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A THIRTEENTH CENTURY 'PROOF' OF THE PARALLEL POSTULATE

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SUMMARIES

Proposition I.14 of Witelo's Perspectiva purports to provide a proof of the claim contained in Euclid's fifth postulate. The Latin text of the proposition is presented and translated into English; a commentary on the nature of the 'proof' is also provided.

In teorema I.14 din Perspectiva sacrisă de Witelo, întâlnim o 'demonstrație' a postulatului numărul cinci al lui Euclid. Textul Latin al teoremei împreună cu o traducere engleză sunt prezentate în această lucrare; deasemeni un comentariu asupra naturii demonstrației este inclus.

במשפט I.14 המופיע בחבורו של וויטלו, Perspectiva, מוצעת כביכול, הוכחה לפוסטולט מספר המש של אויקליד. הטקסט הלטיני של המשפט יחד עם תרגומו לאנגלית בהוספת ניתוח, ההוכחה וטבעה מוגשים לעיונו של הקורא.

In his widely read and gargantuan treatise, *Perspectiva*, Witelo presents a 'proof' of the parallel postulate [Witelo 1572, 8]. The Latin text, drawn from an edition of Book I of the *Perspectiva* which I have prepared [Unguru 1970, 392-393] and which will appear as volume XV of the series *Studia Copernicana* [Unguru 1977], follows:

[Propositio] 14. *Si linea recta super duas rectas ceciderit feceritque angulos coalternos inequales, aut duos intrinsecos minores duobus rectis, vel extrinsecum inequalem intrinseco, illas duas lineas ad minorum angulorum partem concurrere est necesse, ad aliam vero partem impossibile; et si lineae concurrunt necesse est dictos angulos aliquo propositorum modorum se habere.*

Sint [Fig. 15] due lineae AB et CD quas secet

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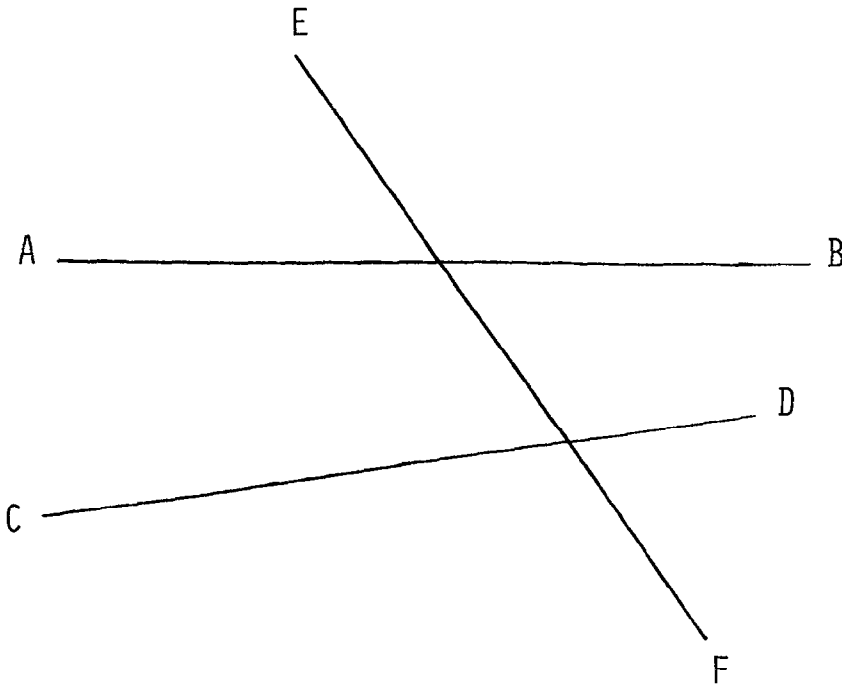


FIGURE 15

linea EF secundum quod proponitur.

Dico quoniam lineae AB et CD concurrent. Si enim non concurrent, patet quod sunt equidistantes. Ergo, per $29^{\text{am}} I^i$, sequitur contrarium ypothesis, quod est inconveniens. Concurrent ergo. Ad partem vero minorum angulorum concurrere est necessarium, quoniam si ad partem maiorum angulorum concurrent, sequetur angulum extrinsecum trigoni tanto fieri minorem angulo intrinseco, quod est contra 16^{am} et $32^{\text{am}} I^i$. Et quia, per premissas probationes ad partes minorum angulorum concurrunt. Si, ex concessio, ad partes maiorum angulorum concurrerent, sequeretur duas rectas lineas superficiem includere, quod est impossibile. Est ergo impossibile ut ad partes maiorum angulorum concurrant, quod est propositum primum. Sed et si detur quod ille lineae concurrant, necesse est angulos aliquo propositorum modorum se habere, per $32^{\text{am}} I^i$. Patet ergo totum quod proponebatur, servata semper ypothesi [1].

