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Electroweak-scale mirror fermions, $\mu \to e\gamma$ and $\tau \to \mu\gamma$

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Abstract

The Lepton Flavor Violating (LFV) processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are estimated in a model of electroweak-scale right-handed neutrinos. The present bounds on the branching ratios, $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$, $B(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$ (BaBar) and $< 4.5 \times 10^{-8}$ (Belle), put strong constraints on the parameters of the model. This constraint links low energy rare decay processes to high-energy phenomena (e.g., decay lengths of the mirror charged leptons which are important in the search for the telltale like-sign dilepton events present in the model of electroweak-scale right-handed neutrinos). The model can be tested at future colliders (LHC, ILC) and at MEG and/or B factories.

1. Introduction

The nature and origin of neutrino masses are undoubtedly among the most important questions in particle physics, experimentally and theoretically. Neutrino oscillation experiments and astrophysical arguments indicate that neutrinos that interact with normal matter at tree level are very light compared with all known charged particles, typically with masses less than an electronvolt. There exist plausible models of light neutrino masses which could, in principle, be tested experimentally. One of such models is the famous see-saw mechanism [1] where a lepton-number conserving ($\Delta L = 0$) Dirac neutrino mass term, $m_D \bar{\nu}_L \nu_R$, is combined with a lepton-number violating ($\Delta L = 2$) Majorana mass term, $M_R \nu_R^T \sigma_2 \nu_R$ where it is usually assumed that $M_R \gg m_D$, to yield a "tiny" mass eigenvalue $-m_D^2/M_R$ and a "very large" one M_R. Since in a generic framework, the only "knowledge" that one has at the present time is the smallness of the ratio $-m_D^2/M_R$, the question of how small m_D is and how large M_R could be is rather model-dependent. The most popular scenario is one in which m_D is related to the electroweak symmetry breaking scale while M_R is in general related to some grand unified scale. It is in particular the relationship of the Majorana scale M_R to some new physics that is of great interest since its probe would reveal not only the see-saw mechanism but also what type of new physics one might be dealing with.

A very interesting connection between M_R and the scale above which parity is restored was made by [2] in which the Standard Model (SM) is extended to $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. In this model, the finiteness, albeit small, of the light neutrino mass m_ν is related to the finiteness of the $SU(2)_R$ gauge boson masses $M_{W_R} \gg M_{W_L}$ with m_ν vanishing in the limit $M_{W_R} \to \infty$. The V–A nature of the weak interactions of the SM fermions is recovered in this limit. It is extremely intriguing that parity restoration is linked in the left–right (LR) symmetric model to the non-vanishing value of the neutrino mass. The strength of V + A interactions vanishes in the limit of zero neutrino masses.

Can parity restoration be accomplished within the SM gauge sector? This issue was addressed in [3,4] where the gauge group is simply the SM $SU(2) \otimes U(1)$. (Notice that the subscripts L and Y are deliberately omitted for reasons to be given below.) However, one now has, for every SM left-handed doublet such as, e.g., $(\nu_e, e)_L$, a *heavy* mirror *right-handed* doublet, e.g., $(\nu_e^M, e^M)_R$ [5]. Similarly, for every SM singlet such as, e.g., e_R, one has a *heavy left-handed* mirror singlet e_L^M. (The content for the quarks and their mirror counterparts are listed in [3].) One word of caution is in order here. What we mean by "mirror" is simply the aforementioned assignments for fermions and nothing else, e.g., there is *no* mirror gauge group, etc.

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There are two remarks we would like to make concerning these types of mirror fermions. First, experimental constraints on mirror quarks and leptons masses are not very well studied since they depend on the specifics of decay modes. However, as discussed in [3], one can safely conclude that the right-handed neutrinos have to be more massive than $M_Z/2$ because of constraints from the width of the Z boson. The lower bound on the charged mirror lepton masses can be taken to be around ~ 100 GeV (long-lived heavy leptons) [6] if not lower. The bounds on the mirror quarks are murkier but one can assume that they might be heavier than ~ 200 GeV (from bounds on long-lived quarks) [6]. The second remark has to do with electroweak precision parameter constraints. As mentioned in [3], the positive contribution of mirror fermions to the S parameter can be compensated by a negative contribution from the Higgs triplet sector present in the model [7]. In [7], it was shown that, depending on the mass splitting inside the triplet Higgs scalar(s), its contribution to S can be negative and relatively large in magnitude, and can offset the positive contributions coming from the mirror fermions. Furthermore, it is well known that Majorana fermions can also have a negative contribution to S [8]. Notice that it is straightforward to satisfy the ρ-parameter constraint [3]. The above remarks and other phenomenological issues concerning mirror fermions will be dealt with elsewhere.

Since ν_R's are now members of $SU(2)$ doublets, it was shown in [3] that the Majorana masses M_R coming from the term $M_R \nu_R^T \sigma_2 \nu_R$ is now related to the electroweak breaking scale $\Lambda_{\mathrm{EW}} \sim 246$ GeV, rendering the detection of a low-scale ν_R at colliders and observing lepton-number violating processes a real possibility. In this scenario, one can directly test the see-saw mechanism at collider energies as well as the Majorana nature of ν_R's.

