Computers and Mathematics with Applications 58 (2009) 2398–2401

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

The homotopy perturbation method for discontinued problems arising in nanotechnology

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ARTICLE INFO

Keywords:
Discretized mKdV lattice equation
Homotopy perturbation method
Approximate solution

ABSTRACT

Continuum hypothesis on nanoscales is invalid, and a differential–difference model is considered as an alternative approach to describing discontinued problems. This paper applies the homotopy perturbation method to a nonlinear differential–difference equation arising in nanotechnology. Comparison of the approximate solution with the exact one reveals that the method is very effective.

1. Introduction

According to E-infinity theory [1–3], space at the quantum scale is not a continuum, and it is clear that nanotechnology possesses a considerable richness which bridges the gap between the discrete and the continuum [4–6]. On nanoscales, He et al. [7] found experimentally an uncertainty phenomenon similar to Heisenberg's uncertainty principle in quantum mechanics. Continuum hypothesis on the nanoscales becomes, therefore, invalid. He and Zhu [8] suggested some differential–difference models describing fascinating phenomena arising in heat/electron conduction and flow in carbon nanotubes, among which we will study the following model:

$$\frac{\mathrm{d}u_n}{\mathrm{d}t} = (u_{n+1} - u_{n-1}) \sum_{k=1}^{m} (\alpha_k + \beta_k (u_n)^k) \tag{1}$$

where α_k and β_k are constants. Physical interpretation is given in Ref. [8]. Eq. (1) includes the well-known discretized mKdV lattice equation [9]:

$$\frac{\mathrm{d}u_n}{\mathrm{d}t} = (\alpha - u_n^2)(u_{n+1} - u_{n-1}) \tag{2}$$

where the subscript n in Eq. (1) represents the nth lattice.

In this paper we will study analytically Eq. (2) using the homotopy perturbation method [10–14]. Previously such equations were solved by the exp-function method [15–17] and the variational iteration method [18].

2. Homotopy perturbation method

Dr. Davood Domiri Ganji pointed out "*Wherever a nonlinear equation is found, Dr. He's Homotopy perturbation method will be the primary tool of discovery*" (see ScienceWatch.com website in February, 2008). Since the appearance of the homotopy

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doi:10.1016/j.camwa.2009.03.048

Table 1
Table (when $\alpha = 1, d = 0.1, t = 1$).

n	Approximate solution	Exact solution	Absolute error	Relative error
15	0.09315671550	0.09322206784	0.0006535234000	0.0007010393731
5	0.06024368408	0.06019410003	0.00004958405	0.0008237360468
4	0.05358059812	0.05347954390	0.0001010542200	0.001889586423
3	0.04616017532	0.04600622678	0.0001539485400	0.003346254425
−3	−0.009797356980	−0.009999228009	0.0002018710300	0.02018866155
−4	−0.01958193014	−0.01973559643	0.0001536662900	0.007786250114
−5	−0.02899431132	−0.02909509651	0.0001007851900	0.00346399229
−15	−0.08597636659	−0.08590324656	0.00007312004000	0.0008511906469

perturbation method in 1999 [10], it has become a universal mathematical tool for nonlinear equations [19–35]. Hereby we will use the method to search for the traveling wave solution of Eq. (2), we first construct a homotopy as follows:

$$(1-p)\left(\frac{\mathrm{d}u_n}{\mathrm{d}t} - \frac{\mathrm{d}u_{n0}}{\mathrm{d}t}\right) + p\left[\frac{\mathrm{d}u_n}{\mathrm{d}t} - (\alpha - u_n^2)(u_{n+1} - u_{n-1})\right] = 0, \tag{3}$$

where u_{n0} is the initial guess which includes some possible unknown parameters [11,12]. For simplicity, we begin with

$$u_{n0}(n,t) = u_{n0}(n,0) \tag{4}$$

$$u_n(n,t) = U_n(n,t) = u_{n0} + pu_{n1} + p^2u_{n2} + p^3u_{n3} + \cdots, \tag{5}$$

where u_{ni}, $(i = 1, 2, 3, \ldots)$ are functions of (n, t) yet to be determined. Substituting Eq. (5) into Eq. (3), and equating the coefficients of the terms with the identical powers of p, we have

$$-\alpha u_{(n+1)0} + \frac{\mathrm{d}u_{n0}}{\mathrm{d}t} + \frac{\mathrm{d}u_{n1}}{\mathrm{d}t} - u_{n0}^2u_{(n-1)0} + \alpha u_{(n-1)0} + u_{n0}^2u_{(n+1)0} = 0, \tag{6}$$

$$2u_{n0}u_{n1}u_{(n+1)0} - u_{n0}^2u_{(n-1)1} + \alpha u_{(n-1)1} - \alpha u_{(n+1)1} + \frac{\mathrm{d}u_{n2}}{\mathrm{d}t} + u_{n0}^2u_{(n+1)1} - 2u_{n0}u_{n1}u_{(n-1)0} = 0. \tag{7}$$

If the 3-term approximation is sufficient, we will obtain:

$$u_n(n,t) = \lim_{p=1} U_n(n,t) = \sum_{k=0}^{2} u_{nk}(n,t). \tag{8}$$

In order to illustrate the effectiveness of the method, we consider the following initial condition:

$$u_{n0}(n,t) = \sqrt{\alpha}\tanh(d)\tanh(nd), \tag{9}$$

where d is an arbitrary constant.