And here is my translation [Unguru 1970, 72-73]:

[Proposition] 14. *If a straight line falling on two straight lines makes the two alternate angles unequal, or two interior [angles on the same side] less than two right [angles], or an exterior [angle] unequal to the interior [and opposite angle on the same side], those two lines must meet on the side of the smaller angles, [this being] impossible on the other side; and [reciprocally] if the lines meet, the said angles must exhibit themselves in any of the proposed manners.*

Let there be [Fig. 15] two lines AB and CD which line EF cuts as proposed.

I say that the lines AB and CD will meet. For if they do not meet, it is clear that they are parallel. Hence, by I, 29 [Euclid] [2], there follows [something] which is contrary to the hypothesis [3], and this is unsuitable. Hence they do meet. It is necessary [for them] to meet on the side of the smaller angles, because if they meet on the side of the greater angles it will follow that an exterior angle of the triangle has been made of such size as the interior angle which is smaller [4], which is against I, 16 and 32 [Euclid]. And so, by the previous proofs, they meet on the side of the smaller angles. Had they met [still assuming this possible], by way of concession, on the sides of the greater angles, it would have followed that two straight lines enclose a surface [5], which is impossible. It is therefore impossible that they should meet on the sides of the greater angles, which is what was proposed first. But even if it is given that those lines meet, it is necessary that the angles arrange themselves in one of the proposed manners, by I, 32 [Euclid] [6]. Therefore all that was proposed is clear, [and] the hypothesis is always saved.

What can one say about this totally unacceptable proof, except that Witelo's name must now be added to the long and 'glorious' list of mathematicians who tried their logical powers in 'proving' the fifth postulate? Moreover, his is one of the most primitive 'proofs' I am aware of. There is no sign of any mathematical sophistication whatever; what meets the eye is mere and trivial logical blunder, i.e., immediate and transparent reliance on what has to be proved. This is a straightforward *petitio principii*. I have no satisfactory explanation for this elementary mistake, which, incidentally, is not entirely compatible (in spite of superficial appearances to the contrary) with Witelo's otherwise general lack of skill as a mathematician

[Unguru 1972]. That Witelo erred in the context he did is no catastrophe in itself (he is in very respectable company indeed); that he erred in this particular way is rather difficult to comprehend. Perhaps this is the reason why, despite Witelo's citing in his 'proof' only I.29, I.16, and I.32 *Elements*, Risner, in his edition, refers to I.27, I.28 *Elements* and to Proclus's lemma to I.16, all of which logically precede I.29 [7].

Finally, it has been remarked that the medieval mathematician was preoccupied with matters of logical structure, that he concentrated on the fundamental assumptions of geometry, and that his use of the *Elements* was primarily for didactic purposes [8]. This is true, with respect to the medieval Euclid. On the other hand, although the merits of the *Perspectiva* are mainly (but not exclusively) didactical and expository, and although proposition I.14 appears in a purely mathematical book formally patterned on the *Elements* and meant to supply the mathematical foundations needed in the study of geometrical optics in the remaining nine optical books comprising the treatise, Witelo's lack of consideration of rudimentary logical issues in his 'proof' would seem to put him beyond the pale of *mathematici Latini*. But this would be unfair. After all, Witelo did not set out to write a new edition of the *Elements* meant to replace Euclid's famous treatise with the same title, notwithstanding the fact that Witelo's work was precisely so used in some medieval universities [Unguru 1972, 506-507].

Witelo's mathematical book is basically a "service-text," a book having its *raison d'être* in the services it can lend to the following nine optical books for the development of their (mostly) quasi-mathematical demonstrations. It was precisely because he relied on Euclid, whose results he generally took for granted, that Witelo got himself into logical muddles. Accepting the *Elements* on the one hand and attempting to prove sometimes propositions contained therein or (as in this case) a postulate could not but lead to vicious circles. Why, then, did he choose to follow such a procedure? Possibly because he wanted Book I to stand by itself and to contain whatever was needed mathematically for Books II-X. Occasionally this meant including constituent parts of the *Elements*. Furthermore, since Book I is a geometrical treatise with definitions, postulates, and propositions, everything except the postulates had to be proved. Why, then, not include postulate 5 as such among his postulates? Because, I think, like so many a mathematician before him, Witelo was not satisfied with its postulational character. This skepticism about the nature of an Euclidean fundamental assumption, in itself a worthy feature, ultimately led him, owing to the character of Book I, to an elementary logical error.