The Dirac mass term now comes from the *mixing* between the SM left-handed doublet and the mirror right-handed doublet through the coupling with a singlet scalar field which develops a non-vanishing vacuum-expectation value (VEV) v_S, giving $m_D = g_{\mathrm{Sl}} v_S$, where g_{Sl} is the Yukawa coupling. The light neutrinos becomes massless in the limit $g_{\mathrm{Sl}} \to 0$, i.e., when the SM particles and their mirror counterparts decouple. It is this mixing which will prove important for the LFV processes considered in this Letter.

What is parity violation? Basically, this is equivalent to the question of why the weak interactions of SM particles are of the V–A type. This rather old question never really disappears and occasionally has new twists to it, especially in light of the upcoming explorations by the LHC and hopefully also by the ILC. For the SM, this is built in because of the absence of right-handed neutrinos. In the LR model, this is due to $M_{W_R} > M_{W_L}$. Parity is restored for $E \gg M_{W_R}$. For $E \gg M_{W_R} > M_{W_L}$, the $\mathrm{V}+\mathrm{A}$ interactions have equal strength to that of the V–A interactions and parity is restored.

In the model of electroweak-scale right-handed neutrinos, the question of parity violation takes on a slightly different meaning as compared with the LR model. Here, $SU(2) \otimes U(1)$ as a gauge theory is actually a vector-like model in the sense that fermions of both chiralities are present. In consequence, "parity restoration" refers to the existence of the mirror fermions in our model. Furthermore, the SM fermions have V–A weak interactions at tree-level but, because of the existence of a *mixing* between SM and mirror fermions, receive $\mathrm{V}+\mathrm{A}$ interactions through one-loop diagrams which are of course suppressed with respect to the tree-level V–A interactions. As with the LR model, these (radiative) $\mathrm{V}+\mathrm{A}$ contributions vanish in the limit $g_{\mathrm{Sl}} \to 0$ which also implies a vanishing Dirac mass m_D and hence a vanishing light neutrino mass. Here, one expects deviations of the SM couplings of the SM quarks and leptons due to the mixing between SM fermions and their mirror counterparts. This includes corrections to the electroweak processes involving electrons and neutrinos as well as lepton flavor violating processes such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$. It is the latter processes that we will concentrate on in this Letter. Details of the former processes will be presented elsewhere [9].

We will see below that there is a deep connection between the mixing parameters involved in LFV processes and the Dirac neutrino mass matrix which participates in the see-saw mechanism. Since these same parameters [3] participate in the determination of the decay length of the mirror charged leptons, l_R^M, it will be shown below that were these LFV processes to be detected the decay length would be microscopic. If the charged mirror leptons were to be found with "macroscopic" decay lengths, LFV processes would be practically unobservable in our model. One word of reminder is in order here on why it is important to gain a good understanding on the size of the decay lengths of the mirror leptons. Because the model presented in [3] deals with right-handed neutrinos with electroweak-scale masses, it is possible to probe the Majorana nature of ν_R's and the see-saw mechanism at present and future colliders (Tevatron, LHC, ILC). The telltale signatures would be SM like-sign dileptons [10] which are produced, e.g., in processes such as

$$q + \bar{q} \to Z \to \nu_R + \nu_R. \tag{1}$$

Since ν_R's are Majorana particles, they can have transitions such as $\nu_R \to l_R^{M,\mp} + W^{\pm}$. A heavier ν_R can decay into a lighter l_R^M and one can have

$$\begin{aligned} \nu_R + \nu_R &\to l_R^{M,\mp} + l_R^{M,\mp} + W^{\pm} + W^{\pm} \\ &\to l_L^{\mp} + l_L^{\mp} + W^{\pm} + W^{\pm} + \phi_S + \phi_S, \end{aligned} \tag{2}$$

where $l_R^{M,\mp} \to l_L^{\mp} + \phi_S$ and where ϕ_S, the singlet scalar field in the model, would constitute the missing energy.

2. Brief review of the electroweak scale right-handed neutrino model

In our model, the dominant contribution to the process $l_i \to l_j \gamma$ comes from the diagrams shown in Fig. 1. The Feynman rules for the diagrams can be extracted from the Lagrangians given below. Let us first write down the Lagrangian of our model and enumerate the particle content and various symmetries.

The gauge group is $SU(2) \otimes U(1)$.

Below is a list of the fermion content. Because of the problem at hand, namely the processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$, we shall concentrate mainly on the leptons in this Letter.

- $(2, Y/2 = -1/2)$ (Lepton doublets):

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_i, \tag{3}$$

$$l_R^M = \begin{pmatrix} \nu_R^M \\ e_R^M \end{pmatrix}_i, \tag{4}$$

where i stands for the family number. In [3], ν_R^M is identified with the right-handed neutrino, a fact which gives rise to an electroweak-scale right-handed neutrino.

