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## Beyond generalized Proca theories

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## ABSTRACT

We consider higher-order derivative interactions beyond second-order generalized Proca theories that propagate only the three desired polarizations of a massive vector field besides the two tensor polarizations from gravity. These new interactions follow the similar construction criteria to those arising in the extension of scalar–tensor Horndeski theories to Gleyzes–Langlois–Piazza–Vernizzi (GLPV) theories. On the isotropic cosmological background, we show the existence of a constraint with a vanishing Hamiltonian that removes the would-be Ostrogradski ghost. We study the behavior of linear perturbations on top of the isotropic cosmological background in the presence of a matter perfect fluid and find the same number of propagating degrees of freedom as in generalized Proca theories (two tensor polarizations, two transverse vector modes, and two scalar modes). Moreover, we obtain the conditions for the avoidance of ghosts and Laplacian instabilities of tensor, vector, and scalar perturbations. We observe key differences in the scalar sound speed, which is mixed with the matter sound speed outside the domain of generalized Proca theories.

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## 1. Introduction

General Relativity (GR) is still the fundamental theory for describing the gravitational interactions even after a century. Cosmological observations [1–3] led to the standard model yielding an accelerated expansion of the late Universe driven by the cosmological constant. The standard model of particle physics describes the strong and electro-weak interactions with an exquisite experimental success marking the milestone in high-energy physics. It is still a big challenge to unify gravity with the known forces in Nature and to merge these two standard models into a single theory. Moreover, employing the usual techniques of quantum field theory, we are not able to explain the small observed value of the cosmological constant. On the other hand, this has motivated to consider infra-red modifications of gravity which could account for an appropriate screening of the cosmological constant. On a similar footing, one can also consider infra-red gravitational modifications to realize an effective negative pressure against gravity in form of dark energy [4].

The simplest and mostly studied large-distance modification of gravity is attributed to an additional scalar field beyond the

standard model of particle physics, e.g., the DGP braneworld [5], Galileons [6], and massive gravity [7]. The scalar field arising in such theories can have non-trivial self-interactions but also it can be generally coupled to gravity [8,9]. These interactions have to be constructed with great caution to guarantee the absence of ghost-like Ostrogradski instability [10], which otherwise would yield an unbounded Hamiltonian from below.

It is well known that matter fields have to be coupled to the Lovelock invariants or to the divergence-free tensors constructed from the Lovelock invariants. Hence they can for instance couple to the volume element  $\sqrt{-g}$  and to the Ricci scalar  $R$  which are the only two non-trivial Lovelock invariants, since the Gauss–Bonnet term is topological in four dimensions. Furthermore, they can couple to the divergence-free metric  $g_{\mu\nu}$ , Einstein tensor  $G_{\mu\nu}$ , and the double dual Riemann tensor  $L_{\mu\nu\alpha\beta}$ . In flat space–time the ghost-free scalar interactions with derivatives acting on them are known as the Galileon interactions [6]. If one would naively promote the partial derivatives to covariant derivatives, this procedure would yield the equations of motion higher than second order [10]. The appearance of higher-order derivative terms can be avoided by introducing non-minimal couplings to gravity through the Lovelock invariants or the divergence-free tensors.

Horndeski theories [11] constitute the most general scalar–tensor interactions with second-order equations of motion. In these theories there is only one scalar degree of freedom (DOF)

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besides two graviton polarizations without having the Ostrogradski instability [12]. It is a natural question to ask whether abandoning the requirement of second-order equations of motion inevitably alters the propagating DOF. Allowing interactions beyond the Horndeski domain will introduce derivative interactions higher than second order. However, this does not necessarily mean that the number of propagating DOF increases. Exactly this spirit was followed in GLPV theories [13], where they expressed the Horndeski Lagrangian in terms of the 3 + 1 Arnowitt–Deser–Misner (ADM) decomposition of space–time in the unitary gauge [14] and did not impose the two conditions that Horndeski theories obey. The Hamiltonian analysis in the unitary gauge revealed that there is still only one scalar DOF [15]. The cosmology and the spherically symmetric solutions in GLPV theories have been extensively studied in Refs. [16–19]. The ghost freedom beyond the unitary gauge and beyond a conformal and disformal transformation is still an ongoing research investigation in the literature [20–25].

Even if the large-distance modifications of gravity through a scalar field are simpler, considerations in form of a vector field can yield interesting phenomenology for the cosmic expansion and growth of large-scale structures. Furthermore, the presence of the vector field might explain the anomalies reported in CMB observations [26]. For a gauge-invariant vector field, the only new interaction is via a coupling of the field strength tensor to the double dual Riemann tensor. Unfortunately, the existence of derivative self-interactions similar to those arising for covariant Galileons is forbidden for a massless, Lorentz-invariant vector field coupled to gravity [27].

However, this negative result does not apply to massive vector fields, for which one can successfully construct derivative self-interactions due to the broken  $U(1)$  symmetry. The idea was to construct interactions with only three propagating degrees of freedom, out of which two would correspond to the transverse and one to the longitudinal mode of the vector field. This was systematically constructed in Ref. [28] together with the Hessian and Hamiltonian analysis. The key point is the requirement that the longitudinal mode belongs to the class of Galileon/Horndeski theories. This constitutes the generalized Proca theories up to the quintic Lagrangian on curved space–time with second-order equations of motion, which is guaranteed by the presence of non-minimal couplings to the Lovelock invariants in the same spirit as in the scalar Horndeski theories [28–31].

One can also construct the sixth-order derivative interactions, if one allows for trivial interaction terms for the longitudinal mode [30,31]. Its generalization to curved space–time contains the double dual Riemann tensor, which keeps the equations of motion up to second order [31]. In fact, this sixth-order Lagrangian accommodates similar vector–tensor theories constructed by Horndeski in 1976 [32]. We refer the reader to Refs. [33–41] for related works. The second-order massive vector theories up to the sixth-order Lagrangian studied in Refs. [31,40,41] constitute the generalized Proca theories.

It is a natural follow-up question to ask whether or not the extension of generalized Proca theories is possible in such a way that there are still three propagating vector DOF even with derivatives higher than second order. In the GLPV extension of Horndeski theories, the Lagrangians of two additional scalar derivative interactions can be expressed in terms of the anti-symmetric Levi-Civita tensor. Outside the domain of generalized Proca theories, one can also construct generalized Lagrangians by using the Levi-Civita tensor. It is then expected that, in beyond-generalized Proca theories, the longitudinal vector mode would have some correspondence with the scalar mode in GLPV theories, but there will be also new interactions corresponding to the purely intrinsic vector modes.

In this Letter, we will propose candidates for new beyond-generalized Proca Lagrangians in Sec. 2 to study the possibility of the healthy extension of generalized Proca theories. In Sec. 3 we derive the background equations of motion on the flat Friedmann–Lemaître–Robertson–Walker (FLRW) background and the associated Hamiltonian  $\mathcal{H}$ . We see that, even in the presence of these new interactions, there exists a second class constraint ( $\mathcal{H} = 0$ ) that removes the Ostrogradski ghost. In Sec. 4 we consider linear cosmological perturbations on the flat FLRW background and show that the number of DOF in beyond-generalized Proca theories is not altered relative to that in generalized Proca theories. We also study what kinds of differences arise for the stability of perturbations by extending generalized Proca theories to beyond-generalized Proca theories. Sec. 5 is devoted to conclusions and future outlook.

