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# Five adjustable parameter fit of quark and lepton masses and mixings

H.B. Nielsen [a,b], Y. Takanishi [a,b]

[a] *Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, D-22603 Hamburg, Germany*
[b] *The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

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**Abstract**

We develop a model of ours fitting the quark and lepton masses and mixing angles by removing from the model a Higgs field previously introduced to organise a large atmospheric mixing angle for neutrino oscillations. Due to the off-diagonal elements dominating in the seesaw neutrino mass matrix the large atmospheric mixing angle comes essentially by itself. It turns out that we have now only five adjustable Higgs field vacuum expectation values needed to fit all the masses and mixings *order of magnitudewise* taking into account the renormalisation group runnings in all sectors. The CHOOZ angle comes out close to the experimental bound.



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## 1. Introduction

In previous models [1,2] based on a series of approximately conserved quantum numbers taken to be gauged quantum numbers broken by Higgs fields with "small" expectation values (VEVs) we managed to fit to typically better than a factor two—on the average ±60%—all the masses and mixing angles of quarks and leptons known so far. This we did by means of six adjustable Higgs VEVs (not counting the Weinberg–Salam Higgs, which is fixed from the Fermi constant) assuming that all coupling constants are of order of magnitude unity, only Higgs fields having VEVs of orders of magnitude different from unity at Planck scale. The adjustable Higgs field VEVs were put up to break a series of gauge symmetries—it is enough for the model prediction purpose to consider Abelian gauge fields, actually two for each family of quarks and leptons—and one of them, $\phi_{B-L}$, fits the overall seesaw neutrino scale, while two, $\rho$ and $\omega$, connect the first and second family by breaking the to these families associated four $U(1)$ gauge groups down to only two. Now, however, there were in the previous version of our model three Higgs

*E-mail addresses:* hbech@mail.desy.de (H.B. Nielsen), yasutaka@mail.desy.de (Y. Takanishi).


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fields, $W$, $T$ and $\chi$, which in a similar way broke two linear combinations of the $U(1)$ gauge groups associated with the families number 2 and 3. But that is seemingly one field too many. It is the main purpose of the present article to point out that the model can be improved by removing from existence one of those, last mentioned three fields, namely the one called $\chi$. For making this avoidance of the field, $\chi$, to fit we only need to make an adjustment of the quantum numbers of the seesaw scale giving Higgs field $\phi_{B-L}$, which we in the new version call $\phi_{\mathrm{SS}}$.

Our newest modification of removing the field, $\chi$, is really only of essential significance for the neutrino oscillation parameters because this field only occurred in very insignificant matrix elements for the mass matrices of the charged quarks and leptons. By fitting now to the neutrinos with fewer parameters in a different way there will of course come a slightly different pull for the parameters than before also in the charged sector, though.

## 2. The model

The backbone of the model is the assignment of the in Table 1 described quantum numbers under the group

$$SMG \times U(1)^5, \tag{1}$$

supposed to be conceived of as the gauge group assigning two $U(1)$ gauge groups to each family of quarks and leptons. In our scheme with three seesaw neutrinos, a family consists of a usual Standard Model family enriched by one right-handed neutrino (to be used as seesaw neutrino). For each family the one of these $U(1)$ gauge groups couples just like to the weak hypercharge, i.e., with $y/2$ coupling, but *only to the family in question*. The other one couples to the $B-L$ = "baryon number" − "lepton number" again only for the *family in question*. That is to say we speculate of there being altogether six $U(1)$ gauge fields, but now one of them—actually the diagonal subgroup of the three of them—*is* the weak hypercharge gauge group for the whole system and thus we only have added five such groups.

The model can quite naturally be embellished by the addition to the group of two extra $SU(2)$ and two extra $SU(3)$ making the whole Standard Model group including a $B-L$ charge, which is gauged, become replicated into one copy for each family of quarks and leptons. So each family gets its own gauge fields of all the types in the Standard Model and in addition a gauged $B-L$ charge also separate for each family. However, we do not need this embellishment for the mass protections that provide the small hierarchies, which we fit. For that purpose the Abelian quantum numbers in Table 1 are sufficient. The embellished model has the gauge group,

$$\underset{i=1,2,3}{\times}\left(SMG_i \times U(1)_{B-L,i}\right), \tag{2}$$

which we call the family replicated gauge group.

The quantum numbers of the fermions and of the Higgs fields which we have chosen to have in our by now a bit simplified model—because of the deletion of one Higgs field—are listed in Table 1. Here the charges, $y_i/2$, are the weak hypercharges for the three families $i = 1, 2, 3$, while the baryon number minus lepton number charges $B-L$ for the three families separately are denoted $(B-L)_i$, also with $i = 1, 2, 3$.

The quantum numbers for the embellished model can if one likes be constructed rather easily by means of the relation between the non-Abelian representation, i.e., of $SU(2)$ and $SU(3)$, and the weak hypercharge $y/2$ in the Standard Model. One namely simply requires to have the same relation valid for each value of the family denoting index $i$ separately. That is to say, that the representation of $SU(2)_i$ and $SU(3)_i$ for an arbitrary particle, quark, lepton, or Higgs, are simply postulated in our embellished version of the model to be the same as that quark or lepton Weyl field in the Standard Model that has the value $y/2 = y_i/2$ for its weak hypercharge (actually, though, we count it mod 1 in the case $\phi_{\mathrm{SS}}$).