- $(1, Y/2 = -1)$ (Lepton singlets):

$$e_{iR};\; e_{iL}^M. \tag{5}$$

Next, we write down the Lagrangian involving the mirror leptons and their interactions with the SM leptons. In so doing, we will also include for clarity the term that gives the Majorana mass to the right-handed neutrinos although it is not needed for the present discussion. Also for completeness we will write down the Lagrangian for the SM leptons in order to make a comparison. We have the following interactions.

- Charged current interactions:

$$\mathcal{L}^{CC} = \mathcal{L}_{SM}^{CC} + \mathcal{L}_M^{CC}, \tag{6}$$

where

$$\mathcal{L}_{SM}^{CC} = -\left(\frac{g}{2\sqrt{2}}\right)\sum_i \bar{\psi}_i^{SM}\gamma^\mu(1-\gamma_5)\left[T^+W_\mu^+ + T^-W_\mu^-\right]\psi_i^{SM}, \tag{7}$$

$$\mathcal{L}_M^{CC} = -\left(\frac{g}{2\sqrt{2}}\right)\sum_i \bar{\psi}_i^M\gamma^\mu(1+\gamma_5)\left[T^+W_\mu^+ + T^-W_\mu^-\right]\psi_i^M, \tag{8}$$

and

$$\psi^{SM} = \begin{pmatrix} \nu \\ e \end{pmatrix}_i, \tag{9}$$

$$\psi^M = \begin{pmatrix} \nu^M \\ e^M \end{pmatrix}_i. \tag{10}$$

Above the subscripts SM and M refer to SM and mirror particles respectively. Notice that Eq. (7) has $(1-\gamma_5)$ as opposed to $(1+\gamma_5)$ of Eq. (8).

- Neutral current interactions:

$$\mathcal{L}^{NC} = \mathcal{L}_{SM}^{NC} + \mathcal{L}_M^{NC}, \tag{11}$$

where

$$\mathcal{L}_{SM}^{NC} = -\left(\frac{g}{4\cos\theta_W}\right)Z_\mu\left\{\sum_i \bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_i + \sum_i \bar{e}_i\gamma^\mu\left[\left(-1+4\sin^2\theta_W\right)+\gamma_5\right]e_i\right\}, \tag{12}$$

$$\mathcal{L}_M^{NC} = -\left(\frac{g}{4\cos\theta_W}\right)Z_\mu\left\{\sum_i \bar{\nu}_i^M\gamma^\mu(1+\gamma_5)\nu_i^M + \sum_i \bar{e}_i^M\gamma^\mu\left[\left(-1+4\sin^2\theta_W\right)-\gamma_5\right]e_i^M\right\}. \tag{13}$$

- Electromagnetic interactions:

$$\mathcal{L}^{EM} = e\sum_i\left(\bar{e}_i\gamma^\mu e_i + \bar{e}_i^M\gamma^\mu e_i^M\right)A_\mu. \tag{14}$$

- Yukawa interactions which contribute to the neutrino Dirac mass term:

This is the most important part which contributes to the aforementioned LFV rare processes. The gauge-invariant Yukawa Lagrangian can be written as follows:

$$\mathcal{L}_S = -\bar{l}_L^0 g_{Sl} l_R^{0,M}\phi_S + \text{H.c.} = -\left(\bar{\nu}_L^0 g_{Sl}\nu_R^{0,M} + \bar{e}_L^0 g_{Sl} e_R^{0,M}\right)\phi_S + \text{H.c.}, \tag{15}$$

where ϕ_S is the singlet scalar field whose vacuum expectation value (VEV) gives rise to the neutrino Dirac mass term [3]. This is combined with the Majorana mass term for the right-handed neutrinos in a see-saw mechanism as shown in [3]. In (15), ν_L^0, $\nu_R^{0,M}$, e_L^0, and $e_R^{0,M}$ denote column vectors with n components for n families and g_{Sl} denotes an $n \times n$ coupling matrix. From hereon, we will take for definiteness $n = 3$.

- Yukawa interactions involving $SU(2)_L$ singlets, e_R and e_L^M:

$$\mathcal{L}'_S = -\left(\bar{e}_R^0 g'_{Sl} e_L^{0,M}\right)\phi_S + \text{H.c.}, \tag{16}$$

where g'_{Sl} is a 3×3 matrix for 3 families.

- Yukawa interactions giving rise to the Majorana mass for the right-handed neutrinos:

$$\mathcal{L}_M = l_R^{M,T}\sigma_2\tau_2 g_M\tilde{\chi} l_R^M, \tag{17}$$

where

$$\tilde{\chi} = \frac{1}{\sqrt{2}}\vec{\tau}\cdot\vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix}, \tag{18}$$

and g_M is a 3×3 matrix for 3 families. Notice that Eq. (17) is written down for completeness in this Letter. As shown in [3], this term gives rise to an electroweak-scale Majorana mass for the right-handed neutrinos when the $SU(2)_L$-triplet Higgs field develops a VEV, $\langle\chi^0\rangle = v_M$, of the order $O(\Lambda_{EW})$. (The subtleties associated with such a large triplet VEV are discussed in [3].) As of now, there is no direct evidence for Higgs triplets, and for that matter, also for Higgs doublet(s). However, one of the characteristics of this model is the existence of particles such as doubly-charged Higgs bosons. Present limits of around 100 GeV are rather model-dependent. A study of the Higgs sector of our model is under investigation [11].