## 2. Extension of generalized Proca theories to beyond-generalized Proca theories

The generalized Proca theories are characterized by second-order interactions with two transverse and one longitudinal polarizations of a vector field  $A^\mu$  coupled to gravity. Introducing the field tensor  $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ , where  $\nabla_\mu$  is the covariant derivative operator, the four-dimensional action of generalized Proca theories is given by

$$S_{\text{gen.Proca}} = \int d^4x \sqrt{-g} \sum_{i=2}^6 \mathcal{L}_i, \quad (2.1)$$

where  $g$  is the determinant of the metric tensor  $g_{\mu\nu}$ , and

$$\mathcal{L}_2 = G_2(X, F, Y), \quad (2.2)$$

$$\mathcal{L}_3 = G_3 \nabla_\mu A^\mu, \quad (2.3)$$

$$\mathcal{L}_4 = G_4 R + G_{4,X} \left[ (\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right], \quad (2.4)$$

$$\begin{aligned} \mathcal{L}_5 = & G_5 G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,X} [(\nabla_\mu A^\mu)^3 \\ & - 3 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2 \nabla_\rho A_\sigma \nabla^\rho A^\sigma \nabla^\sigma A_\gamma] \\ & - g_5 \tilde{F}^{\alpha\mu} \tilde{F}^{\beta\nu} \nabla_\alpha A_\beta, \end{aligned} \quad (2.5)$$

$$\begin{aligned} \mathcal{L}_6 = & G_6 L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta \\ & + \frac{1}{2} G_{6,X} \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu. \end{aligned} \quad (2.6)$$

The function  $G_2$  depends on the following three quantities

$$X = -\frac{A_\mu A^\mu}{2}, \quad F = -\frac{F_{\mu\nu} F^{\mu\nu}}{4}, \quad Y = A^\mu A^\nu F_{\mu\alpha} F_{\nu\alpha}, \quad (2.7)$$

while  $G_{3,4,5,6}$  and  $g_5$  are arbitrary functions of  $X$  with the notation  $G_{i,X} \equiv \partial G_i / \partial X$ . The vector field is coupled to the Ricci scalar  $R$  and the Einstein tensor  $G_{\mu\nu}$  through the functions  $G_4(X)$  and  $G_5(X)$ . The  $L^{\mu\nu\alpha\beta}$  and  $\tilde{F}^{\mu\nu}$  are the double dual Riemann tensor and the dual strength tensor defined, respectively, by

$$L^{\mu\nu\alpha\beta} = \frac{1}{4} \mathcal{E}^{\mu\nu\rho\sigma} \mathcal{E}^{\alpha\beta\gamma\delta} R_{\rho\sigma\gamma\delta}, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \mathcal{E}^{\mu\nu\alpha\beta} F_{\alpha\beta}, \quad (2.8)$$

where  $R_{\rho\sigma\gamma\delta}$  is the Riemann tensor and  $\mathcal{E}^{\mu\nu\rho\sigma}$  is the Levi-Civita tensor obeying the normalization  $\mathcal{E}^{\mu\nu\rho\sigma} \mathcal{E}_{\mu\nu\rho\sigma} = -4!$ . We can potentially include the dependence of the quantity  $F^{\mu\nu} \tilde{F}_{\mu\nu}$  in the function  $G_2$  [28,36]. If we impose the parity invariance, however, it does not contribute to the perturbations at linear order, so we do not take into account such dependence in  $G_2$ .

The action (2.1) was constructed to keep the equations of motion up to second order to avoid the appearance of an extra DOF

besides two transverse and one longitudinal modes of the vector field [28]. Each Lagrangian density can be expressed in terms of the Levi-Civita tensor  $\mathcal{E}_{\mu_1\mu_2\mu_3\mu_4}$  and the first derivatives of  $A^\mu$ . The anti-symmetric property of  $\mathcal{E}_{\mu_1\mu_2\mu_3\mu_4}$  allows us to eliminate the terms containing time derivatives of the temporal vector component  $A^0$ , such that the additional DOF does not propagate. In this set up the derivatives of  $A^\mu$  higher than first-order are not taken into account, as they give rise to the derivatives of a scalar field  $\pi$  higher than second order in the Lagrangian by taking the limit  $A^\mu \rightarrow \nabla^\mu \pi$ .

The action (2.1) consists of three parts. The first part corresponds to the Lagrangian densities (with the index  $i = 0, 1, 2, 3$ ):

$$\mathcal{L}_{i+2}^{\text{Ga}} = g_{i+2} \delta_{\alpha_1 \dots \alpha_i \gamma_{i+1} \dots \gamma_4}^{\beta_1 \dots \beta_i \gamma_{i+1} \dots \gamma_4} \nabla_{\beta_1} A^{\alpha_1} \dots \nabla_{\beta_i} A^{\alpha_i}, \quad (2.9)$$

where  $g_{i+2}$  are functions of  $X$  and we have introduced the operator  $\hat{\delta}_{\alpha_1 \dots \alpha_i \gamma_{i+1} \dots \gamma_4}^{\beta_1 \dots \beta_i \gamma_{i+1} \dots \gamma_4} = \mathcal{E}_{\alpha_1 \dots \alpha_i \gamma_{i+1} \dots \gamma_4} \mathcal{E}^{\beta_1 \dots \beta_i \gamma_{i+1} \dots \gamma_4}$ . They recover those of Minkowski Galileons from the scalar part  $\pi$  of  $A^\mu$  for the functions  $g_{2,3,4,5} \propto X$ . The second part arises from the terms derived by exchanging some of the indices in  $\mathcal{L}_{4,5}^{\text{Ga}}$ , i.e.,

$$\mathcal{L}_4^V = \tilde{h}_4 \hat{\delta}_{\alpha_1 \alpha_2 \gamma_3 \gamma_4}^{\beta_1 \beta_2 \gamma_3 \gamma_4} \nabla_{\beta_1} A_{\beta_2} \nabla^{\alpha_1} A^{\alpha_2}, \quad (2.10)$$

$$\mathcal{L}_5^V = \tilde{h}_5 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \gamma_4}^{\beta_1 \beta_2 \beta_3 \gamma_4} \nabla^{\alpha_1} A^{\alpha_2} \nabla_{\beta_1} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3}, \quad (2.11)$$

with again  $\hat{\delta}_{\alpha_1 \alpha_2 \gamma_3 \gamma_4}^{\beta_1 \beta_2 \gamma_3 \gamma_4} = \mathcal{E}_{\alpha_1 \alpha_2 \gamma_3 \gamma_4} \mathcal{E}^{\beta_1 \beta_2 \gamma_3 \gamma_4}$  and general functions  $\tilde{h}_4$  and  $\tilde{h}_5$  depending on  $X$ . These interactions can be regarded as the intrinsic vector modes that vanish in the scalar limit  $A^\mu \rightarrow \nabla^\mu \pi$ . The Lagrangian density  $\mathcal{L}_6$  contains the intrinsic vector contribution

$$\mathcal{L}_6^V = \tilde{h}_6 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{\beta_1 \beta_2 \beta_3 \beta_4} \nabla_{\beta_1} A_{\beta_2} \nabla^{\alpha_1} A^{\alpha_2} \nabla_{\beta_3} A^{\alpha_3} \nabla_{\beta_4} A^{\alpha_4}. \quad (2.12)$$

The third part corresponds to the non-minimal coupling terms  $G_4(X)R$ ,  $G_5(X)G_{\mu\nu}\nabla^\mu A^\nu$ , and  $G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu\nabla_\alpha A_\beta$ , which are required to keep the equations of motion up to second order [28, 31].

If we try to make the minimal extension of the above generalized Proca theories, we can take into account terms containing the products of  $A^{\alpha_1} A_{\beta_1}$  and the first derivatives of  $A^\mu$ . Let us consider the following new Lagrangian densities

$$\mathcal{L}_4^{\text{N}} = f_4 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \gamma_4}^{\beta_1 \beta_2 \beta_3 \gamma_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3}, \quad (2.13)$$

$$\mathcal{L}_5^{\text{N}} = f_5 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{\beta_1 \beta_2 \beta_3 \beta_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}, \quad (2.14)$$

$$\tilde{\mathcal{L}}_5^{\text{N}} = \tilde{f}_5 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{\beta_1 \beta_2 \beta_3 \beta_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A^{\alpha_3} \nabla_{\beta_2} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}, \quad (2.15)$$