This embellished model has the beauty of having the biggest gauge groups under some restrictions such as: only transforming the known particles and the seesaw neutrinos, not unifying Standard Model irreducible representations, and having no anomalies [2,3].

Table 1
All $U(1)$ quantum charges in the family replicated model. The symbols for the fermions shall be considered to mean "proto"-particles. Non-Abelian representations are given by a rule from the Abelian ones (see Section 2)

| | $SMG_1$ | $SMG_2$ | $SMG_3$ | $U_{B-L,1}$ | $U_{B-L,2}$ | $U_{B-L,3}$ |
|---|---|---|---|---|---|---|
| $u_L, d_L$ | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $u_R$ | $\frac{2}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $d_R$ | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $e_L, \nu_{e_L}$ | $-\frac{1}{2}$ | 0 | 0 | $-1$ | 0 | 0 |
| $e_R$ | $-1$ | 0 | 0 | $-1$ | 0 | 0 |
| $\nu_{e_R}$ | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $c_L, s_L$ | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $c_R$ | 0 | $\frac{2}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $s_R$ | 0 | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |
| $\mu_L, \nu_{\mu_L}$ | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ | 0 |
| $\mu_R$ | 0 | $-1$ | 0 | 0 | $-1$ | 0 |
| $\nu_{\mu_R}$ | 0 | 0 | 0 | 0 | $-1$ | 0 |
| $t_L, b_L$ | 0 | 0 | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{3}$ |
| $t_R$ | 0 | 0 | $\frac{2}{3}$ | 0 | 0 | $\frac{1}{3}$ |
| $b_R$ | 0 | 0 | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ |
| $\tau_L, \nu_{\tau_L}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ |
| $\tau_R$ | 0 | 0 | $-1$ | 0 | 0 | $-1$ |
| $\nu_{\tau_R}$ | 0 | 0 | 0 | 0 | 0 | $-1$ |
| $\phi_{\mathrm{WS}}$ | 0 | $\frac{2}{3}$ | $-\frac{1}{6}$ | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ |
| $\omega$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | 0 | 0 | 0 |
| $\rho$ | 0 | 0 | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 |
| $W$ | 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $T$ | 0 | $-\frac{1}{6}$ | $\frac{1}{6}$ | 0 | 0 | 0 |
| $\phi_{\mathrm{SS}}$ | 0 | 1 | $-1$ | 0 | 2 | 0 |

### *2.1. Mass matrices*

With the system of quantum numbers in Table 1 one can easily evaluate, for a given mass matrix element, the numbers of Higgs field VEVs of the different types needed to perform the transition between the corresponding left- and right-handed Weyl fields. The products of Higgs fields needed to achieve the quantum number transitions between left- and right-handed fermions are after removal of the Higgs field in the previous version of our model, called $\chi$, completely unique. We do no longer have several possibilities of achieving the same transition so that a choice of the one giving the dominant matrix element would be needed. Rather it is simple linear algebra with the charges to compute the needed powers for our different fields. The main point of the present type of model is that the order of magnitudes of the mass matrix elements are determined by the number and order of magnitudes of the Higgs field vacuum expectation value factors needed. We namely take all the Yukawa couplings (and also other couplings) in our model and the masses of particles that are not mass-protected to be of order unity in some fundamental scale (which we usually imagine to be the Planck scale, and actually take to be that in the renormalisation group runnings of the effective couplings in our model). Thereby of course the whole calculation we can do will only give order of magnitudewise results! Thus one shall imagine that every expression for a mass matrix element in addition to the Higgs vacuum expectation value factors should be provided with an unknown factor of order unity (which we in the calculation take to be a random number of order unity), then later logarithmically averaging the results. We have, however, not to complicate the expressions too much left out as to be understood these factors $\lambda_{ij}^{(q)}$, where $q \in \{\text{up}, \text{down}, \text{charged lepton}, \text{Dirac neutrino}, \text{Majorana neutrino}\}$, of order unity. The order unity factors physically represent that all the enormously many couplings and masses of not mass protected particles are realistically unknown so that the only realistic assumption about them is that they are of order of magnitude of unity (in fundamental units). In our numerical evaluation of the predictions of our model we take the "order unity factors" just in front of each matrix element in our mass matrices as dummy integration variables. In fact we integrate the logarithm of, say, a predicted mass or mixing angle (as at first a function of our $9 \times 4 + 6 = 42$ order of unity parameters) over the real and imaginary parts of these parameters with a weight which is normalised so that the corresponding integral of one would be unity. The weight is to be chosen so that it emphasises the "order of one factor" being indeed of order unity. This integral is then used as our prediction for the logarithm of the mass or mixing in question. This integration over the real and imaginary parts of the "order unity factors" is in practice performed by a Monte Carlo method. Had we simply put the order one factor equal to unity, we would in some simple cases have got roughly right results, however, in many cases subdeterminants might become exactly zero by such procedure and we would have got severely wrong results—even order of magnitudewise—compared to our physical picture of the fundamental couplings being in reality not exactly unity but *only* of such order.