The above mixings between the SM fermions and their mirror counterparts also give rise to mass mixings in the charged lepton sector (as well as in the quark sector). A brief review of the points made in [3] goes as follows. Take, for example, one family of fermions. When ϕ_S develops a VEV v_S, one obtains a Dirac mass for the neutrino and a mass mixing between the SM and mirror charged leptons as follows (assuming $g'_{S1} = g_{S1}$)

$$m_\nu^D = g_{S1} v_S, \tag{19}$$

for the Dirac neutrino and

$$M_l = \begin{pmatrix} m_l & m_\nu^D \\ m_\nu^D & m_{lM} \end{pmatrix}, \tag{20}$$

for the charged SM and mirror leptons. In Eq. (20), m_l is a generic notation for the mass of an SM charged lepton obtained from the coupling to the SM Higgs doublet. Similarly, m_{lM} is a generic notation for the mirror charged lepton mass obtained in the same manner. The off-diagonal element in Eq. (20) comes from the cross coupling to the singlet scalar field through Eq. (15). The diagonalization of (20) gives the following eigenvalues for the charged (SM and mirror) lepton masses

$$\tilde{m}_l = m_l - \frac{(m_\nu^D)^2}{m_{lM} - m_l}, \tag{21a}$$

$$\tilde{m}_{lM} = m_{lM} + \frac{(m_\nu^D)^2}{m_{lM} - m_l}. \tag{21b}$$

As shown in [3], the Majorana mass of the right-handed neutrino $M_R = g_M v_M$, where $v_M \sim O(\Lambda_{\text{EW}})$. The see-saw mechanism then gives a light neutrino a mass $(m_\nu^D)^2/M_R$ of $O(< 1\ \text{eV})$ and a heavy neutrino of mass $M_Z/2 < M_R < O(\Lambda_{\text{EW}})$. This makes the right-handed neutrinos detectable by future collider experiments. This also implies that $m_\nu^D \sim 10^5$ eV. Since the mirror lepton mass is of the order of the electroweak scale [3], it follows that the second terms of the right-hand side of (21a), (21b) are tiny compared with the first terms and can be ignored. Hence, for all practical purposes, the masses of the charged fermions are those obtained in an SM way, i.e., through the Yukawa couplings with the SM Higgs doublet(s). As a result, the mass eigenstates for the charged (SM and mirror) leptons are principally obtained in this way. This will be the (rather good) approximation that we will use below in the discussion of LFV rare processes involving charged leptons.

In what follows, we will make use of the remarks made above concerning mass eigenstates for the charged leptons. To express (15) in terms of mass eigenstates, let us define

$$e_L^0 = U_L^l e_L; \qquad e_R^{0,M} = U_R^{lM} e_R^M. \tag{22}$$

Using (22), one can now rewrite the charged lepton part of (15) in terms of the mass eigenstates as follows

$$\mathcal{L}_{S,\text{charged}} = -(\bar{e}_L U^L e_R^M)\phi_S + \text{H.c.}, \tag{23}$$

where

$$U^L = U_L^{l,\dagger} g_{S1} U_R^{lM}. \tag{24}$$

From [3], one can deduce from (15) the Dirac mass matrix of the neutrino sector when $\langle\phi_S\rangle = v_S$ as follows

$$m_\nu^D = v_S g_{S1}. \tag{25}$$

In terms of (25), the matrix U^L can now be written as

$$U^L = U_L^{l,\dagger} \left(\frac{m_\nu^D}{v_S}\right) U_R^{lM}. \tag{26}$$

One can see that the matrix U^L which mixes different families of SM charged leptons with those of the mirror leptons now involves the Dirac neutrino mass matrix. LFV processes to be discussed in this Letter will in consequence indirectly probe the Dirac part of the neutrino mass matrix. In fact, one can invert Eq. (26) to obtain

$$\frac{m_\nu^D}{v_S} = U_L^l U^L U_R^{lM,\dagger}. \tag{27}$$

As discussed in [3], let us recall that the see-saw mechanism yields the following mass matrix for the light neutrino sector

$$m_{\nu,\text{light}} = -m_\nu^{D,T} M_R^{-1} m_\nu^D, \tag{28}$$

while the mass matrix for the heavy right-handed sector is simply M_R. Once again, let us notice that $U^L \to 0$ in the limit $m_{\nu,\text{light}}(m_\nu^D) \to 0$, and there will be no mixing between SM and mirror fermions.