with the functions  $f_{4,5}$  and  $\tilde{f}_5$  depending on  $X$ . If we take the limit  $A^\mu \rightarrow \nabla^\mu \pi$ , the Lagrangian densities  $\mathcal{L}_4^{\text{N}}$  and  $\mathcal{L}_5^{\text{N}}$  for the scalar field  $\pi$  are equivalent to those appearing in GLPV theories [13]. Thus, the above construction of new derivative interactions is analogous to the GLPV extension of scalar Horndeski theories, but in our case the situation is more involved due to the existence of transverse vector modes. We also need to take into account the intrinsic vector term  $\tilde{\mathcal{L}}_5^{\text{N}}$  derived after exchanging the indices  $\beta_2$  and  $\alpha_3$  in  $\mathcal{L}_5^{\text{N}}$ . Note, that we did not include the term  $\tilde{\mathcal{L}}_4^{\text{N}} = \tilde{f}_4 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \gamma_4}^{\beta_1 \beta_2 \beta_3 \gamma_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A^{\alpha_3} \nabla_{\beta_2} A_{\beta_3}$ , since it is already included in  $\mathcal{L}_2$ . For the sixth-order interaction, we run out of the indices to make the product  $A^{\alpha_1} A_{\beta_1}$ . Instead, we consider the following Lagrangian density

$$\mathcal{L}_6^{\text{N}} = \tilde{f}_6 \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{\beta_1 \beta_2 \beta_3 \beta_4} \nabla_{\beta_1} A_{\beta_2} \nabla^{\alpha_1} A^{\alpha_2} \nabla_{\beta_3} A^{\alpha_3} \nabla_{\beta_4} A^{\alpha_4}, \quad (2.16)$$

with  $\tilde{f}_6(X)$ . This is of the same form as Eq. (2.12), but the difference from generalized Proca theories is that the relative coefficient to the non-minimal coupling term  $L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu\nabla_\alpha A_\beta$  is detuned

in beyond-generalized Proca theories, which generates derivatives higher than second order. Then, the new Lagrangian densities in our set up are given by

$$\mathcal{L}^{\text{N}} = \mathcal{L}_4^{\text{N}} + \mathcal{L}_5^{\text{N}} + \tilde{\mathcal{L}}_5^{\text{N}} + \mathcal{L}_6^{\text{N}}. \quad (2.17)$$

To study the effect of derivative interactions in beyond-generalized Proca theories, we consider the following action

$$S = \int d^4x \sqrt{-g} \left( \sum_{i=2}^6 \mathcal{L}_i + \mathcal{L}^{\text{N}} + \mathcal{L}_M \right), \quad (2.18)$$

where  $\mathcal{L}_M$  is the matter Lagrangian density.

In the following, we would like to analyze the possible number of propagating DOF in beyond-generalized Proca theories explained above. The worry is that the new terms (2.17) might induce the propagation of a ghostly DOF associated with the Ostrogradski instability. For this purpose, we shall focus on the study for both the background (Sec. 3) and the linear perturbation (Sec. 4) on top of the isotropic FLRW background.

Note that this first analysis does not necessarily guarantee the absence of ghostly DOF on more general backgrounds. For a complete proof of the absence of extra DOF, the full 3 + 1 ADM Hamiltonian analysis is needed without fixing the gauge.

### 3. Background equations of motion and the Hamiltonian

#### 3.1. Background and perturbed quantities

To derive the background and perturbation equations of motion on the isotropic cosmological background, we consider the general perturbed metric in the form [42]

$$ds^2 = -(1 + 2\alpha) dt^2 + 2(\chi_{|i} + V_i) dt dx^i + a^2(t) [(1 + 2\psi)\delta_{ij} + 2E_{ij} + 2F_{i|j} + h_{ij}] dx^i dx^j, \quad (3.1)$$

where  $\alpha, \chi, \psi, E$  are scalar metric perturbations,  $V_i, F_i$  are vector perturbations, and  $h_{ij}$  is the tensor perturbation. The index “|” represents the covariant derivative with respect to the three-dimensional spatial metric. Expanding the action (2.18) up to first order in scalar perturbations, we can obtain the background equations of motion on the flat FLRW background described by the line element  $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ . The linear perturbation equations also follow from the action (2.18) expanded up to second order in scalar, vector, and tensor perturbations. Before doing so, we first remove redundant gauge DOFs.

Under a scalar gauge transformation  $t \rightarrow t + \delta t$  and  $x^i \rightarrow x^i + \delta^{ij}\delta x_{|j}$ , the scalar perturbations  $\psi$  and  $E$  transform, respectively, as  $\psi \rightarrow \psi - H\delta t$  and  $E \rightarrow E - \delta x$  [43], where  $H = \dot{a}/a$  is the Hubble expansion rate and a dot represents a derivative with respect to  $t$ . Under a vector gauge transformation  $x^i \rightarrow x^i + \delta x^i$ , the vector perturbation  $F_i$  transforms as  $F_i \rightarrow F_i - \delta x_i$ . If we choose the flat gauge

$$\psi = 0, \quad E = 0, \quad F_i = 0, \quad (3.2)$$

then the time slicing  $\delta t$ , the spatial threading  $\delta x$ , and the infinitesimal vector  $\delta x_i$  are unambiguously fixed.

In what follows, we shall derive the equations of motion for the background and cosmological perturbations under the gauge choice (3.2). By fixing the gauge in this way, we already removed the extra gauge DOFs from the beginning. We have also expanded the action (2.18) up to second order in perturbations without fixing the gauge from the beginning and have derived the equations of motion from the general gauge-invariant Lagrangian. Choosing the flat gauge (3.2) in the equations of motion at the end, we confirmed that the resulting dynamical equations for tensor, vector,

and scalar perturbations are equivalent to those derived by fixing the gauge from the beginning in the Lagrangian.

The vector perturbation satisfies the transverse condition  $\partial^i V_i = 0$ , where  $\partial^i$  represents the spatial derivative. The tensor perturbation  $h_{ij}$  obeys the transverse and traceless conditions  $\partial^i h_{ij} = 0$  and  $h_i^i = 0$ . We express the temporal and spatial components of the vector field  $A^\mu$ , as

$$A^0 = \phi(t) + \delta\phi, \quad A^i = \frac{1}{a^2(t)} \delta^{ij} (\partial_j \chi_V + E_j), \quad (3.3)$$

where  $\phi(t)$  is the background value of the temporal vector component,  $\delta\phi$  and  $\chi_V$  are the scalar perturbations, and  $E_j$  is the intrinsic vector perturbation obeying the transverse condition  $\partial^j E_j = 0$ .

For the matter sector, we take into account a perfect fluid described by the Schutz–Sorkin action [44]:

$$S_M = - \int d^4x \left[ \sqrt{-g} \rho_M(n) + J^\mu \left( \partial_\mu \ell + \sum_{i=1}^2 \mathcal{A}_i \partial_\mu \mathcal{B}_i \right) \right]. \quad (3.4)$$

The energy density  $\rho_M$  depends on the fluid number density  $n = \sqrt{J_\mu J^\mu} / \sqrt{g}$ , where the temporal and spatial components of  $J^\mu$  can be decomposed, respectively, as

$$J^0 = \mathcal{N}_0 + \delta J, \quad J^i = \frac{1}{a^2} \delta^{ik} (\partial_k \delta j + W_k), \quad (3.5)$$

where  $\mathcal{N}_0$  is a constant associated with the total background particle number (related with the background number density  $n_0$  as  $\mathcal{N}_0 = n_0 a^3$ ),  $\delta J$  and  $\delta j$  are the scalar perturbations, and  $W_k$  is the vector perturbation satisfying  $\partial^k W_k = 0$ .

The scalar quantity  $\ell$  can be decomposed as  $\ell = \ell_0 - \rho_{M,n} v$ , where the background value  $\ell_0$  obeys the relation  $\partial_0 \ell_0 = -\rho_{M,n} \equiv -\partial \rho_M / \partial n$  and  $v$  is the perturbation associated with the velocity potential. Then, we can write  $\ell$  in the form  $\ell = - \int^t \rho_{M,n}(\tilde{t}) d\tilde{t} - \rho_{M,n} v$ .

