4th International Conference on New Horizons in Education

A hierarchical-precedences model for the analysis of teaching process

Josef Botlík<sup>a</sup>, Milena Botlíková<sup>b</sup>, Zuzana Palová<sup>c</sup>

<sup>a</sup>...... Silesian University in Opava, School of Business Administration in Karvina, Univerzitní nám. 1934/3, 73340 Karviná, Czech Republic
<sup>b</sup>...... Silesian University in Opava, School of Business Administration in Karvina, Univerzitní nám. 1934/3, 73340 Karviná, Czech Republic
<sup>c</sup>...... Silesian University in Opava, School of Business Administration in Karvina, Univerzitní nám. 1934/3, 73340 Karviná, Czech Republic

Abstract

Mapping the learning process and its analysis are relatively complicated activities. This paper demonstrates the use of precedence for non-standard analysis. Precedential model and precedence analysis are based on a comparison of elements in the system and the search precedence in the linkages between these elements. Links are defined for the elements of the system and precedence is set for attributes of elements. Multiple precedence and consensus in precedence are examined and modeled using the identified precedence. This way we can find the critical points and critical activities in the educational process.

© 2013 The Authors. Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
Selection and peer-review under responsibility of The Association of Science, Education and Technology-TASET, Sakarya Universitesi, Turkey.

Keywords: precedence, incidence, model, learning process, schedule of teaching

1. INTRODUCTION

The learning process is essentially composed of a set of elements and the links between them. A good definition of these elements and links can be for modeling and analyzing to use of classical methods of systems theory, systems analysis, operational analysis and graph theory. Standard methods and optimization tasks provide a similar set of results, which is nowadays usually applied. To increase efficiency analysis and modeling is a need to find new, non-standard methods and procedures that extend the methods used previously. These new methods must fulfill several conditions:

- Universal application
- Simple applicability to existing structures

* Corresponding author. +420 596 398 242; fax: +420 596 312 069.
E-mail address: botlik@opf.slu.cz
Existing methodologies and mathematical apparatus
At the Silesian University in Opava, School of Business Administration in Karvina there are existed projects that are investigating the use of precedence for new types of modeling and analysis. Precedence analysis is based on the basic definition of the general system. The system is defined as a set of elements and links between them. Each element has assigned set of attributes. Precedence analysis compares the values of selected attributes of the elements that have defined common link. Comparison can be absolute values of the attributes or purpose defined changeover value: relative increases over, the absolute increases over time, relative increases in space, absolute increases in space, deviations from the average values, deviations from the maximum values, deviations from the minimum values, etc. By comparing these values we get between soever defined pairs of elements of the direction of flow values for a given attribute. On the basis of these flows we set precedence and a sequence of elements according to the values of the attributes in the system - and succession precedence. These precedence and succession we process by existing mathematical apparatus described closer Borje (1981). These tools further developed as Diestel (2005), who described the application of chart theory and Bakshi (2008), which dealt with practical applications. In the Czech Republic followed the example Borje for example Unčovský (1991), who generalized the incidence analysis in graph theory. These theoretical foundations follows research carried out in the grant SGS 5/2013 at the Silesian University in Opava, where is examined from mainly analyzes Botlík, Botlíková (see reference for 2010-2013), currently the analyze involved Andrýsek (2013), Palová (2013), Václavíková (2012) and others.

2. MODEL FOR ANALYZES

The basic model is multi-level; the structure is evident from Fig. 1†. The first level consists of a set of incidence matrices. Incidence matrices are defined for different types of links. The basic set consists of matrices for transitions between years, matrices for links between sections, matrices for links between objects and other.

Incidence matrix is a square matrix that records the existence of the link. Matrix has a dimension of \((m+i)\times(m+i)\) where \(m\) is the number of elements in the system and \(i\) is the number of the defined surrounding. For each element of the surrounding is defined by exactly one row and one column of the matrix. Matrix elements have the value "1" if the row element has linkage with columnar. It is a symmetric matrix according to the main diagonal. With the implementation of the matrices to model we change the structure of the model. Incidence matrix is easily modifiable and universal. The second level consists of the matrix of values of attributes. Matrix has a dimension of \((m+i)\times n\), where \(m\) is the number of elements of the system and \(i\) is the number of the surrounding of analyzed attributes. In the absence of the attribute values in the relevant surroundings attribute value is counted using a set of keys (min, max, average, etc.) from the attribute values of elements of the system. The values of one attribute are therefore described by the row vectors. Matrix of values of attribute is again easily modifiable and universal. The third level consists of a set of precedents matrix. Precedence matrix is generated for the attribute from vector of attribute values and the incidence matrix. At this level are compared values of the selected attribute between each pair of elements. Subsequently, based on the value of the element of incidence matrix it is determined whether there is between elements the appropriate type of linkage. In case of a linkage is determined by binding orientation direction from an element with a lower value to an element with a higher value of the attribute. In some cases, it is at this level performed correction of values. For example, due to ensuring of acyclic of chart are values of attributes corrected for insignificant value item which will benefit the selected elements according to some qualitative criteria (e.g. by averaging the scores of student publications by the teacher, etc.). Over the set of precedence matrices are performed basic system operations. Composition