With the order of unity factor removed our mass matrices look as follows:

The up-type quarks:

$$M_U \simeq \frac{\langle(\phi_{\mathrm{WS}})^\dagger\rangle}{\sqrt{2}} \begin{pmatrix} (\omega^\dagger)^3 W^\dagger T^2 & \omega\rho^\dagger W^\dagger T^2 & \omega\rho^\dagger (W^\dagger)^2 T \\ (\omega^\dagger)^4 \rho W^\dagger T^2 & W^\dagger T^2 & (W^\dagger)^2 T \\ (\omega^\dagger)^4 \rho & 1 & W^\dagger T^\dagger \end{pmatrix}. \tag{3}$$

The down-type quarks:

$$M_D \simeq \frac{\langle\phi_{\mathrm{WS}}\rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W (T^\dagger)^2 & \omega\rho^\dagger W (T^\dagger)^2 & \omega\rho^\dagger T^3 \\ \omega^2 \rho W (T^\dagger)^2 & W (T^\dagger)^2 & T^3 \\ \omega^2 \rho W^2 (T^\dagger)^4 & W^2 (T^\dagger)^4 & W T \end{pmatrix}. \tag{4}$$

The charged leptons:

$$M_E \simeq \frac{\langle\phi_{\mathrm{WS}}\rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W (T^\dagger)^2 & (\omega^\dagger)^3 \rho^3 W (T^\dagger)^2 & (\omega^\dagger)^3 \rho^3 W^4 (T^\dagger)^5 \\ \omega^6 (\rho^\dagger)^3 W (T^\dagger)^2 & W (T^\dagger)^2 & W^4 (T^\dagger)^5 \\ \omega^6 (\rho^\dagger)^3 (W^\dagger)^2 T^4 & (W^\dagger)^2 T^4 & W T \end{pmatrix}. \tag{5}$$

The Dirac neutrinos:

$$M_\nu^D \simeq \frac{\langle(\phi_{\mathrm{WS}})^\dagger\rangle}{\sqrt{2}} \begin{pmatrix} (\omega^\dagger)^3 W^\dagger T^2 & (\omega^\dagger)^3 \rho^3 W^\dagger T^2 & (\omega^\dagger)^3 \rho^3 W^2 (T^\dagger)^7 \\ (\rho^\dagger)^3 W^\dagger T^2 & W^\dagger T^2 & W^2 (T^\dagger)^7 \\ (\rho^\dagger)^3 (W^\dagger)^4 T^8 & (W^\dagger)^4 T^8 & W^\dagger T^\dagger \end{pmatrix}. \tag{6}$$

The Majorana (right-handed) neutrinos:

$$M_R \simeq \langle\phi_{\mathrm{SS}}\rangle \begin{pmatrix} (\rho^\dagger)^6 T^6 & (\rho^\dagger)^3 T^6 & (\rho^\dagger)^3 W^3 (T^\dagger)^3 \\ (\rho^\dagger)^3 T^6 & T^6 & W^3 (T^\dagger)^3 \\ (\rho^\dagger)^3 W^3 (T^\dagger)^3 & W^3 (T^\dagger)^3 & W^6 (T^\dagger)^{12} \end{pmatrix}. \tag{7}$$

The philosophy of the model is that these mass matrices correspond to effective Yukawa couplings to be identified with running Yukawa couplings at the fundamental/Planck scale for the Higgs field $\phi_{\rm WS}$ in the case of the first three mass matrices and for $\phi_{\rm SS}$ in the case of the right-handed neutrino mass matrix. Therefore these effective Yukawa couplings have in principle to be run down by the beta-functions to the scale of observation, see Section 3. It is also important that we include the "running" of the irrelevant operator of dimension 5 giving the neutrino oscillation masses.

The right-handed neutrino couplings—or mass matrix—is used for producing an effective mass matrix for the left-handed neutrinos which we after the five-dimensional running down mentioned take as the ones observed in neutrino oscillations [4,5]:

$$M_{\rm eff} \simeq M_\nu^D M_R^{-1} \left(M_\nu^D\right)^{\rm T}. \tag{8}$$

*2.2. $M_{\rm eff}$ crude calculation*

Taking into account the various orders of magnitude of our fitting parameters, the expectation values in vacuum of Higgs fields, it is not difficult to perform the calculations crudely and very often only one or two terms dominate a given quantity. Even the slightly more complicated case of the effective matrix for the left-handed neutrinos—the neutrino oscillation mass matrix—we compute crudely:

$$\begin{aligned}
M_{\rm eff} &\sim \frac{\langle\phi_{\rm WS}\rangle^2}{4\langle\phi_{\rm SS}\rangle W^6T^6\rho^6} \overbrace{\begin{pmatrix} \omega^3WT^2 & \omega^3\rho^3WT^2 & \omega^3\rho^3W^2T^7 \\ \rho^3WT^2 & WT^2 & W^2T^7 \\ \rho^3W^4T^8 & W^4T^8 & WT \end{pmatrix}}^{M_\nu^D \text{ part}} \\
&\quad \times \underbrace{\begin{pmatrix} W^6 & W^6\rho^3 & \sqrt{2}\,W^3T^3\rho^3 \\ W^6\rho^3 & W^6\rho^6 & \sqrt{2}\,W^3T^3\rho^6 \\ \sqrt{2}\,W^3T^3\rho^3 & \sqrt{2}\,W^3T^3\rho^6 & \sqrt{2}\,T^6\rho^6 \end{pmatrix}}_{M_R^{-1} \text{ part}} \underbrace{\begin{pmatrix} \omega^3WT^2 & \rho^3WT^2 & \rho^3W^4T^8 \\ \omega^3\rho^3WT^2 & WT^2 & W^4T^8 \\ \omega^3\rho^3W^2T^7 & W^2T^7 & WT \end{pmatrix}}_{(M_\nu^D)^{\rm T} \text{ part}} \\
&\sim \frac{\langle\phi_{\rm WS}\rangle^2}{4\langle\phi_{\rm SS}\rangle W^6T^6\rho^6} \begin{pmatrix} \omega^6W^8T^4 & \sqrt{2}\,\omega^3\rho^3W^8T^4 & \sqrt{2}\,\omega^3\rho^3W^5T^6 \\ \sqrt{2}\,\omega^3\rho^3W^8T^4 & 2\rho^6W^8T^4 & 2\rho^6W^5T^6 \\ \sqrt{2}\,\omega^3\rho^3W^5T^6 & 2\rho^6W^5T^6 & \sqrt{2}\,\rho^6W^2T^8 \end{pmatrix}.
\end{aligned} \tag{9}$$

Here we used the following rules of calculation aimed at taking into account the random numbers of order unity not written explicitly, but meant to be conceived of as standing as factors in front of each term in the mass matrices:

(1) When $p$ terms with the same set of "VEV-factors", i.e., of the types $W$, $T$, $\omega$, $\rho$ ($\phi_{\rm WS}$ and $\phi_{\rm SS}$) we imagine random phases in the addition (in the complex plane) and take the result as $\sqrt{p}$ times the term that were $p$ times repeated.
(2) We only take the dominant or couple of dominant terms with respect to the VEV-factors.

## 3. Renormalisation group equations

From the Planck scale down to the seesaw scale or rather from where our gauge group is broken down to $SMG \times U(1)_{B-L}$ we use the one-loop renormalisation group running of the Yukawa coupling constant matrices, $Y_U$, $Y_D$, $Y_E$, $Y_\nu$ and $Y_R$ (being proportional to the mass matrices $M_U$, $M_D$, $M_E$, $M_\nu^D$ and $M_R$, respectively), and the gauge couplings [6]:

$$16\pi^2\frac{dg_1}{dt}=\frac{41}{10}g_1^3, \tag{10}$$

$$16\pi^2\frac{dg_2}{dt}=-\frac{19}{16}g_2^3, \tag{11}$$

$$16\pi^2\frac{dg_3}{dt}=-7g_3^3, \tag{12}$$

$$16\pi^2\frac{dY_U}{dt}=\frac{3}{2}\big(Y_U(Y_U)^\dagger-Y_D(Y_D)^\dagger\big)Y_U+\left\{Y_S-\left(\frac{17}{20}g_1^2+\frac{9}{4}g_2^2+8g_3^2\right)\right\}Y_U, \tag{13}$$

$$16\pi^2\frac{dY_D}{dt}=\frac{3}{2}\big(Y_D(Y_D)^\dagger-Y_U(Y_U)^\dagger\big)Y_D+\left\{Y_S-\left(\frac{1}{4}g_1^2+\frac{9}{4}g_2^2+8g_3^2\right)\right\}Y_D, \tag{14}$$

$$16\pi^2\frac{dY_E}{dt}=\frac{3}{2}\big(Y_E(Y_E)^\dagger-Y_\nu(Y_\nu)^\dagger\big)Y_E+\left\{Y_S-\left(\frac{9}{4}g_1^2+\frac{9}{4}g_2^2\right)\right\}Y_E, \tag{15}$$

$$16\pi^2\frac{dY_\nu}{dt}=\frac{3}{2}\big(Y_\nu(Y_\nu)^\dagger-Y_E(Y_E)^\dagger\big)Y_\nu+\left\{Y_S-\left(\frac{9}{20}g_1^2+\frac{9}{4}g_2^2\right)\right\}Y_\nu, \tag{16}$$

$$16\pi^2\frac{dY_R}{dt}=\big((Y_\nu)^\dagger Y_\nu\big)Y_R+Y_R\big((Y_\nu)^\dagger Y_\nu\big)^{\mathrm{T}}, \tag{17}$$

$$Y_S=\mathrm{Tr}\big(3Y_U^\dagger Y_U+3Y_D^\dagger Y_D+Y_E^\dagger Y_E+Y_\nu^\dagger Y_\nu\big), \tag{18}$$

where $t=\ln\mu$ and $\mu$ is the renormalisation point.