The terms  $\mathcal{A}_i$  and  $\mathcal{B}_i$  in Eq. (3.4) correspond to vector perturbations obeying the transverse conditions. It is sufficient to consider the  $x, y$  components of  $\mathcal{A}_i$  whose perturbations depend on  $t$  and  $z$  alone, i.e.,  $\mathcal{A}_1 = \delta \mathcal{A}_1(t, z)$  and  $\mathcal{A}_2 = \delta \mathcal{A}_2(t, z)$ . One can extract the required property of the vector mode by choosing  $\mathcal{B}_1 = x + \delta \mathcal{B}_1(t, z)$  and  $\mathcal{B}_2 = y + \delta \mathcal{B}_2(t, z)$ . Varying the matter action (3.4) with respect to  $J^\mu$ , it follows that

$$J_\mu = \frac{n \sqrt{-g}}{\rho_{M,n}} \left( \partial_\mu \ell + \sum_{i=1}^2 \mathcal{A}_i \partial_\mu \mathcal{B}_i \right), \quad (3.6)$$

which is related with the fluid four-velocity  $u_\mu$ , as  $u_\mu = J_\mu / (n \sqrt{-g})$ . The spatial part of  $u_\mu$  can be expressed as

$$u_i = -\partial_i v + v_i, \quad (3.7)$$

where  $v_i$  is the transverse vector perturbation associated with  $\delta \mathcal{A}_i$ , as  $\delta \mathcal{A}_i = \rho_{M,n} v_i$ .

At the background level, the fluid action (3.4) reads

$$S_M^{(0)} = \int d^4x \sqrt{-g} P_M(n_0), \quad P_M(n_0) = n_0 \rho_{M,n} - \rho_M, \quad (3.8)$$

where  $P_M$  corresponds to the fluid pressure. As far as the scalar perturbation is concerned, the perfect fluid can be also described by the k-essence action [45]

$$S_M = \int d^4x \sqrt{-g} P_M(Z), \quad Z = -\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma, \quad (3.9)$$

where the pressure  $P_M$  depends on the kinetic term of a scalar field  $\sigma$  (see also Refs. [46]). At the background level the matter energy density is given by  $\rho_M = 2Z P_{M,Z} - P_M$ , so there is the

correspondence  $n_0 \rho_{M,n} \rightarrow 2Z P_{M,Z} = \rho_M + P_M$ . From the k-essence action (3.9) we obtain the density perturbation  $\delta \rho_M$ , the pressure perturbation  $\delta P_M$ , and the velocity potential  $v$ , respectively, as

$$\begin{aligned} \delta \rho_M &= (P_{M,Z} + 2Z P_{M,ZZ}) \delta Z, \\ \delta P_M &= P_{M,Z} \delta Z, \\ v &= \frac{\delta \sigma}{\dot{\sigma}}, \end{aligned} \quad (3.10)$$

where  $\delta Z$  corresponds to

$$\delta Z = \dot{\sigma} \delta \sigma - \dot{\sigma}^2 \alpha. \quad (3.11)$$

As far as the tensor and scalar perturbations are concerned, we can employ either the Schutz–Sorkin action or the k-essence action, but for the computation of vector perturbations we need to resort to the Schutz–Sorkin action.

We shall expand the action (2.18) together with the Schutz–Sorkin action (3.4) up to second-order in perturbations on the flat FLRW background to discuss the propagating DOF. In doing so, we perform the following field redefinitions:

$$\begin{aligned} Z_i &= E_i + \phi(t) V_i, \\ \psi &= \chi_V + \phi(t) \chi, \\ \delta \rho_M &= \frac{\rho_{M,n}}{a^3} \delta J, \end{aligned} \quad (3.12)$$

where  $Z_i$  and  $\psi$  correspond to the vector and scalar parts of  $A_i$  respectively, and  $\delta \rho_M$  is the matter density perturbation. The vector field  $Z_i$  obeys the transverse condition  $\partial^i Z_i = 0$ , so there are two independent components. At first order, the perturbation  $\delta n$  of the fluid number density is equivalent to  $\delta \rho_M / \rho_{M,n}$ .

### 3.2. Background equations

Expanding the action (2.18) up to first order in scalar perturbations, the resulting first-order action is given by

$$S^{(1)} = a^3 (\mathcal{C}_1 \alpha + \mathcal{C}_2 \delta\phi + \mathcal{C}_3 v), \quad (3.13)$$

where we introduced the following short-cuts for convenience

$$\begin{aligned} \mathcal{C}_1 &= G_2 + G_{2,X} \phi^2 + 3G_{3,X} H \phi^3 + 6(G_4 + G_{4,XX} \phi^4) H^2 \\ &\quad - (G_{5,X} + G_{5,XX} \phi^2) H^3 \phi^3 - \rho_M \\ &\quad + 6 \left[ 3f_4 + f_{4,X} \phi^2 + H \phi (3f_5 + f_{5,X} \phi^2) \right] H^2 \phi^4, \end{aligned} \quad (3.14)$$

$$\begin{aligned} \mathcal{C}_2 &= \phi \{ G_{2,X} + 3G_{3,X} H \phi + 6(G_{4,X} + G_{4,XX} \phi^2) H^2 \\ &\quad - (3G_{5,X} + G_{5,XX} \phi^2) H^3 \phi \\ &\quad + 6[4f_4 + f_{4,X} \phi^2 + (5f_5 + f_{5,X} \phi^2) H \phi] H^2 \phi^2 \}, \end{aligned} \quad (3.15)$$

$$\mathcal{C}_3 = -\frac{\mathcal{N}_0}{a^3} \left( \dot{\rho}_{M,n} + 3H \frac{\mathcal{N}_0}{a^3} \rho_{M,nn} \right), \quad (3.16)$$

where  $H = \dot{a}/a$  is the Hubble expansion rate. Variations of the action (3.13) with respect to  $\alpha, \delta\phi, v$  give rise to the background equations

$$\mathcal{C}_i = 0 \quad (i = 1, 2, 3), \quad (3.17)$$

respectively. On using the properties  $\mathcal{N}_0 = n_0 a^3$  and  $n_0 \rho_{M,n} = \rho_M + P_M$ , the third equation ( $\mathcal{C}_3 = 0$ ) corresponds to the matter continuity equation

$$\dot{\rho}_M + 3H (\rho_M + P_M) = 0. \quad (3.18)$$



In the k-essence description of the perfect fluid, the third term on the r.h.s. of Eq. (3.13) is replaced by  $a^3 P_{M,Z} \dot{\sigma} \delta \sigma$ . Variation with respect to  $\delta \sigma$  leads to the matter equation of motion  $\frac{d}{dt}(a^3 P_{M,Z} \dot{\sigma}) = 0$ , i.e.,

$$\left( P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ} \right) \ddot{\sigma} + 3H P_{M,Z} \dot{\sigma} = 0. \quad (3.19)$$

Using the correspondence  $\rho_M = 2Z P_{M,Z} - P_M$ , the continuity Eq. (3.18) follows from Eq. (3.19).

The terms containing  $f_4$  and  $f_5$  in Eqs. (3.14) and (3.15) correspond to the new terms arising from the Lagrangians (2.13) and (2.14). They originate from the longitudinal component of the vector field, so it is expected that the equations of motion can be written in terms of the quantities similar to those appearing in GLPV theories [13]. To see the correspondence with GLPV theories, we introduce the following quantities

$$\begin{aligned} A_2 &= G_2, & A_3 &= (2X)^{3/2} E_{3,X}, \\ A_4 &= -G_4 + 2X G_{4,X} + 4X^2 f_4, \\ A_5 &= -\sqrt{2} X^{3/2} \left( \frac{1}{3} G_{5,X} - 4X f_5 \right), \\ B_4 &= G_4, & B_5 &= (2X)^{1/2} E_5, \end{aligned} \quad (3.20)$$

where  $E_3(X)$  and  $E_5(X)$  are auxiliary functions [14] satisfying

$$G_3 = E_3 + 2X E_{3,X}, \quad G_{5,X} = \frac{E_5}{2X} + E_{5,X}. \quad (3.21)$$

Then, the two background equations  $\mathcal{C}_1 = 0$  and  $\mathcal{C}_2 = 0$  can be written in compact forms:

$$A_2 - 6H^2 A_4 - 12H^3 A_5 = \rho_M, \quad (3.22)$$

$$\phi \left( A_{2,X} + 3H A_{3,X} + 6H^2 A_{4,X} + 6H^3 A_{5,X} \right) = 0. \quad (3.23)$$

Taking the time derivative of Eq. (3.22) and using Eq. (3.18), it follows that

$$\dot{A}_3 + 4\dot{H} A_4 + 4H \dot{A}_4 + 12H \dot{H} A_5 + 6H^2 \dot{A}_5 = \rho_M + P_M. \quad (3.24)$$

The background Eqs. (3.22) and (3.24) are of the same forms as those in GLPV theories (see Eqs. (2.15) and (2.16) of Ref. [17]) with the particular relation (3.23). In GLPV theories the constraint (3.23) is absent, but in beyond-generalized Proca theories the relation (3.23) gives the constraint on the background trajectory with  $\phi$  always related to  $H$  [40] (e.g., analogous to the tracker solution [47] found for scalar Galileons).

