An auto-switched chaos system

Xinhua Wu\textsuperscript{a,b*}, Zhonglin Wang\textsuperscript{a}

\textsuperscript{a}Department of Physics and Electronics, Binzhou University, Binzhou, 256603, China
\textsuperscript{b}Institute of theoretical physics, Binzhou University, Binzhou, 256603, China

Abstract

In order to generate complex chaotic attractor, an auto-switched chaotic system which consists of two subsystems is constructed. The switched chaotic system can change its behavior automatically from one to another via a data selector. The system is also implemented based on FPGA by EDA Technology, and the experiments shows a good agree with simulation.

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1. Introduction

In 1963, Lorenz found a three-dimensional autonomous chaotic system, which generated the well known Lorenz chaotic attractor\textsuperscript{[1]}. The Lorenz system has been extensively studied in the fields of chaotic theory and dynamical systems\textsuperscript{[2-3]}. However, the switched chaotic system has more complex dynamical behaviors than common chaotic system. It has been studied with increasing interest due to its theoretical and practical applications in technological fields, such as secure communication, laser, nonlinear circuits, neural networks, control and synchronization\textsuperscript{[4-10]}. There are two main methods to generate the switched system. One is by manually operation\textsuperscript{[11-13]}, the other is switching automatically\textsuperscript{[14-15]}. The latter is better.

This paper proposes an auto-switched chaotic system, which is based on a chaotic system\textsuperscript{[4]} and can change its behaviour automatically from one to another. It is also proved by the Lyapunov exponent spectrum, bifurcation diagram and experiments based on FPGA by EDA technology.

2. Construction of an auto-switched chaotic system

Consider the following three-dimensional autonomous chaotic system based on a chaotic system\textsuperscript{[4]}:

* Corresponding author. Tel.: +86-13854398209.
E-mail address: bzcong@126.com
\[
\begin{aligned}
\begin{cases}
\dot{x} = a(x - y) \\
\dot{y} = -cy + xz \\
\dot{z} = -bz + xf(x)
\end{cases}
\end{aligned}
\]  

(1)

where \( f(x) = \begin{cases} 
  x & x \geq 0 \\
  y & x < 0 
\end{cases} \), then an auto-switched chaotic system is obtained. When the state variable \( x \) of the auto-switched system (1) satisfies \( x \geq 0 \), the function \( f(x) \) is \( x \), thus it runs on one subsystem.

\[
\begin{aligned}
\begin{cases}
\dot{x} = a(x - y) \\
\dot{y} = -cy + xz \\
\dot{z} = -bz + x^3
\end{cases}
\end{aligned}
\]  

(2)

When \( x < 0 \) it runs on other subsystem

\[
\begin{aligned}
\begin{cases}
\dot{x} = a(x - y) \\
\dot{y} = -cy + xz \\
\dot{z} = -bz + xy
\end{cases}
\end{aligned}
\]  

(3)

When \( a = 20, b = 2, c = 28 \), subsystem (2) and subsystem (3) both have a chaotic attractor. The auto-switched chaotic system (1) transform its behavior randomly between subsystem (2) and (3) as the state \( x \) is varied. The chaotic attractor of auto-switched system (1) is shown in Fig 1, in which black parts express the orbits of subsystem (2) and blue parts express the orbits of subsystem (3).

![Fig. 1](image_url)

Fig. 1  The chaotic attractor of the auto-switched chaotic system (1) with \( a = 20, b = 2, c = 28 \).

3. The basic properties of the auto-switched chaotic system

3.1 Symmetry

The subsystem (2) and (3) are both symmetrical about the z-axis for its invariance under the coordinate transform \((x, y, z) \rightarrow (-x, -y, z)\), but the switched chaotic system (1) is not symmetrical about the z-axis. The symmetry property of the switched chaotic system is changed by the function \( f(x) \).

3.2 Dissipation

For system (1), (2) and (3), we all have

\[
\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = a - b - c
\]  

(4)

Therefore, all of the three systems are dissipative if and only if \( a - c - b > 0 \). Under this condition, three systems converges exponentially
\[ \frac{dv}{dt} = e^{-(b+c-a)t} \]  

(5)

That is, the volume element at time \( t \) is \( V_0 e^{-(b+c-a)t} \), with the initial volume element being \( V_0 \). All orbits of these systems are confined to a specific subset of zero volume as \( t \to \infty \). Then, the existence of an attractor is proved.

3.3 Equilibria and stability

The equilibria of these systems can be found by letting the left-hand side of Eq.(1) equal zero. If \( h(b+c) > 0 \), these systems have three equilibria: \( S_1(\sqrt{56}, \sqrt{56}, 28) \), \( S_2(-\sqrt{56}, -\sqrt{56}, 28) \) and \( S_0(0, 0, 0) \). Linearizing system (1) at the equilibrium \( s_0 \), the Jacobian matrix is obtained as follows:

\[
J_0 = \begin{bmatrix}
a & -a & 0 \\
0 & -c & 0 \\
0 & 0 & -b
\end{bmatrix}, \text{ When } a = 20, b = 2, c = 28, J = \begin{bmatrix}
20 & -20 & 0 \\
0 & -28 & 0 \\
0 & 0 & -2
\end{bmatrix}.
\]

The corresponding eigenvalues are: \( \lambda_1 = -2 \), \( \lambda_2 = -28 \), \( \lambda_3 = 20 \). Here \( \lambda_3 \) is a positive real number, \( \lambda_1 \) and \( \lambda_2 \) are two negative real numbers. Therefore, the equilibrium \( s_0 \) is a saddle point and is unstable.

