( d \) is the height of cell, \( h \) the thickness of the cell, \( \kappa \) the thermal diffusivity, \( H_0 \) the strength of external magnetic field and \( M_s \) the saturation magnetization. \( Dh=10 \) in the current experiments.
3. Results and Discussion

In Fig. 2, the heat transfer characteristics of $AR=1.0$, $1.5$, and $2.0$ with and without the imposed magnetic field, are depicted as the local $Nu$ vs. $Ra$. It is noted that the experimental results are taken for the steady state conditions and the local $Nu$ is calculated based on the mean heat flux to the cell and the local temperature gradient measured at the position of the temperature measuring point in the cell by thermocouples. As seen, by imposing the magnetic field the heat transfer curve shift toward the left, indicating the increase of the heat transfer rate. The heat transfer measurements suggest that magnetic field has a destabilizing effect. These results show close similarity with those of Yamaguchi et al [3-4] and Wen and Su [6]. Second instability mode is also observed in the cases of aspect ratios 1.0 and 1.5, but not in 2.0.

Table 1 summarizes the critical Rayleigh numbers at the first and second flow transitions. The first critical Rayleigh number, $Ra_{c1}$, is estimated by extrapolating the data by the following relation: $Nu = l + k (Ra - Ra_{c1})^{1/2}$. The critical Rayleigh numbers in the present Hele-Shaw cells of different $AR$, with and without the imposed magnetic field, are different from those in the two-dimensional square cavity case [4]. The very small thickness in the third dimension in the case of Hele-Shaw cell flow has great influence on the characteristics of flow instability. Increasing the aspect ratio destabilizes the flow.

Figure 3 shows the liquid crystal images of the magnetic natural-convection flows in a Hele-Shaw cell of $AR=1.5$. The isotherms are shown on the top and temperature difference between two adjacent isotherms is constant in each image. Figure 3(a) presents the liquid crystal images before the onset of transition without the externally imposed vertical magnetic field. As shown, the fluid is stationary and the nearly even-spaced isotherms demonstrate that heat transfer through the cell is by conduction only. Figure 3(b) and 3(c) illustrate the flow fields at higher $Ras$ with $Ram = 3.8 \times 10^5$ (which corresponds to $H_0 = 4.55$ G), experiencing the first and second mode instabilities, respectively. In Fig. 3(b), if $\Delta T$ is increased beyond a critical value under this magnetic field, the fluid becomes unstable and a pair of counter-rotating vortices is clearly observed with the isotherms. The circulation of fluid in the cell convects warm fluid upward and cold fluid downward and accommodates a higher heat transfer rate across the cell. The flow features are similar to those in the shadowgraphs by Wen, Chen & Yang[5] and liquid crystal images by Wen and Su [6] for $AR=1$, and those corresponding to the second instability mode in the numerical simulations of Yamaguchi et al. [4]. Note that a single convection vortex is generated at the first mode in the two-dimensional square cavity case studied [4], instead of a pair of vortices in the present Hele-Shaw cell case. If $\Delta T$ is further increased, the flow undergoes a second transition to turbulence. The core of the cell is then almost isothermal, with the major temperature variation adjacent to the top and bottom walls, as shown in Fig.3(c). The isotherms in Fig. 3(c) were obtained by averaging twenty liquid crystal images of turbulent flows under the same $Ra$ and $Ram$. The flow characteristics of $AR=1.0$, $1.5$ and $2.0$ shows similar features, except that the second mode instability is not observed for $AR=2.0$ under the current experimental temperature and magnetic field ranges.
Fig. 2. Heat transfer characteristics. Green square, red circle and yellow triangle symbols present data of cases $Ra_m=0$, $3.8 \times 10^5$, and $9.6 \times 10^5$, respectively, and they are fitted with dash lines.

Table 1 Critical Rayleigh numbers at the first and second flow transitions.

<table>
<thead>
<tr>
<th></th>
<th>$AR=1.0$</th>
<th>$AR=1.5$</th>
<th>$AR=2.0$</th>
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<tr>
<td>$Ra_m=0$</td>
<td>1770</td>
<td>5960</td>
<td>1530</td>
</tr>
<tr>
<td>$Ra_m=3.8 \times 10^5$</td>
<td>1330</td>
<td>4490</td>
<td>1040</td>
</tr>
<tr>
<td>$Ra_m=9.6 \times 10^5$</td>
<td>790</td>
<td>3000</td>
<td>390</td>
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Fig. 3. Flow fields obtained from liquid crystal thermography. $AR=1.5$.  
(a) $Ra_m=0$, $Ra=1400$; (b) $Ra_m=3.8 \times 10^5$, $Ra=2000$; (c) $Ra_m=3.8 \times 10^5$, $Ra=4000$. 
4. Conclusion

The macroscopic magnetic flow fields in the Hele-Shaw cell of $AR=1.0, 1.5,$ and $2.0$ are visualized with liquid crystal thermography. A pair of symmetric counter-rotating vortices interpreted from the isotherm plots of liquid crystal thermography is observed for the first instability mode. Results obtained from heat transfer measurements suggest that the magnetic field has a destabilizing hydrodynamic effect on the flow and are consistent with flow visualizations. Increasing the aspect ratio destabilizes the flow. Second instability mode is only observed in the cases of $AR= 1.0$ and $1.5$.

References