From Eq. (3.20) there are two particular relations

$$\begin{aligned} A_4 + B_4 - 2X B_{4,X} &= 4X^2 f_4, \\ A_5 + \frac{1}{3} X B_{5,X} &= (2X)^{5/2} f_5. \end{aligned} \quad (3.25)$$

In generalized Proca theories the Lagrangians  $\mathcal{L}_4^N$  and  $\mathcal{L}_5^N$  are absent, so that  $f_4 = 0$  and  $f_5 = 0$ . In this case, the functions  $B_4$  and  $B_5$  are related with  $A_4$  and  $A_5$  according to the relations  $A_4 + B_4 - 2X B_{4,X} = 0$  and  $A_5 + X B_{5,X} / 3 = 0$ . In beyond-generalized Proca theories the functions  $f_4$  and  $f_5$  are non-zero, so there are two more free functions  $B_4$  and  $B_5$  than those in generalized Proca theories. This situation is analogous to the extension of Horndeski theories to GLPV theories [13]. We recall that the Lagrangians  $\mathcal{L}_6$ ,  $\tilde{\mathcal{L}}_5^N$ , and  $\mathcal{L}_6^N$ , which correspond to the intrinsic vector mode, do not contribute to the background equations of motion.

Since the background Eqs. (3.22)–(3.24) do not contain the functions  $B_4$  and  $B_5$ , beyond-generalized Proca theories cannot be distinguished from generalized Proca theories at the background level (as it happens in the GLPV extension of Horndeski theories [17,48]). However, as we will discuss in Sec. 4, this situation is different at the level of cosmological perturbations.

### 3.3. Hamiltonian

The discussion in Sec. 3.2 shows that, at the background level, beyond-generalized Proca theories do not give rise to additional ghostly DOF to that in generalized Proca theories. It is also possible to see the absence of the Ostrogradski ghost by computing the Hamiltonian of the system. In doing so, we consider the line element

$$ds^2 = -N^2(t) dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (3.26)$$

which contains the lapse function  $N(t)$ . For the vector field given by  $A^\mu = (\phi(t)/N(t), 0, 0, 0)$ , the action (2.18) reduces to  $S = \int d^4x L$ , with

$$\begin{aligned} L &= Na^3 G_2 - a^3 G_{3,X} \phi^2 \dot{\phi} - \frac{6a\dot{a}^2 G_4}{N} + \frac{6a\dot{a}^2 G_{4,X} \phi^2}{N} \\ &\quad - \frac{G_{5,X} \dot{a}^3 \phi^3}{N^2} + \frac{6a\dot{a}^2 f_4 \phi^4}{N} + \frac{6\dot{a}^3 f_5 \phi^5}{N^2} + Na^3 P_M, \end{aligned} \quad (3.27)$$

where we have carried out the integration by parts. Since the Lagrangian (3.27) does not contain the time derivative of  $N$ , there exists a Hamiltonian constraint. In fact, the variation of  $L$  with respect to  $N$  leads to

$$\frac{\partial L}{\partial N} = -\frac{\mathcal{H}}{N} = 0, \quad (3.28)$$

where  $\mathcal{H} = \Pi^\mu \dot{O}_\mu - L$  is the Hamiltonian with  $\Pi^\mu = \partial L / \partial \dot{O}_\mu$  and  $O_\mu = (N(t), \phi(t), a(t))$ . The explicit form of  $\mathcal{H}$  is given by

$$\begin{aligned} \mathcal{H} &= -Na^3 \left( G_2 + 6H^2 G_4 - 6G_{4,X} H^2 \phi^2 + 2G_{5,X} H^3 \phi^3 \right. \\ &\quad \left. - 6f_4 H^2 \phi^4 - 12f_5 H^3 \phi^5 - \rho_M \right), \end{aligned} \quad (3.29)$$

which does not contain any time derivatives of  $\phi$ . Equation (3.28) shows that  $\mathcal{H} = 0$  exactly. Hence there is no Ostrogradski instability associated with the Hamiltonian unbounded from below. Existence of the constraint (3.28) removes the would-be ghostly DOF associated with the time derivatives of  $\phi$ .

The Hamiltonian constraint  $\mathcal{H} = 0$  follows from the background Eqs. (3.17). In fact, after eliminating the term  $G_{2,X}$  from the two equations  $\mathcal{C}_1 = 0$ ,  $\mathcal{C}_2 = 0$  and setting  $N = 1$ , we obtain the constraint equation  $\mathcal{H} = 0$ . Moreover, varying the Lagrangian (3.27) with respect to  $\phi$ , the resulting equation of motion is equivalent to  $\mathcal{C}_2 = 0$ .

What we have shown in this section is by no means a full proof of the absence of extra ghostly DOF on arbitrary backgrounds. A full ADM Hamiltonian analysis is needed for this purpose. Even though this proof is not the goal of the present work, we will consider linear perturbations on the FLRW background in Sec. 4 and investigate the propagating DOF.

## 4. Dynamics of linear perturbations

In this section we expand the action (2.18) up to second order in tensor, vector, and scalar perturbations to study the number of DOFs as well as no-ghost and stability conditions for linear cosmological perturbations.

### 4.1. Tensor perturbations

We begin with the derivation of the second-order action for tensor perturbations  $h_{ij}$ . We can express  $h_{ij}$  in terms of two polarization modes  $h_+$  and  $h_\times$ , as  $h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$ . The unit bases  $e_{ij}^+$  and  $e_{ij}^\times$  satisfy the normalization conditions  $e_{ij}^+(\mathbf{k}) e_{ij}^+(-\mathbf{k})^* = 1$ ,

$e_{ij}^\times(\mathbf{k})e_{ij}^\times(-\mathbf{k})^* = 1$ , and  $e_{ij}^+(\mathbf{k})e_{ij}^\times(-\mathbf{k})^* = 0$  in Fourier space, where  $\mathbf{k}$  is the comoving wave number. Expanding the action (2.18) up to quadratic order in tensor perturbations, the second-order action reads

$$S_T^{(2)} = \sum_{\lambda=+, \times} \int dt d^3x a^3 \frac{q_T}{8} \left[ \dot{h}_\lambda^2 - \frac{c_T^2}{a^2} (\partial h_\lambda)^2 \right], \quad (4.1)$$

where

$$q_T = 2G_4 - 2G_{4,X}\phi^2 + G_{5,X}H\phi^3 - 2f_4\phi^4 - 6f_5H\phi^5 \\ = -2(A_4 + 3HA_5), \quad (4.2)$$

$$c_T^2 = \frac{2G_4 + G_{5,X}\phi^2\dot{\phi}}{q_T} = -\frac{2B_4 + \dot{B}_5}{2(A_4 + 3HA_5)}. \quad (4.3)$$

In the second equalities of Eqs. (4.2) and (4.3) we have used the quantities defined by Eq. (3.20). The Lagrangians  $\mathcal{L}_4^N$  and  $\mathcal{L}_5^N$  lead to the modification of  $q_T$ , which on the other hand can be expressed in terms of  $A_4$  and  $A_5$  alone. The numerator of  $c_T^2$  contains the terms  $B_4$  and  $\dot{B}_5$ , so beyond-generalized Proca theories give rise to the tensor propagation speed different from that in generalized Proca theories. The expressions of  $q_T$  and  $c_T^2$  are of the same forms as those in GLPV theories [48]. The action (4.1) does not contain the derivative terms higher than second order, so the dynamical DOF of the tensor mode remain two.