For the $SU(2)_L$ singlets, one has

$$e_R^0 = U_R^l e_R; \qquad e_L^{0,M} = U_L^{lM} e_L^M, \tag{29}$$

(16) is rewritten as

$$\mathcal{L}'_S = -(\bar{e}_R U^R e_L^M)\phi_S + \text{H.c.}, \tag{30}$$

where

$$U^R = U_R^{l,\dagger} g'_{S1} U_L^{lM}. \tag{31}$$

Although it is not necessary to do so, one can further simplify the problem by assuming that $g'_{S1} = g_{S1}$ in which case we obtain

$$U^R = U_R^{l,\dagger} \left(\frac{m_\nu^D}{v_S}\right) U_L^{lM}. \tag{32}$$

Similarly to (27), one can invert (32) to obtain

$$\frac{m_\nu^D}{v_S} = U_R^l U^R U_L^{lM,\dagger}. \tag{33}$$

From the above discussion, one cannot fail but notice the deep connection between the Dirac part of the neutrino mass matrix m_ν^D and the matrices which are involved in LFV processes in our model, namely U^L and U^R.

3. The processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$

The processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ can now be computed in our model by using the interaction Lagrangians listed above. The Feynman rules needed for Fig. 1 can be read off Eqs. (8), (13), (14), (23), (30). Notice that there is also a contribution to these LFV processes coming from diagrams with a W and a light neutrino propagating inside the loop. But this is entirely negligible as noticed by [12].

Fig. 1 is the diagram which contains a magnetic moment term (i.e., proportional to $\sigma_{\mu\nu}$) and will be the one that we concentrate on. (Two other diagrams with the photon line attached

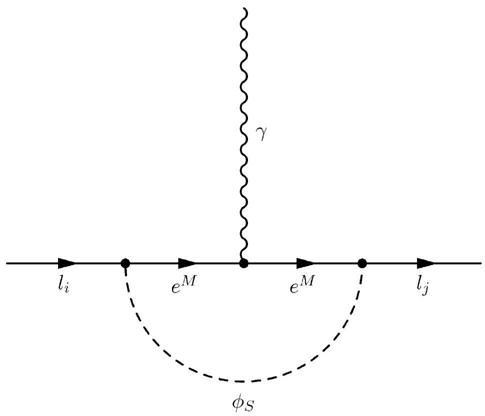

Fig. 1. Dominant diagram for the LFV process $l_i \to l_j + \gamma$.

to the external legs need not be considered, being proportional to γ_μ, see, e.g., [13].)

The general form of the amplitude can be written as

$$T(l_i \to l_j\gamma) = \epsilon^\lambda \bar{u}_{l_j}(p-q)\left\{iq^\nu\sigma_{\lambda\nu}\left[c_L^{(l_i)}\left(\frac{1-\gamma_5}{2}\right) + c_R^{(l_i)}\left(\frac{1+\gamma_5}{2}\right)\right]\right\}u_{l_i}(p). \quad (34)$$

The decay rate and the branching ratio are respectively

$$\Gamma(l_i \to l_j\gamma) = \frac{m_{l_i}^3}{16\pi}\left(\left|c_L^{(l_i)} + c_R^{(l_i)}\right|^2 + \left|c_L^{(l_i)} - c_R^{(l_i)}\right|^2\right), \quad (35)$$

$$B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu)} = \frac{12\pi^2}{m_\mu^2 G_F^2}\left(\left|c_L^{(\mu)} + c_R^{(\mu)}\right|^2 + \left|c_L^{(\mu)} - c_R^{(\mu)}\right|^2\right), \quad (36)$$

$$\frac{B(\tau \to \mu\gamma)}{B(\tau \to \mu\bar{\nu}_\mu\nu_\tau)} = \frac{\Gamma(\tau \to \mu\gamma)}{\Gamma(\tau \to \mu\bar{\nu}_\mu\nu_\tau)} = \frac{12\pi^2}{m_\tau^2 G_F^2}\left(\left|c_L^{(\tau)} + c_R^{(\tau)}\right|^2 + \left|c_L^{(\tau)} - c_R^{(\tau)}\right|^2\right). \quad (37)$$

$c_L^{(\mu)}$, $c_R^{(\mu)}$, $c_L^{(\tau)}$ and $c_R^{(\tau)}$ are computed from the diagram shown in Fig. 1 to be

$$c_L^{(\mu)} = \frac{1}{64\pi^2}\sum_i \frac{U_{i\mu}^{R*}U_{ei}^L}{m_i}, \quad (38)$$

$$c_R^{(\mu)} = \frac{1}{64\pi^2}\sum_i \frac{U_{i\mu}^{L*}U_{ei}^R}{m_i}, \quad (39)$$

$$c_L^{(\tau)} = \frac{1}{64\pi^2}\sum_i \frac{U_{i\tau}^{R*}U_{\mu i}^L}{m_i}, \quad (40)$$

$$c_R^{(\tau)} = \frac{1}{64\pi^2}\sum_i \frac{U_{i\tau}^{L*}U_{\mu i}^R}{m_i}, \quad (41)$$

where m_i are the masses of the charged mirror leptons. (In obtaining the above results, we looked at the coefficient of the term $p \cdot \epsilon$, where p is the momentum of the decaying particle and ϵ is the polarization of the photon. Also, as discussed in [3], $m_S \ll m_i$.) Notice in (36) and (37) that whereas $B(\mu \to e\bar{\nu}_e\nu_\mu) \sim 100\%$, one has $B(\tau \to \mu\bar{\nu}_\mu\nu_\tau) \sim 17.4\%$.