NOTES

1. Cf. Frederic Risner's text in his edition [Witelo 1572, 8]
2. As known, I.29 is the first proposition of the *Elements* proved by relying on the parallel postulate!
3. I.e., that the alternate angles would be equal, or that the sum of the two interior angles on the same side would be equal to two right angles, etc.
4. This is rather obscure. On Witelo's own terms, however, a contradiction would follow if the straight lines were to meet "on the side of the greater angles." For example, it would follow that the smaller angles are greater than two right angles and the greater angles are less than two right angles.
5. The way it stands, this claim seems to make no sense, since the same (false) conclusion could be drawn irrespective of the side on which *AB* and *CD* meet. What Witelo seems to have in mind is that since he 'proved' that the two lines meet on the side of the smaller angles, had they *also* met on the side of the greater angles, then they would enclose a surface, which is impossible. If this is indeed what he meant, then Witelo seems to have forgotten here the uniqueness of a line drawn between any two points, as ensured by postulate 1.
6. Here, as above where he mentions I.32 *Elements*, Witelo relies on a proposition in whose proof appeal is made to I.29, which, as we saw, appeals in its turn to postulate 5.
7. See [Witelo 1572, 8], where Risner says: "È 27.28 p 1 element. Lemma Procli ad 16 p 1 elem." I have argued in [Unguru 1970] that Risner's references at the end of the enunciations of various propositions should not be taken always to identify real Witelian sources, but that they should also be seen as indications of *other* sources where the same, or related, propositions could be found by the interested reader. In this case, there is no positive evidence for the availability of Proclus's *Commentary on the First Book of Euclid's Elements* in Latin in the thirteenth century. In this connection see [M. Grabmann 1936, 2, 413-423], [M. Steinschneider 1956, 17], [J. G. Wenrich 1842], [L. J. Rosan 1949, 223] and [C. Baeumker 1908, 235].
8. These points are made in a penetrating and basic article [Murdoch 1968] in which John Murdoch takes off in his analysis from another fundamental study [Clagett 1953].

REFERENCES

- Baeumker, Clemens 1908 Witelo, ein Philosoph und Naturforscher des XIII. Jahrhunderts *Beiträge zur Geschichte der Philosophie des Mittelalters* 3 (2) Münster

- Clagett, Marshall 1953 The medieval Latin translations from the Arabic of the *Elements* of Euclid with special emphasis on the versions of Adelard of Bath *Isis* 44, 16-42
- Grabmann, Martin 1936 *Mittelalterliches Geistesleben* Vol. 2 München
- Murdoch, John E 1968 The medieval Euclid: Salient aspects of the translations of the *Elements* by Adelard of Bath and Campanus of Novera *Revue de Synthèse* 89 (IIIe S.), nos. 49-52, 67-94
- Proclus 1970 *A Commentary on the First Book of Euclid's Elements* (Princeton)
- Rosan, L J 1949 *The Philosophy of Proclus, The Final Phase of Ancient Thought* (New York)
- Steinschneider, Moritz 1956 *Die Europäischen Übersetzungen aus dem Arabischen bis Mitte des 17. Jahrhunderts* (Graz)
- Unguru, Sabetai 1970 *Witelo as a Mathematician: A Study in XIIIth Century Mathematics including a Critical Edition and English Translation of the Mathematical Book of Witelo's Perspectiva* Dissertation, University of Wisconsin
- 1972 Witelo and Thirteenth-Century Mathematics: An Assessment of his contributions *Isis* 63, 496-508
- 1977 *Witelonis Perspectivae Liber Primus Book I of Witelo's Perspectiva: An English Translation with Introduction and Commentary and a Latin Edition of the Mathematical Book of Witelo's Perspectiva* The Polish Academy of Sciences Press (Warsaw)
- Weinrich, J G 1842 *De Auctorum Graecorum versionibus et commentariis Syriacis Arabicis Armeniacis Persicisque* (Leipzig)
- Witelo 1572 *Vitellonis Thuringopoloni opticae libri decem* (Basel)