Solving $u_1(n,t)$ and $u_2(n,t)$, from Eqs. (7) and (8), results in

$$u_{n1}(n,t) = \frac{2\alpha^{\frac{3}{2}}(\cosh(d)^2 - 1)t}{\cosh(d)^2\cosh(nd)^2}, \tag{10}$$

$$u_{n2}(n,t) = \frac{4t^2\alpha^{\frac{5}{2}}\sinh(nd)\sinh(d)(\cosh(d)^4 + \cosh(d)^2\cosh(nd)^2 - 2\cosh(d)^2 - \cosh(nd)^2 + 1)}{\cosh(d)^3\cosh(nd)^3(\sinh(nd)^2\sinh(d)^2 - \cosh(nd)^2\cosh(d)^2)}. \tag{11}$$

The 3-term approximation reads

$$u_n(n,t) = u_{n0}(n,t) + u_{n1}(n,t) + u_{n2}(n,t) \tag{12}$$

where $u_0(n,t)$, $u_1(n,t)$ and $u_2(n,t)$ are expressed, respectively, as Eqs. (9), (10), and (11).

Table 1 illustrates remarkable accuracy of the approximate solution. Fig. 1 reveals that the system tends to equilibrium $u_n(n,t) \to \alpha^{1/2}\tanh(d)$ when $n > 20$, and Fig. 2 shows the absolute error of the obtained solution.

3. Discussion and conclusion

The solution procedure is simple while the obtained result is of high accuracy, which can be further improved if the solution procedure is continued to higher orders. From Fig. 2, we can see that, unlike other numerical methods where the absolute error is usually noisy, the absolute error of our solution is almost not noisy. This is one of the unique characters of the homotopy perturbation method.

If the studied equation describes the flow in a carbon nanotube, then the velocity of the flow tends to the maximum when $n \approx 20$, this can explain some fascinating phenomena of nanohydrodynamics.

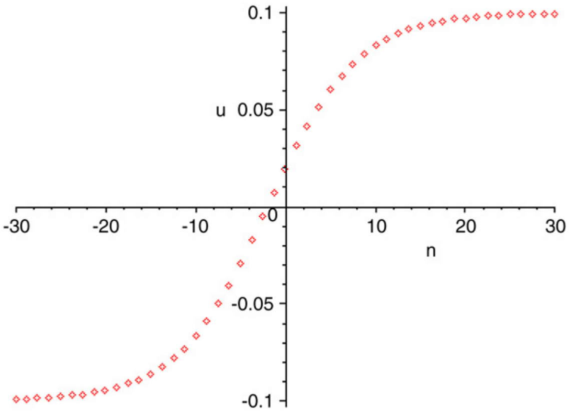

Fig. 1. Approximate solution ($\alpha = 1$, $d = 0.1$, and $t = 1$).

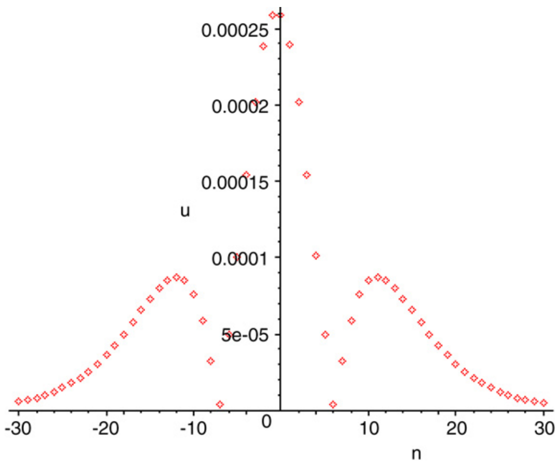

Fig. 2. Absolute error ($\alpha = 1$, $d = 0.1$, and $t = 1$).

Acknowledgements

The project was supported by the National Natural Science Foundation of China under the Grant No.10771195, 973 Project under Grant No.2006CB708304, the Natural Science Foundation of Zhejiang Province under Grant No.Y607128, and the Natural Science Foundation of Zhejiang Lishui University under the Grant No.KZ070016.

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