However, below the seesaw scale the right-handed neutrino are no more relevant and the Dirac neutrino terms in the renormalisation group equations should be removed, and also the Dirac neutrino Yukawa couplings themselves are not accessible anymore. That means that, from the seesaw scale down to the weak scale, the only leptonic Yukawa beta-functions should be changed as follows:

$$16\pi^2\frac{dY_E}{dt}=\frac{3}{2}\big(Y_E(Y_E)^\dagger\big)Y_E+\left\{Y_S-\left(\frac{9}{4}g_1^2+\frac{9}{4}g_2^2\right)\right\}Y_E. \tag{19}$$

Note that the quantity, $Y_S$, must be also changed below the seesaw scale:

$$Y_S=\mathrm{Tr}\big(3Y_U^\dagger Y_U+3Y_D^\dagger Y_D+Y_E^\dagger Y_E\big). \tag{20}$$

Really we stopped the running down according to formula (16) differently for the different matrix elements in the $Y_\nu$ matrix corresponding to the right-handed neutrino mass supposed most important for the matrix element in question.

Starting the running in an analogous way, we further should evolve the effective neutrino mass matrix considered as a five-dimensional non-renormalisable term [7] from the different right-handed neutrino masses to the weak scale[1] (180 GeV) depending on the terms:

$$16\pi^2\frac{dM_{\mathrm{eff}}}{dt}=\big(-3g_2^2+2\lambda+2Y_S\big)M_{\mathrm{eff}}-\frac{3}{2}\big(M_{\mathrm{eff}}\big(Y_EY_E^\dagger\big)+\big(Y_EY_E^\dagger\big)^{\mathrm{T}}M_{\mathrm{eff}}\big), \tag{21}$$

where $Y_S$ defined in Eq. (20) and in this energy range the Higgs self-coupling constant running equation is

$$16\pi^2\frac{d\lambda}{dt}=12\lambda^2-\left(\frac{9}{5}g_1^2+9g_2^2\right)\lambda+\frac{9}{4}\left(\frac{3}{25}g_1^4+\frac{2}{5}g_1^2g_2^2+g_2^4\right)+4Y_S\lambda-4H_S, \tag{22}$$

[1] We take the weak scale as 180 GeV, for simplicity. At this scale we calculated the pole mass of top quark, too, using $M_t=m_t(M)(1+\frac{4}{3}\frac{\alpha_s(M)}{\pi})$.

with

$$H_S = \mathrm{Tr}\Big\{3\big(Y_U^\dagger Y_U\big)^2 + 3\big(Y_D^\dagger Y_D\big)^2 + \big(Y_E^\dagger Y_E\big)^2\Big\}. \tag{23}$$

The mass of the Standard Model Higgs boson is given by $M_H^2 = \lambda\langle\phi_{\mathrm{WS}}\rangle^2$ and, for definiteness, we take $M_H = 115$ GeV at weak scale.

From 180 GeV down to 1 GeV—experimental scale[2] (1 GeV)—we have evaluated the beta-functions with only the gauge coupling constans. In order to run the renormalisation group equations, we use the following initial values:

$$U(1):\quad g_1(M_Z) = 0.462, \qquad g_1(M_{\mathrm{Planck}}) = 0.614, \tag{24}$$

$$SU(2):\quad g_2(M_Z) = 0.651, \qquad g_2(M_{\mathrm{Planck}}) = 0.504, \tag{25}$$

$$SU(3):\quad g_3(M_Z) = 1.22, \qquad g_3(M_{\mathrm{Planck}}) = 0.491. \tag{26}$$

Note that we have ignored the influence of the $B - L$ gauge coupling constants; however, this effect should not be significant, i.e., Planck scale to the seesaw scale ($\approx 10^{16}$ GeV) is only $10^3$ order of magnitude difference. Therefore, it should be good enough for our order magnitude calculations.

## 4. Neutrino oscillations

The Sudbury Neutrino Observatory (SNO) collaboration has reported [8] recently the measurement of the neutral current of the active $^8$B solar neutrino flux and related it to measurements of the day and night solar neutrino energy spectra and rates. Moreover, they presented improved determinations of the charged current and neutrino–electron scattering rate. It turns out that, global analyses [9] of solar neutrino data—combination of the SNO results with previous measurements from other experiments [10–16]—have confirmed that the Large Mixing Angle MSW (LMA-MSW) solution [17] gives the best fit to the data and that the LOW solution is allowed now only at $2.5\,\sigma$ and Small Mixing Angle MSW (SMA-MSW) solution at $3.7\,\sigma$, respectively. Not only that we "know" the solution of the solar neutrino puzzle but also that these recent results tell us that the bi-maximal solution ($\tan^2\theta_\odot = \tan^2\theta_{\mathrm{atm}} = 1.0$) is strongly disfavoured at the $3.3\,\sigma$ C.L. for the LMA-MSW solution [18].