Next, linearizing the system (1) about the other equilibrium \( s_1 \) yields the following Jacobian matrix:

\[
J = \begin{bmatrix}
20 & -20 & 0 \\
28 & -28 & \sqrt{56} \\
2\sqrt{56} & 0 & -2
\end{bmatrix}
\]

The corresponding eigenvalues are \( \lambda_1 = -16.8979 \), \( \lambda_{2,3} = 3.449 \pm 10.984j \). Here \( \lambda_1 \) is a negative real number, \( \lambda_2 \) and \( \lambda_3 \) become a pair of complex conjugate eigenvalues with positive real parts. The equilibrium point \( s_1 \) is a saddle-focus point; this equilibrium point is unstable. For equilibrium point \( s_2 \), its corresponding Jacobian matrix is:

\[
J = \begin{bmatrix}
20 & -20 & 0 \\
28 & -28 & -\sqrt{56} \\
-\sqrt{56} & -\sqrt{56} & -2
\end{bmatrix}
\]

The eigenvalues are \( \lambda_1 = -18.614 \), \( \lambda_{2,3} = 4.307 \pm 10.0981j \). Results show that \( \lambda_1 \) is a negative real number, \( \lambda_2 \) and \( \lambda_3 \) form a complex conjugate pair and their real parts are positive. Equilibrium point \( s_2 \) is also a saddle-focus point; this equilibrium point is unstable.

The above analyses show that the three equilibrium points of the nonlinear system are all saddle-focus nodes.

3.4 Bifurcation analysis

The new chaotic system (1) can be characterized with its Lyapunov exponents computed by Wolf method. For equilibrium points: \( LE_3 < LE_2 < LE_1 < 0 \); for periodic orbits: \( LE_3 < LE_2 < 0, LE_1 = 0 \); for quasi-periodic orbits: \( LE_3 < 0, LE_2 = LE_1 = 0 \); while for chaotic orbits: \( LE_3 < 0, LE_2 = 0, LE_1 > 0, LE_3 + LE_2 + LE_1 < 0 \).

In order to observe the effect of the parameters on the dynamics of the chaotic system, we fix the parameters \( a = 20, c = 28 \), and let the parameter \( b \) vary in the interval [0 9].
The bifurcation diagram and the Lyapunov exponents are shown in Fig. 2.

![Bifurcation Diagram and Lyapunov Exponents](image)

Fig 2 Lyapunov exponents and bifurcation of the system (1) when \( a = 20 \), \( c = 28 \), and vary \( b \).

From Fig. 2 (a) and (b), one can see that the Lyapunov exponents spectrum are completely consistent with the bifurcation diagram.

When \( b \in [0, 9.45] \), the largest Lyapunov exponent is positive, implying that the system shows chaotic behaviour. Fig. 1 depicts the chaotic attractor for \( b = 2 \).

When \( b \in [9.45, 10] \), the three Lyapunov exponents are all less than zero, the system is equilibrium points, Fig. 3 displays equilibrium points for \( b = 9 \).

![Equilibrium Points](image)

Fig. 3 The equilibrium points of system (1) with \( a = 20 \), \( b = 9 \), \( c = 28 \).

### 4. Circuit realization of the switched chaotic system based on FPGA

The circuit of system (1) is designed by the DSP Builder tool from Altera Company in America. It is based on FPGA and the circuit frame figure is shown in Fig. 4.

Adder, delay, multiplication, amplifier and data selector are selected from the DSP Builder component library, and the digital integrator is designed according to the relationship between first order differential coefficient and difference.

We can implement the system (1) model of the FPGA circuit in Matlab/Simulink and generate hardware description language automatically. The system (1) can be mapped into a rock-bottom hardware realized chaotic attractor based on FPGA directly. The attractors have been obtained by using D/A converter as shown in Fig. 5 and Fig. 6. Experiment results are in accordance with simulation results.
Fig. 4. Circuit realized the switched chaotic system in the Simulink by DSP-Builder blocks

Fig. 5. Experimental observations of the system (1) with $a = 20, b = 2, c = 28$

Fig. 6. Experimental observations of the system (1) with $a = 20, b = 9, c = 28$

5. Conclusions

A novel auto-switched chaotic system is proposed based on Cai system, the system consists of two subsystems which can change its behaviour automatically from one to another. Lyapunov exponent
spectrum, bifurcation diagram and Poincaré map of the system are analysed, a digital circuit experimentation is carried out upon FPGA by EDA technology. The result is agree with simulation.

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