#### 4.2. Vector perturbations

Let us proceed to the discussion of vector perturbations. Due to the transverse conditions of the vector mode (e.g.,  $\partial^i Z_i = 0$ ), we can choose the components of these fields as  $Z_i = (Z_1(t, z), Z_2(t, z), 0)$  without losing the generality. The second-order matter action  $(S_M^{(2)})_V$  of the vector mode is the same as that derived in Refs. [40,41]. Varying the action  $(S_M^{(2)})_V$  with respect to  $W_i, \delta\mathcal{A}_i, \delta\mathcal{B}_i$ , we obtain the following relations

$$W_i = \mathcal{N}_0 (v_i - V_i), \quad (4.4)$$

$$\delta\mathcal{A}_i = \rho_{M,n} v_i = C_i, \quad (4.5)$$

where  $C_i$  are constants in time, and

$$v_i = V_i - a^2 \delta\mathcal{B}_i. \quad (4.6)$$

After integrating out the fields  $W_i$  and  $\delta\mathcal{A}_i$ , the full second-order action derived by expanding Eq. (2.18) in vector perturbations reads

$$S_V^{(2)} = \int dt d^3x \sum_{i=1}^2 \left[ \frac{aq_V}{2} \dot{Z}_i^2 - \frac{1}{2a} \alpha_1 (\partial Z_i)^2 - \frac{a}{2\phi^2} \alpha_2 Z_i^2 \right. \\ \left. + \frac{\phi}{2a} \alpha_3 \partial V_i \partial Z_i + \frac{q_T}{4a} (\partial V_i)^2 + \frac{1}{2} a (\rho_M + P_M) v_i^2 \right], \quad (4.7)$$

where

$$q_V = G_{2,F} + 2G_{2,Y}\phi^2 - 4g_5H\phi + 2G_6H^2 \\ + 2G_{6,X}H^2\phi^2 + 4\tilde{f}_6H^2\phi^2, \quad (4.8)$$

$$\alpha_1 = q_V + 2[G_6\dot{H} - G_{2,Y}\phi^2 - \tilde{f}_5H\phi^3 \\ - (H\phi - \dot{\phi})(G_{6,X}H\phi - g_5 + 2\tilde{f}_6H\phi)], \quad (4.9)$$

$$\alpha_2 = 4G_4\dot{H} - 4G_{4,X}H\phi\dot{\phi} + 2G_{5,X}H^2\phi^2\dot{\phi} \\ + \rho_M + P_M, \quad (4.10)$$

$$\alpha_3 = 2G_{4,X} - G_{5,X}H\phi + 2f_4\phi^2 + 6f_5H\phi^3 \\ = \frac{2}{\phi^2} (A_4 + B_4 + 3HA_5). \quad (4.11)$$

The structure of the action (4.7) is the same as that derived in generalized Proca theories [40,41] with the different coefficients  $q_V, \alpha_1, \alpha_2, \alpha_3$ . Hence the new Lagrangians (2.13)–(2.16) do not give rise to any additional DOF associated with vector perturbations.

Varying the action (4.7) with respect to  $V_i$  yields

$$\frac{q_T}{2} \frac{k^2}{a^2} V_i = -(\rho_M + P_M) v_i - \frac{\alpha_3 \phi}{2} \frac{k^2}{a^2} Z_i, \quad (4.12)$$

and similarly with respect to  $Z_i$ :

$$\ddot{Z}_i + \left( H + \frac{\dot{q}_V}{q_V} \right) \dot{Z}_i + \frac{1}{q_V} \left( \alpha_1 \frac{k^2}{a^2} + \frac{\alpha_2}{\phi^2} \right) Z_i \\ - \frac{\alpha_3 \phi}{2q_V} \frac{k^2}{a^2} V_i = 0. \quad (4.13)$$

In the small-scale limit ( $k \rightarrow \infty$ ) we can neglect the matter contribution in Eq. (4.12), so we obtain the approximate relation  $V_i \simeq -(\alpha_3 \phi / q_T) Z_i$ . Substituting this into Eq. (4.13), the dynamical vector field  $Z_i$  obeys

$$\ddot{Z}_i + \left( H + \frac{\dot{q}_V}{q_V} \right) \dot{Z}_i + c_V^2 \frac{k^2}{a^2} Z_i \simeq 0, \quad (4.14)$$

where the vector propagation speed squared is given by

$$c_V^2 = \frac{\alpha_3^2 \phi^2}{2q_T q_V} + \frac{\alpha_1}{q_V} \\ = 1 + \frac{2(A_4 + B_4 + 3HA_5)^2}{\phi^2 q_T q_V} \\ + \frac{2(G_6\dot{H} - G_{2,Y}\phi^2 - \tilde{f}_5H\phi^3)}{q_V} \\ - \frac{2(H\phi - \dot{\phi})(G_{6,X}H\phi - g_5 + 2\tilde{f}_6H\phi)}{q_V}. \quad (4.15)$$

To avoid the ghost and the Laplacian instability on small scales, we require the conditions  $q_V > 0$  and  $c_V^2 > 0$ . All the new Lagrangian densities (2.13)–(2.16) affect  $c_V$  through the changes of coefficients (4.8)–(4.11), while  $q_V$  is only modified by the term  $\mathcal{L}_6^N$ . In spite of these modifications, the DOF of vector perturbations remain two as those in generalized Proca theories.

#### 4.3. Scalar perturbations

For scalar perturbations, we first expand the Schutz–Sorkin action (3.4) up to second order by using the matter perturbation  $\delta\rho_M$  defined in Eq. (3.12). Varying this action with respect to  $\delta j$ , we obtain

$$\partial\delta j = -a^3 n_0 (\partial v + \partial\chi). \quad (4.16)$$

On using this relation and the background equation of motion, the second-order matter action reduces to

$$(S_M)_S^{(2)} = \int dt d^3x a^3 \left[ -\frac{n_0 \rho_{M,n}}{2a^2} (\partial v)^2 + n_0 \rho_{M,n} v \frac{\partial^2 \chi}{a^2} \right. \\ \left. + \dot{v} \delta\rho_M - 3Hc_M^2 v \delta\rho_M - \frac{c_M^2}{2n_0 \rho_{M,n}} (\delta\rho_M)^2 - \alpha \delta\rho_M \right], \quad (4.17)$$

where  $c_M^2$  is the matter sound speed squared defined by

$$c_M^2 \equiv \frac{P_{M,n}}{\rho_{M,n}} = \frac{n_0 \rho_{M,nn}}{\rho_{M,n}}. \quad (4.18)$$

Expansion of the full action (2.18) up to second order in scalar perturbations gives

$$\begin{aligned}
S_S^{(2)} = \int dt d^3x a^3 \left\{ \left( w_1 \alpha + \frac{w_2 \delta \phi}{\phi} \right) \frac{\partial^2 \chi}{a^2} - w_3 \frac{(\partial \alpha)^2}{a^2} \right. \\
+ w_4 \alpha^2 - \frac{w_3}{4} \frac{(\partial \delta \phi)^2}{a^2 \phi^2} + w_5 \frac{(\delta \phi)^2}{\phi^2} - \frac{w_3}{4 \phi^2} \frac{(\partial \dot{\psi})^2}{a^2} \\
+ \frac{w_7}{2} \frac{(\partial \psi)^2}{a^2} - (3H w_1 - 2w_4) \alpha \frac{\delta \phi}{\phi} \\
+ \alpha \left[ w_3 \frac{\partial^2 (\delta \phi)}{a^2 \phi} + w_3 \frac{\partial^2 \dot{\psi}}{a^2 \phi} - w_6 \frac{\partial^2 \psi}{a^2} \right] \\
\left. - (w_8 \psi - w_3 \dot{\psi}) \frac{\partial^2 (\delta \phi)}{2a^2 \phi^2} \right\} + (S_M)_S^{(2)}, \quad (4.19)
\end{aligned}$$

where we introduced the following variables for compactness

$$w_1 = -A_{3,X} \phi^2 + 4H(A_4 - A_{4,X} \phi^2) + 6H^2(2A_5 - A_{5,X} \phi^2), \quad (4.20)$$

$$w_2 = w_1 + 2Hq_T, \quad (4.21)$$

$$w_3 = -2\phi^2 q_V, \quad (4.22)$$

$$w_4 = 3H(w_2 - Hq_T) + w_5, \quad (4.23)$$

$$w_5 = \frac{1}{2} \phi^4 \left( A_{2,XX} + 3HA_{3,XX} + 6H^2 A_{4,XX} + 6H^3 A_{5,XX} \right), \quad (4.24)$$