The best fit values of the mass squared difference and mixing angle parameters in the two flavour LMA-MSW solution somehow depends on the analysis method, but we take the following point as fit values (see Table 2):

$$\Delta m_\odot^2 = 5.0 \times 10^{-5}\ \mathrm{eV}^2 \quad \text{and} \quad \tan^2\theta_\odot = 0.34. \tag{27}$$

The atmospheric neutrino parameters are the following according to the Super-Kamiokande results [19]:

$$\Delta m_{\mathrm{atm}}^2 = 2.5 \times 10^{-3}\ \mathrm{eV}^2 \quad \text{and} \quad \tan^2\theta_{\mathrm{atm}} = 1.0. \tag{28}$$

## 5. Results

The calculation using random numbers and performed numerically was used to fit the masses and mixing angles to the phenomenological estimates by minimising what we call "goodness of fit",

$$\mathrm{g.o.f.} \equiv \sum_i \left[\ln\left(\frac{m_{i,\mathrm{pred}}}{m_{i,\mathrm{exp}}}\right)\right]^2, \tag{29}$$

[2] We set the experimentally observable scale as 1 GeV, thus the charged fermion masses and mixing angles are compared to "measurements" at this scale, except the top pole mass.

Table 2
Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass. Note that we use the square roots of the neutrino data in this table, as the fitted neutrino mass and mixing parameters, in our goodness of fit (g.o.f.) definition, Eq. (29)

| | Fitted | Experimental |
|---|---|---|
| $m_u$ | 4.4 MeV | 4 MeV |
| $m_d$ | 4.3 MeV | 9 MeV |
| $m_e$ | 1.6 MeV | 0.5 MeV |
| $m_c$ | 0.64 GeV | 1.4 GeV |
| $m_s$ | 295 MeV | 200 MeV |
| $m_\mu$ | 111 MeV | 105 MeV |
| $M_t$ | 202 GeV | 180 GeV |
| $m_b$ | 5.7 GeV | 6.3 GeV |
| $m_\tau$ | 1.46 GeV | 1.78 GeV |
| $V_{us}$ | 0.11 | 0.22 |
| $V_{cb}$ | 0.026 | 0.041 |
| $V_{ub}$ | 0.0027 | 0.0035 |
| $\Delta m^2_\odot$ | $9.0 \times 10^{-5}\ \mathrm{eV}^2$ | $5.0 \times 10^{-5}\ \mathrm{eV}^2$ |
| $\Delta m^2_{\mathrm{atm}}$ | $1.7 \times 10^{-3}\ \mathrm{eV}^2$ | $2.5 \times 10^{-3}\ \mathrm{eV}^2$ |
| $\tan^2\theta_\odot$ | 0.26 | 0.34 |
| $\tan^2\theta_{\mathrm{atm}}$ | 0.65 | 1.0 |
| $\tan^2\theta_{\mathrm{chooz}}$ | $2.9 \times 10^{-2}$ | $\lesssim 2.6 \times 10^{-2}$ |
| g.o.f. | 3.63 | – |

a kind of $\chi^2$ for the case that we have only order of magnitude accuracy. The result of the fitting is presented in Table 2. The results presented there were obtained from the following values of the set of Higgs VEVs—where the Higgs field VEVs for the fields $W$, $T$, $\omega$ and $\rho$ causing the breaking to the diagonal subgroup $SMG \times U(1)_{B-L}$ are quoted with the VEV in "fundamental units", while they are for $\phi_{\mathrm{SS}}$ (and the not fitted $\phi_{\mathrm{WS}}$) given in GeV units:

$$\begin{aligned} &\langle W\rangle = 0.157, \qquad \langle T\rangle = 0.0766, \qquad \langle\omega\rangle = 0.244, \qquad \langle\rho\rangle = 0.265,\\ &\langle\phi_{\mathrm{SS}}\rangle = 5.25\times 10^{15}\ \mathrm{GeV}, \qquad \langle\phi_{\mathrm{WS}}\rangle = 246\ \mathrm{GeV}. \end{aligned} \tag{30}$$

The results of the best fit, with the VEVs in Eq. (30), are shown in Table 2 and the fit has g.o.f. $= 3.63$. To see a typical error, say average error, compared to the experimental values we should divide this value with the number of predictions ($17 - 5 = 12$) and then take the square root of it: $\sqrt{3.63/12} = 0.55$. This means that the 12 degrees of freedom have each of them a logarithmic deviation of about 55%, i.e., we have fitted all quantities with a typical error of a factor $\exp(\sqrt{3.63/12}) \simeq 1.73$ up or down. This agrees with theoretically predicted deviations [20]. However, our worst fitting value is now the electron mass which we predict/fit a factor 3 too heavy.

We should here emphasise this point: even though we reduced the number of free parameters by one—now only five VEVs—we are able to fit all fermion masses and mixing angles with an average factor 1.73 of deviation as in previous work.