A few special cases are worth noticing.

- $g_{Sl} = g'_{Sl}$, $U_L^l = U_R^l$ and $U_R^{l^M} = U_L^{l^M}$:

The last two above equalities simply imply that one *assumes* that the mass matrices for the SM charged leptons and those of the mirror leptons are such that the "left" and "right" diagonalization matrices are *identical*. With these assumptions, one obtains

$$U^L = U^R = U^E. \quad (42)$$

With these assumptions, one obtains

$$c_L^{(\mu)} = c_R^{(\mu)} = \frac{1}{64\pi^2}\sum_i \frac{U_{i\mu}^{E*}U_{ei}^E}{m_i}, \quad (43a)$$

$$c_L^{(\tau)} = c_R^{(\tau)} = \frac{1}{64\pi^2}\sum_i \frac{U_{i\tau}^{E*}U_{\mu i}^E}{m_i}. \quad (43b)$$

It is also convenient to rewrite the above expressions in terms of a mass m_E (more like an average of the three mirror lepton masses) as follows

$$c_L^{(\mu)} = c_R^{(\mu)} = \frac{1}{64\pi^2}\frac{1}{m_E}\sum_i\left(\frac{m_E}{m_i}\right)\left(U_{i\mu}^{E*}U_{ei}^E\right), \quad (44a)$$

$$c_L^{(\tau)} = c_R^{(\tau)} = \frac{1}{64\pi^2}\frac{1}{m_E}\sum_i\left(\frac{m_E}{m_i}\right)\left(U_{i\tau}^{E*}U_{\mu i}^E\right). \quad (44b)$$

What are the implications of the above assumption?

- Degenerate charged mirror leptons: $m_i = m_E$.

In this case one is left with the factors $\sum_i U_{i\mu}^{E*}U_{ei}^E$ and $\sum_i U_{i\tau}^{E*}U_{\mu i}^E$ which would *vanish identically* if g_{Sl} (or equivalently m_ν^D/v_S) were proportional to the unit matrix because in this case the matrix U^E would be *unitary*. One would then have $c_L^{(\mu)} = c_R^{(\mu)} = 0$ and $c_L^{(\tau)} = c_R^{(\tau)} = 0$, implying $B(\mu \to e\gamma) = 0$ and $B(\tau \to \mu\gamma) = 0$. However U^E would no longer be *unitary* if m_ν^D/v_S were *not* proportional to the unit matrix, a fact which implies a non-vanishing value for the aforementioned branching ratios. Once again, one notices the implication of the form of neutrino mass matrices on LFV processes.

- Non-degenerate charged mirror leptons: $m_1 = m_E$, $m_2 = m_E + \delta m_2$, and $m_3 = m_E + \delta m_3$ with the assumption $|\delta m_{2,3}| \ll m_E$.

The non-degenerate case implies that the above branching ratios can be non-vanishing even if m_ν^D/v_S were proportional to the unit matrix.

4. Constraints on the model from $B(\mu \to e\gamma)_{\text{exp}}$ and $B(\tau \to \mu\gamma)_{\text{exp}}$

The experimental constraints we will use here are those from BaBar Collaboration [14] and Belle Collaboration [15] for $B(\tau \to \mu\gamma)_{\text{exp}}$ and from PDG [6] for $B(\mu \to e\gamma)_{\text{exp}}$.

They are $B(\tau \to \mu\gamma)_{\rm exp} < 6.8 \times 10^{-8}$ for BaBar Collaboration [14] and $B(\tau \to \mu\gamma)_{\rm exp} < 4.5 \times 10^{-8}$ for Belle Collaboration [15]; $B(\mu \to e\gamma)_{\rm exp} < 1.2 \times 10^{-11}$ [6]. Also we will be using $B(\tau \to \mu\bar{\nu}_\mu\nu_\tau) \sim 17.4\%$.

Although the most general discussion would involve the non-degenerate case with $U^L \neq U^R$, it is probably more illuminating to first investigate the non-degenerate scenario with $U^L = U^R$. For this, we will use Eq. (44). We obtain for $B(\mu \to e\gamma)$

$$B(\mu \to e\gamma) = \left(\frac{3}{256\pi^2}\right)\left(\frac{1}{m_\mu^2 G_F^2 m_E^2}\right) \times \left|\sum_i \left(\frac{m_E}{m_i}\right)\left(U_{i\mu}^{E*}U_{ei}^E\right)\right|^2 \tag{45}$$

and for $B(\tau \to \mu\gamma)$

$$B(\tau \to \mu\gamma) = \left(\frac{3}{256\pi^2}\right)\left(\frac{1}{m_\tau^2 G_F^2 m_E^2}\right) \times 0.174 \times \left|\sum_i \left(\frac{m_E}{m_i}\right)\left(U_{i\tau}^{E*}U_{\mu i}^E\right)\right|^2. \tag{46}$$