$$w_6 = -\frac{1}{2\phi} [4H(q_T - 2B_4) - w_8], \quad (4.25)$$

$$w_7 = \frac{2(q_T - 2B_4)}{\phi^2} \dot{H} + \frac{w_8}{2\phi^3} \dot{\phi}, \quad (4.26)$$

$$w_8 = 2w_2 + 4H\phi^2 (2B_{4,X} - HB_{5,X}). \quad (4.27)$$

The coefficients  $w_1, w_2, w_4, w_5$  only contain the functions  $A_i$  and their derivatives, but there exist the functions  $B_{4,5}$  and their derivatives in  $w_6, w_7, w_8$ . Hence the difference from generalized Proca theories arises through the terms containing  $w_6, w_7, w_8$ . In particular we have the following relation

$$\begin{aligned}
w_8 - (w_6 \phi + w_2) &= -4H(A_4 + B_4 - 2XB_{4,X}) \\
&\quad - 4H^2(3A_5 + XB_{5,X}) \\
&= -4H\phi^4 (f_4 + 3H\phi f_5), \quad (4.28)
\end{aligned}$$

where we have used Eq. (3.25). In generalized Proca theories studied in Refs. [40,41] we have that  $f_4 = f_5 = 0$ , so there is the specific relation  $w_8 = w_6 \phi + w_2$ . In beyond-generalized Proca theories,  $w_8$  is different from  $w_6 \phi + w_2$ .

While the presence of the Lagrangians  $\mathcal{L}_4^N$  and  $\mathcal{L}_5^N$  manifests themselves through the modifications of the functions  $B_{4,5}$ , the effect of  $\mathcal{L}_6^N$  arises through the modification of the term  $w_3 = -2\phi^2 q_V$ . The existence of  $\tilde{\mathcal{L}}_5^N$  does not affect the second-order action of scalar perturbations.

The structure of the action (4.19) is the same as that in generalized Proca theories derived in Refs. [40,41], so the new Lagrangian densities (2.13)–(2.16) do not give rise to any additional DOF. As in the GLPV extension of Horndeski theories [13], there are no derivatives higher than second order in the scalar action (4.19). Since this second-order property also holds for tensor and vector perturbations, beyond-generalized Proca theories with the new terms (2.13)–(2.16) are not prone to the Ostrogradski instability on the flat FLRW background.

As we will see in the following, beyond-generalized Proca theories can be distinguished from generalized Proca theories by different evolution of the scalar propagation speed  $c_S$ . This situation should be analogous to that in GLPV theories where the new Lagrangians beyond the Horndeski domain lead to the mixing between  $c_S$  and the matter sound speed  $c_M$  [16,13]. In order to see such a mixing explicitly, it is convenient to employ the k-essence description (3.9) of the perfect fluid. On using the correspondence (3.10) and the field equation of motion (3.19), the second-order matter action (4.17) is equivalent to

$$\begin{aligned}
(S_M)_S^{(2)} = \int dt d^3x a^3 \left[ \frac{1}{2} (P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ}) (\delta\dot{\sigma}^2 - 2\dot{\sigma}\alpha\dot{\sigma}) \right. \\
- \frac{1}{2a^2} P_{M,Z} \left[ (\partial\delta\sigma)^2 + 2\dot{\sigma}\partial\chi\partial\delta\sigma \right] \\
\left. + \frac{1}{2} \dot{\sigma}^2 (P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ}) \alpha^2 \right]. \quad (4.29)
\end{aligned}$$

The last term of Eq. (4.29) gives rise to the contribution to the term  $w_4 \alpha^2$  in Eq. (4.19). One can confirm that direct expansion of the k-essence action (3.9) leads to the second-order action same as Eq. (4.29).

On using Eq. (4.29) and varying the full action (4.19) with respect to  $\alpha, \chi, \delta\phi$  respectively, we obtain the perturbation equations of motion in Fourier space:

$$\begin{aligned}
(P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ}) (\dot{\sigma}\dot{\sigma} - \dot{\sigma}^2 \alpha) \\
+ (3Hw_1 - 2w_4) \frac{\delta\phi}{\phi} - 2w_4 \alpha \\
+ \frac{k^2}{a^2} \left[ \frac{w_3}{\phi} (\dot{\psi} + \delta\phi + 2\alpha\phi) + w_1 \chi - w_6 \psi \right] = 0, \quad (4.30)
\end{aligned}$$

$$P_{M,Z} \dot{\sigma} \delta\sigma + w_1 \alpha + \frac{w_2}{\phi} \delta\phi = 0, \quad (4.31)$$

$$\begin{aligned}
(3Hw_1 - 2w_4) \alpha - 2w_5 \frac{\delta\phi}{\phi} \\
+ \frac{k^2}{a^2} \left[ \frac{w_3}{2\phi} (\dot{\psi} + \delta\phi + 2\alpha\phi) + w_2 \chi - \frac{w_8}{2\phi} \psi \right] = 0. \quad (4.32)
\end{aligned}$$

From Eq. (3.10) the first contributions to Eqs. (4.30) and (4.31) can be written as  $(P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ}) (\dot{\sigma}\dot{\sigma} - \dot{\sigma}^2 \alpha) = \delta\rho_M$  and  $P_{M,Z} \dot{\sigma} \delta\sigma = (\rho_M + P_M)v$ , respectively. On using Eqs. (4.30)–(4.32), we can express the perturbations  $\alpha, \chi, \delta\phi$  in terms of  $\psi, \delta\sigma$  and their derivatives. Substituting those relations into Eq. (4.19), the second-order scalar action reduces to the following form

$$\begin{aligned}
S_S^{(2)} = \int dt d^3x a^3 \left( \dot{\vec{\chi}}^t \mathbf{K} \dot{\vec{\chi}} + \frac{k^2}{a^2} \vec{\chi}^t \mathbf{G} \vec{\chi} \right. \\
\left. - \vec{\chi}^t \mathbf{M} \vec{\chi} - \vec{\chi}^t \mathbf{B} \dot{\vec{\chi}} \right), \quad (4.33)
\end{aligned}$$

where  $\mathbf{K}, \mathbf{G}, \mathbf{M}, \mathbf{B}$  are  $2 \times 2$  matrices ( $\mathbf{M}$  does not contain the  $k^2$  term), and the vector field  $\vec{\chi}$  is defined by

$$\vec{\chi}^t = (\psi, \delta\sigma). \quad (4.34)$$

The form of the action (4.33) explicitly shows that there are only two scalar DOF coming from the field  $\psi$  and the matter field  $\delta\sigma$ .

In the small-scale limit ( $k \rightarrow \infty$ ), the components of the matrices  $\mathbf{K}$  and  $\mathbf{G}$  are given by<sup>1</sup>

<sup>1</sup> If we use the Schutz-Sorkin action itself for the matter sector, the leading-order contributions to  $K_{22}$  and  $G_{22}$  are proportional to  $1/k^2$ . After transforming the Schutz-Sorkin action to the k-essence action, both  $K_{22}$  and  $G_{22}$  do not have the  $k$ -dependence as the components  $K_{11}$  and  $G_{11}$ .

$$\begin{aligned}
K_{11} &= Q_S + \xi_1^2 K_{22}, \\
K_{22} &= \frac{1}{2} \left( P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ} \right), \\
K_{12} &= K_{21} = \xi_1 K_{22},
\end{aligned} \tag{4.35}$$

and

$$\begin{aligned}
G_{11} &= \mathcal{G} + \dot{\mu} + H\mu, \\
G_{22} &= \frac{1}{2} P_{M,Z}, \\
G_{12} &= G_{21} = \xi_2 G_{22},
\end{aligned} \tag{4.36}$$

where we introduced the following quantities

$$\begin{aligned}
Q_S &= \frac{H^2 q_T (3w_1^2 + 4q_T w_4)}{(w_1 - 2w_2)^2 \phi^2}, \\
\xi_1 &= -\frac{w_2 \dot{\sigma}}{(w_1 - 2w_2)\phi}, \\
\xi_2 &= -\frac{(w_8 - w_6\phi)\dot{\sigma}}{(w_1 - 2w_2)\phi}, \\
\mathcal{G} &= \frac{w_1 w_8 (4w_2 w_6\phi - w_1 w_8) - 4w_2^2 w_6^2 \phi^2}{4w_3 (w_1 - 2w_2)^2 \phi^2} - \frac{w_7}{2}, \\
\mu &= \frac{2w_2 w_6\phi - w_1 w_8}{4(w_1 - 2w_2)\phi^2}.
\end{aligned} \tag{4.37}$$

Provided that the matrix  $\mathbf{K}$  is positive definite, the scalar ghosts are absent. Under the no-ghost condition  $K_{22} > 0$  of the fluid, the positivity of  $\mathbf{K}$  is ensured for  $Q_S > 0$ . Since the quantity  $Q_S$  does not contain the term  $q_V$ , the no-ghost condition is not affected by the intrinsic vector mode. We also note that  $Q_S$  is solely expressed in terms of the functions  $A_{3,4,5}$  and their derivatives, so the no-ghost condition is similar to that in generalized Proca theories.