Experimental results on the values of neutrino mixing angles are usually presented in terms of the function $\sin^2 2\theta$ rather than $\tan^2\theta$ (which, contrary to $\sin^2 2\theta$, does not have a maximum at $\theta = \pi/4$ and thus still varies in this region). Transforming from $\tan^2\theta$ variables to $\sin^2 2\theta$ variables, our predictions for the neutrino mixing angles become:

$$\sin^2 2\theta_\odot = 0.66, \tag{31}$$

$$\sin^2 2\theta_{\mathrm{atm}} = 0.96, \tag{32}$$

$$\sin^2 2\theta_{\text{chooz}} = 0.11. \tag{33}$$

We also give here our predicted hierarchical left-handed neutrino masses ($m_i$) and the right-handed neutrino masses ($M_i$) with mass eigenstate indices ($i = 1, 2, 3$):

$$m_1 = 1.4 \times 10^{-3}\ \text{eV}, \qquad M_1 = 1.0 \times 10^6\ \text{GeV}, \tag{34}$$

$$m_2 = 9.6 \times 10^{-3}\ \text{eV}, \qquad M_2 = 6.1 \times 10^9\ \text{GeV}, \tag{35}$$

$$m_3 y = 4.2 \times 10^{-2}\ \text{eV}, \qquad M_3 = 7.8 \times 10^9\ \text{GeV}. \tag{36}$$

Note that our fit of the CHOOZ angel (Eq. (33)) is lying on the borderline of the experimental results which analysed by two flavour method. However, our fit satisfy even $2\sigma$ C.L. limit ($\tan^2\theta_{\text{chooz}} = 3.3 \times 10^{-2}$) based on three flavour analysis [21].

For the $CP$-violation parameter, the Jarlskog triangle area [22], $J_{CP}$, we expect our prediction to have a larger uncertainty than for the quantities, which like the masses are essentially mass matrix elements, because it corresponds to a ratio or product of six such quantities and thus our prediction,

$$J_{CP,\text{pred}} = 3.6 \times 10^{-6} \tag{37}$$

compared to the "experimental" value

$$J_{CP,\text{exp}} = (2\text{–}3.5) \times 10^{-5}, \tag{38}$$

should be considered a success of only $1.5\,\sigma$ deviation.

For the for the neutrinoless double beta-decay relevant specially weighted neutrino mass average

$$\left|\langle m\rangle\right| \equiv \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right|, \tag{39}$$

we obtain the prediction from our fit

$$\left|\langle m\rangle\right| = 3.5 \times 10^{-3}\ \text{eV}. \tag{40}$$

Our proton decay predictions may vary a bit with the philosophy, however, we in any case predict so long lived protons that it is pretty hopeless to see any proton decay. At least we get a proton life time of the order:

$$\tau_{(p\to\pi^0 e^+)} \approx 10^{43}\ \text{years}. \tag{41}$$

The brenching ratio of $\mu \to e + \gamma$ is performed by [23]:

$$\text{Br}(\mu \to e + \gamma) = \frac{3\alpha_{em}}{128\pi}\left(\frac{\Delta m_\odot^2}{M_W^2}\right)^2 \sin^2 2\theta_\odot, \tag{42}$$

where $M_W$ is the mass of the $W^\pm$ bosons. We insert our predictions from Table 2 and Eq. (31) in this formula, then we get

$$\text{Br}(\mu \to e + \gamma) \sim 10^{-56}. \tag{43}$$

## 6. Simple relations

In the approximation of only taking dominant terms and imagining all the quantities extrapolated to the Planck scale—i.e., ignoring renormalisation group running and sub-dominant terms—we can get the rather simple relations:

(1) Family degeneracy for charged quarks and leptons:

$$m_b \approx m_\tau, \qquad m_s \approx m_\mu, \qquad m_u \approx m_d \approx m_e. \tag{44}$$

(However, we avoid the $m_c$ and $m_t$ being degenerate with their families.)

(2) Factorisation of mixing angles for quarks:

$$V_{cb} V_{us} \approx V_{ub}. \tag{45}$$

(3) Neutrino relations:

$$\theta_\odot \theta_{\text{atm}} \approx \theta_{\text{chooz}}, \qquad \left(\frac{\Delta m^2_\odot}{\Delta m^2_{\text{atm}}}\right)^{1/2} \approx \frac{1}{2}\theta^2_{\text{atm}}, \qquad \left(\frac{m_1^2}{\Delta m^2_\odot}\right)^{1/2} \approx \frac{1}{2}\theta^2_\odot. \tag{46}$$

Here $m_1$ is the mass of the lightest left-handed neutrino.

(4) Relations for charged quarks and leptons:

$$m_b^3 \approx m_t m_s m_c, \qquad m_t \approx \text{“Weak scale”}, \qquad V_{cb} \approx \frac{m_s^2}{m_c m_b}. \tag{47}$$

(5) Relations between neutrinos and charged quarks and leptons:

$$\theta_{\text{atm}} \approx \sqrt{2}\,\frac{W^3}{T^2} \approx \sqrt{2}\,\frac{m_c^2}{V_{cb} m_b^2}, \qquad \theta_\odot \approx \frac{1}{\sqrt{2}}\frac{\omega^3}{\rho^3} \approx \frac{1}{\sqrt{2}}\frac{m_d^2}{m_s^2 V_{us}^3}. \tag{48}$$

Note that according to Eq. (46) the neutrino data fit by just by two parameters, $\theta_\odot$ and $\theta_{\text{atm}}$, and one overall scale, $\langle \phi_{\text{SS}} \rangle$. Moreover, according to Eq. (48) the mixing angles are given in term of the quark quantities.