It is also useful to relate one branching ratio to another as follows

$$B(\mu \to e\gamma) = \frac{|\sum_i (\frac{m_E}{m_i})(U_{i\mu}^{E*}U_{ei}^E)|^2}{|\sum_i (\frac{m_E}{m_i})(U_{i\tau}^{E*}U_{\mu i}^E)|^2} \times \frac{B(\tau \to \mu\gamma)}{0.174} \times \left(\frac{m_\tau}{m_\mu}\right)^2. \tag{47}$$

We will illustrate the previous results with two examples: $m_E = 100$ GeV and $m_E = 200$ GeV.

- $m_E = 100$ GeV:

$$\left|\sum_i \left(\frac{m_E}{m_i}\right)\left(U_{i\mu}^{E*}U_{ei}^E\right)\right|^2 < 1.25 \times 10^{-15}, \tag{48}$$

$$\left|\sum_i \left(\frac{m_E}{m_i}\right)\left(U_{i\tau}^{E*}U_{\mu i}^E\right)\right|^2 < \begin{Bmatrix} 7.1 \\ 4.7 \end{Bmatrix} \times 10^{-12}, \tag{49}$$

where the first and second numbers on the right-hand side of Eq. (49) refer to BaBar and Belle Collaborations, respectively.

- $m_E = 200$ GeV:

$$\left|\sum_i \left(\frac{m_E}{m_i}\right)\left(U_{i\mu}^{E*}U_{ei}^E\right)\right|^2 < 5.0 \times 10^{-15}, \tag{50}$$

$$\left|\sum_i \left(\frac{m_E}{m_i}\right)\left(U_{i\tau}^{E*}U_{\mu i}^E\right)\right|^2 < \begin{Bmatrix} 28.4 \\ 18.8 \end{Bmatrix} \times 10^{-12}. \tag{51}$$

As we have mentioned in the beginning of the Letter, the matrix elements of U^E determine the lifetime and hence the decay length of the mirror charged leptons. Also, as we have seen above, some of these elements are constrained by the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$. Let us make some rough assumptions on the above bounds in order to gain some insights into what one might expect.

Let us take the case $m_E = 100$ GeV for definiteness and let us assume:

$$|\delta m_i| \ll m_E. \tag{52}$$

Let us also assume the following hierarchy (Case I) with $i = 1, 2, 3$:

$$U_{ie}^E \sim \lambda^3; \qquad U_{i\mu}^E \sim \lambda^2; \qquad U_{i\tau}^E \sim \lambda. \tag{53}$$

With the above assumptions, one can now rewrite the bounds (49), (48) (taking, e.g., the Belle value) as

$$\lambda^6 < 5.2 \times 10^{-13}, \tag{54}$$

$$\lambda^{10} < 1.4 \times 10^{-16}, \tag{55}$$

where (54) refers to the bound from $\tau \to \mu\gamma$ while (55) comes from the bound on $\mu \to e\gamma$. (54) gives $\lambda < 0.009$. This gives $\lambda^{10} < 10^{-21}$ which satisfies (55). Suppose that the process $\tau \to \mu\gamma$ can be experimentally probed with a branching ratio not too far below the current bound. What does it say about the detectability of $\mu \to e\gamma$? This can be easily estimated by looking at the relation (47)

$$B(\mu \to e\gamma) \approx 1.6 \times 10^3 \lambda^4 B(\tau \to \mu\gamma). \tag{56}$$

With $\lambda < 0.009$, one would have $B(\mu \to e\gamma) \sim 10^{-8} B(\tau \to \mu\gamma) < 4 \times 10^{-16}$ which makes the process $\mu \to e\gamma$ unobservable in the near future, about two orders of magnitude below the sensitivity of the MEG proposal [16].

A more interesting hierarchy (Case II) is as follows ($i = 1, 2, 3$):

$$U_{ie}^E \sim \lambda^3; \qquad U_{i\mu}^E \sim \lambda; \qquad U_{i\tau}^E \sim \lambda^2. \tag{57}$$

We now obtain the following bounds for (49), (48)

$$\lambda^6 < 5.2 \times 10^{-13}, \tag{58}$$

$$\lambda^8 < 1.4 \times 10^{-16}. \tag{59}$$

Note that (58) is identical to (54) since one is simply switching the role of μ and τ, but now the exponent on the left-hand side of (59) is lower 8 instead of 10. Again (58) gives $\lambda < 0.009$. One now has $\lambda^8 < 4.3 \times 10^{-17}$, a factor of three below (59) which is clearly satisfied. There is in addition a *huge* advantage: If $\tau \to \mu\gamma$ is *discovered*, e.g., slightly below the present bound then one *might expect* to discover $\mu \to e\gamma$ with a rate of about a factor of three below its present experimental limit! In fact, the relationship (47) is now

$$B(\mu \to e\gamma) \approx 1.6 \times 10^3 \lambda^2 B(\tau \to \mu\gamma). \tag{60}$$

With $\lambda < 0.009$, one now has $B(\mu \to e\gamma) \sim 10^{-4} B(\tau \to \mu\gamma) < 5 \times 10^{-12}$. The observability of one process implies that of the other. Note that the estimated bound on $B(\mu \to e\gamma)$ is well within the range of the MEG proposal [16].