In the large  $k$  limit, the dominant contributions to the second-order action (4.33) are the first two terms, so the dispersion relation is given by

$$\det \left( c_S^2 \mathbf{K} - \mathbf{G} \right) = 0, \tag{4.38}$$

where  $c_S^2$  is the sound speed squared related with the frequency  $\omega$ , as  $\omega^2 = c_S^2 k^2 / a^2$ . Then,  $c_S^2$  is the solution to the equation

$$\left( c_S^2 K_{11} - G_{11} \right) \left( c_S^2 K_{22} - G_{22} \right) - \left( c_S^2 K_{12} - G_{12} \right)^2 = 0. \tag{4.39}$$

In generalized Proca theories there is the relation  $w_8 = w_6\phi + w_2$  and hence  $\xi_1 = \xi_2$ . Since in this case  $G_{12}/K_{12} = G_{22}/K_{22}$ , we obtain the two decoupled solutions to Eq. (4.39):

$$\begin{aligned}
c_M^2 &= \frac{G_{22}}{K_{22}} = \frac{P_{M,Z}}{P_{M,Z} + \dot{\sigma}^2 P_{M,ZZ}}, \\
c_P^2 &= \frac{1}{Q_S} \left[ G_{11} - (K_{11} - Q_S) \frac{G_{22}}{K_{22}} \right] \\
&= \frac{1}{Q_S} \left[ \mathcal{G} + \dot{\mu} + H\mu - \frac{w_2^2 (\rho_M + P_M)}{2(w_1 - 2w_2)^2 \phi^2} \right],
\end{aligned} \tag{4.41}$$

where  $c_M^2$  is the matter propagation speed squared equivalent to Eq. (4.18). Another sound speed squared  $c_P^2$  coincides with the one derived in Refs. [40,41].

In beyond-generalized Proca theories we have that  $\xi_1 \neq \xi_2$ , in which case there is a mixing between the two scalar propagation speeds. To quantify the deviation from generalized Proca theories, we introduce the following dimensionless quantities

$$\alpha_P \equiv \frac{\xi_2}{\xi_1} - 1 = \frac{w_8 - (w_6\phi + w_2)}{w_2}, \tag{4.42}$$

and

$$\begin{aligned}
\beta_P &\equiv 2c_M^2 \left( \frac{K_{11}}{Q_S} - 1 \right) \alpha_P \\
&= \frac{w_2 (w_8 - w_6\phi - w_2) (\rho_M + P_M)}{(3w_1^2 + 4q_T w_4) q_T H^2}.
\end{aligned} \tag{4.43}$$

Expressing the terms  $G_{22}, G_{11}, K_{11}, K_{22}, G_{12}$  in terms of  $c_M^2$  etc. by using Eqs. (4.40), (4.41), (4.43) as well as the relations  $K_{22} = K_{12}^2 / (K_{11} - Q_S)$  and  $G_{12}/K_{12} = (1 + \alpha_P) G_{22}/K_{22}$ , the two solutions to Eq. (4.39) are given by

$$c_S^2 = \frac{1}{2} \left[ c_M^2 + c_P^2 - \beta_P \pm \sqrt{(c_M^2 - c_P^2 + \beta_P)^2 + 2c_M^2 \alpha_P \beta_P} \right]. \tag{4.44}$$

For non-relativistic matter ( $c_M^2 = 0$ ), the two solutions (4.44) reduce to  $c_S^2 = 0$  and  $c_S^2 = c_P^2 - \beta_P$ . The latter corresponds to the scalar sound speed squared associated with the field  $\psi$ , whose value is different from  $c_P^2$  by the factor  $\beta_P$ . Thus, the sound speed squared is a key quantity to distinguish between generalized Proca theories and beyond-generalized Proca theories. This situation is analogous to the difference between Horndeski and GLPV theories. Since the contribution of the intrinsic vector mode affects  $c_S^2$  through the term  $w_3$  in the quantity  $\mathcal{G}$ , the sound speed in beyond-generalized Proca theories generally differs from that in GLPV theories.

## 5. Conclusions

We proposed the new derivative interactions (2.13)–(2.16) beyond the domain of second-order generalized Proca theories. These Lagrangian densities are constructed in terms of the products of the anti-symmetric Levi-Civita tensor as well as the vector field  $A^\mu$  and its first derivatives. By taking the scalar limit  $A^\mu \rightarrow \nabla^\mu \pi$ , the terms  $\mathcal{L}_4^N$  and  $\mathcal{L}_5^N$  recover derivative interactions of the scalar field  $\pi$  appearing in GLPV theories. Since it is known that GLPV theories do not increase the number of propagating DOF relative to that in Horndeski theories [15,20], our interest is to see what happens by extending generalized Proca theories to those containing the four new interactions (2.13)–(2.16).

We first derived the dynamical equations of motion on the flat FLRW background and the associated Hamiltonian of the system. Even in the presence of new interactions, there is no additional ghostly DOF to that appearing in second-order generalized Proca theories. In fact the existence of a constraint leads to a vanishing Hamiltonian, which explicitly shows the absence of the Ostrogradski ghost.

As a second step, we considered linear cosmological perturbations on top of the flat FLRW background by taking into account a perfect fluid and studied the propagating DOF by expanding the action (2.18) up to second order in perturbations. We showed that the number of DOF is the same as that in generalized Proca theories: two tensor polarizations, two transverse vector modes, and two scalar modes (one longitudinal scalar and one matter fluid). Thus, beyond-generalized Proca theories are not prone to the appearance of additional DOF at the level of linear cosmological perturbations.

We also found that the four new interactions affect the vector propagation speed squared  $c_V^2$ , while the vector no-ghost condition is only modified by the term  $\mathcal{L}_6^N$ . By introducing the quantities given by Eq. (3.20), we obtained the two relations (3.25) analogous to those appearing in the GLPV extension of Horndeski theories. Since the functions  $f_4$  and  $f_5$  do not vanish in beyond-generalized



Proca theories, this leads to the scalar sound speed squared  $c_s^2$  away from the value  $c_p^2$  of generalized Proca theories with the difference weighed by  $\beta_p$ . Thus, the two theories can be distinguished from each other by the different evolution of scalar and vector sound speeds.

There are several issues we did not address in this Letter. While we showed that the number of DOF in beyond-generalized Proca theories is the same as that in generalized Proca theories on the FLRW background, it remains to see whether the same conclusion also holds at the fully non-linear level on general curved backgrounds. In doing so, it will be convenient to express the action (2.18) in terms of quantities appearing in the 3+1 ADM decomposition of space-time (along the line of Ref. [14]). In fact, we showed that the quantities associated with the FLRW background and tensor perturbations in beyond-generalized Proca theories can be expressed in simple forms by using the variables (3.20) similar to those appearing in the ADM formulation of GLPV theories, but the situation is more involved for vector and scalar perturbations. In our case, there should be new contributions to the ADM action of GLPV theories associated with the vector mode. Moreover, it will be of interest to study the cosmological viability of dark energy models in the framework of beyond-generalized Proca theories. These topics will be left for future works.

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