## 7. Theoretical lesson for the mass matrix

We should emphasise that we have obtained the rather good fit for all the neutrino quantities—$\theta_\odot$, $\theta_{\text{atm}}$, $\theta_{\text{chooz}}$, $\Delta m^2_\odot$ and $\Delta m^2_{\text{atm}}$—and the charged masses and mixing angles using only 5 free parameters, namely, five VEVs of Higgs fields. Our effective left-handed neutrino mass matrix, Eq. (9), is of the form

$$\begin{pmatrix} \phi_1^2 & \phi_1\phi_2 & \phi_1\phi_3 \\ \phi_1\phi_2 & \phi_2^2 & \phi_2\phi_3 \\ \phi_1\phi_3 & \phi_2\phi_3 & \phi_3^2 \end{pmatrix}, \tag{49}$$

with different order unity factors, which we call factorised mass matrix. It is as a consequence of this type of form we get a relation (46). In fact, the ratio of mass squared difference, $\Delta m^2_\odot/\Delta m^2_{\text{atm}}$, is characterised by the fourth power of the atmospheric mixing angle at Planck scale. Due to the large atmospheric neutrino mixing angles, it is clear that this fact—mass squared difference being order unity–is not true experimentally if the effects of renormalisation equations are not taken into account on the Dirac- and Majorana-sectors from Planck scale to seesaw scale, and also non-renomalisable five dimensional evolution equation from the seesaw scale to the weak scale. However, with the running included the “factorised” mass matrix is a good candidate for fitting the LMA-MSW solution due to only moderate hierarchy structure of this solution.

## 8. Baryogenesis not so successful

Our prediction of baryogenesis in the Fukugita–Yanagida scheme [24] is unfortunately not so successful: the asymmetry from the decay of the lightest of the seesaw neutrinos is way too small to produce the $B - L$ to give the baryon number asymmetry needed for the light element production in big bang at minute scale. However, the asymmetry from the decay of the two heavy and approximately degenerate seesaw neutrinos is much larger and so our hope would be that they could produce the asymmetry needed; however, it turns out that the wash-out by the processes involving the lightest right-handed neutrino becomes too large and we get too little baryon number at the end because of the rather high value of the $\tilde{m}_1$ [25].

It must though be said that the $\tilde{m}$'s are products or ratios of three mass matrix elements so that the uncertainty is expected to be $\exp(\pm\sqrt{3}\cdot 64\%) = \exp(\pm 111\%)$, meaning a factor 3 up or down. Our problem with the rather high value of $\tilde{m}$'s is partly due to the renormalisation group corrections without which lower $\tilde{m}$'s by about a factor 3 would result. We also hope that taking into account that different flavours will be washed out with different rates [26] will lower the wash-out rate at the end. Somewhat optimistically this could mean that we could use an effective $\tilde{m}_1$ a factor 3 smaller.

Taking into account also the uncertainty in other quantities we might hope for that the baryon number predicted only disagrees by about three standard deviations.

The many questions of discussion involved in calculating the baryon number we should say that we got pretty tight to a viable result, however, only based on the idea that it is *the heavy seesaw neutrinos* that *produced the baryon asymmetry*.

## 9. Conclusion

We have developed our previous model by deleting from the list of assumed Higgs fields with vacuum expectation values breaking the assumed gauge group $SMG \times U(1)^5$ or $\underset{i=1,2,3}{\times}(SMG_i \times U(1)_{B-L,i})$ a Higgs field, $\chi$, which were already suspected to be unwanted by having the same quantum numbers as the combination $W^3(T^\dagger)^9$. We found that we can indeed fit to order of magnitude accuracy the nine charged quark and lepton masses, the two neutrino mass square differences, the five measured mixing angles in addition to matching the $CP$-violation with *only five Higgs field vacuum expectation values being adjusted* to order of magnitude accuracy. In addition this fit manages to pass the test of several experimental bounds: no neutrinoless double beta-decay, no proton decay, and no $\mu \to e + \gamma$ within present experimental accuracy, and most interestingly we predict a sufficiently small CHOOZ angle, $U_{3e}$; however, there is the prediction that we are actually close to the limit so that our model might be realistically falsified by decreasing the CHOOZ angle limit by an order of magnitude.

It is remarkable that we fit such a large number, 17, of genuinely measured quantities with only our 5 parameters order of magnitudewise. Our precise gauge group proposals are not uniquely called for in as far as we could at least include more or less embellishment with non-Abelian groups for the different families or replace some of the Abelian groups by non-Abelian, but the idea that different families have different "chiral" quantum numbers may be hard to avoid without throwing our good agreement out as being accidental.

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