Last but not least, we could also consider the "inverted hierarchy" scenario (Case III):

$$U_{ie}^E \sim \lambda; \qquad U_{i\mu}^E \sim \lambda^2; \qquad U_{i\tau}^E \sim \lambda^3. \tag{61}$$

This case results in for (49), (48)

$$\lambda^{10} < 5.2 \times 10^{-13}, \tag{62}$$

Table 1
Bounds on branching ratios for three mixing scenarios (53), (57), (61)

	$\lambda \leqslant$	$B(\tau \to \mu\gamma) \leqslant$	$B(\mu \to e\gamma) \leqslant$
Case I: (53)	0.009	4.5×10^{-8}	4×10^{-16}
Case II: (57)	0.009	4.5×10^{-8}	5×10^{-12}
Case III: (61)	0.002	1.2×10^{-25}	1.2×10^{-11}

$$\lambda^6 < 1.4 \times 10^{-16}. \tag{63}$$

This case is the mirror of Case I. Using (62), one obtains $\lambda < 0.06$. For λ close to this upper limit, (63) cannot be satisfied. If we satisfy (63) with $\lambda < 0.002$, $B(\tau \to \mu\gamma)$ which is proportional to $\lambda^{10} < 10^{-27}$ will be hopelessly small.

The above constraints (Cases I, II, and III) are listed for convenience in Table 1. Notice that the upper bounds on λ listed in Table 1 are those which satisfy the experimental constraints from *both* $B(\tau \to \mu\gamma)$ and $B(\mu \to e\gamma)$.

Notice that in all of the cases listed above the bound on $B(\tau \to e\gamma) < 1.1 \times 10^{-7}$ [17] which is weaker than the other two is trivially satisfied. That is the primary reason for using the constraints from the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$.

In summary, Case II appears to be the most interesting one in that both LFV branching ratios could in principle be observed. This scenario has also another interesting phenomenological consequence: The primary decay mode of a given mirror charged lepton is into a muon instead of a tau as in the first scenario or an electron as in the third scenario. What would a typical decay length be?

In the simple scenario described above, the primary decay mode of the mirror charged leptons is $\mu + \phi_S$ (assuming, e.g., that ν_R^M is approximately degenerate with its charged counterpart) with a coupling of the order $< 9 \times 10^{-3}$. For example, the decay rate of e_3^M is approximately $\Gamma(e_3^M \to \mu + \phi_S) \sim m_E \lambda^2/(32\pi)$ and a decay length $l = 1/\Gamma(e_3^M \to \mu + \phi_S)$. With the bound on $\lambda < 9 \times 10^{-3}$, one estimates the decay length to be $l > 2445$ fm, which is *microscopic*. A *macroscopic* decay length of the order of a few centimeters would imply a *much smaller* λ rendering the LFV processes discussed here practically *unobservable*.

5. Conclusion

The electroweak-scale right-handed neutrino model [3] has a number of phenomenological implications which could be explored experimentally in the near future. As mentioned in [3], the see-saw mechanism could be directly tested at colliders by searching for like-sign dilepton events coming from the production and decays of a pair of right-handed neutrinos into a pair of like-sign lighter charged mirror leptons. The subsequent decays of those charged mirror leptons into SM leptons, $e_R^M \to e_L + \phi_S$ (e_R^M and e_L are generic notations), provide the desired signals. How far from the beam pipe these decays occur will depend on the strength of the Yukawa interactions written down in Eqs. (23), (30). These Yukawa interactions are found to be proportional to the Dirac neutrino mass matrix which enters the see-saw mechanism and vanish in the limit where the light neutrino mass goes to zero. It turns out that these interactions also generate at one-loop level LFV processes such as the ones discussed here, namely $\mu \to e\gamma$ and $\tau \to \mu\gamma$. (The subject of μ–e conversion will be treated elsewhere.) These LFV processes put some interesting constraints on the model. In one example, it is found that if one process is observed (which by itself is already astonishing), the other will not be far from the current bound. Furthermore, within the framework of this model, a "macroscopic" decay length (a cm or so) of the mirror charged lepton, if discovered, implies that these LFV processes would be practically unobservable. Conversely, if any of these LFV processes is observed, the decay length would be tiny. This would require an extremely careful analysis to distinguish these like-sign dilepton events from possible backgrounds.

In summary, our model contains several phenomena which can be tested at future experimental facilities: (1) LFV processes at MEG and/or B factories; (2) electroweak-scale right-handed neutrinos at future colliders (LHC, ILC).

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