† A_1, A_2, ..., A_k: incidence matrix, P_1, P_2, ..., P_k: precedence matrix, k_1, k_2, k_3: selective vectors, P_1^p, P_2^p, ..., P_k^p: matrix P_rasted to the b-th, a_1, a_2, ..., a_n: system elements, o_1, o_2, ..., o_c: around the system, v_1, v_2, ... element attributes
operations of the selection vector with matrix and unification of selected vectors is defined for the model. Based on these operations, we can determine a set of precedents for the selected element, set precedents on the set of selected elements, existence of precedence multiple lengths between two elements, and frequency of precedence multiple length between the two elements. Turn the orientation of the links, which we will carry through the precedence matrix transposition, we get succession matrix (successor’s matrix). The set of operations, then we can extend the searching set succession to set precedents and vice versa.

Figure 1 hierarchical model
Creating system structures is evident from Fig. 2, precedence formation is Fig. 3

![Diagram of system structures](image1)

**Figure 2 application incidence to the chart**

![Diagram of precedence formation](image2)

**Figure 3 creating precedence using the values of attribute**

### 3. ANALYSIS OF THE LEARNING PROCESS, EXAMPLES

#### 3.1. Preceding set of elements

In complicated structures is usually difficult to find the selected sets. In a learning process, we often encounter problems when we need to find a set of time-dependent objects. This is usually about the links between classes, following in the subjects in the following forms of study (Bachelor, Master, and Doctorate), following in the form of a ensure subject (the links between departments), follow the test process (credit, exam, comprehensive exam), etc. These links can be defined almost unlimited; the only criterion is the existence of quantifiable attribute. The situation shown in Fig. 4, we can imagine such as finding the set of objects which a student must complete in the selected semester (designate B) so that he can graduate immediately following semester (designate A). Apparently simple problem is in complex structures difficult to implement. In precedent model is a simple calculation of (1). The set of objects immediately follow-up of the semester we describe using a selective vector $k$

$$P_{k=k'}$$

(1)
Selective vector has a dimension of the number of elements of the system. Selection we denote so that a corresponding element of vector will have a value of "1" when it is in the set of elements to choose. Operation (1), Borje (1981) indicates that matrix multiplication with the vector\(^1\); operation consists of a selection of column vectors precedence matrix \(P\) that have the same indexes (corresponding to the same elements of the system) as elements of the selection vector and the subsequent unification of these vectors. The result is a vector of \(k'\), which contains the value "1" in the elements that precede the set described by selective vector.

![Figure 4 Previous precedences](image)

3.2. Independent operations

Selective vector can be used for finding a set of independent operations. The basis of the algorithm is that the set of suceedence to set of prececents of some set is another set (2). On Fig. 5 and 6 is an example where the schedule items dropped out educator. We need to find out which subjects may be taught independently of this specific educator and by which subjects we must ensure substitute teaching. The set of subjects taught a teacher is marked \(A\), the set of courses that students have finished and that follow the teacher’s subjects is marked \(B\), set of independent activities is marked \(C\) and the set, which is independent but in the future will be dependent is marked as \(D\). The set \(B\) we obtain by multiplying the selection vector \(k\) with precedence matrix \(P\). The set \(C\) we obtain as the complement of the set \(A\) and set of sucession of set \(B\). The set \(D\) is obtained as the intersection of set \(C\) and set of sucesison of set \(A\).

\[
P^k \neq P^{k'}
\]

(2)

![Figure 5 independent operations](image)

\(^1\) Described in the references, Botlik, Botliková (2010 – 2013)
3.3. A set of multiple links, multiple precedence

A common problem is tracing existing dependencies. Dependencies can be direct, unmediated or dependence through other objects. In the case of finding of direct dependency is a finding of all precedence in the system. In the case of mediated linkages it is about finding multiple-precedence.

Fig. 7 is an example where we have described links between subjects in each semester (link shows that subjects consecutively in the next semester). Using multiple-precedence can identify groups of subjects, among which are different time intervals (2 semesters, 3 semesters, 4 semesters, etc.).
The solution is based on the principle of searching elements among which are multiple-precedence. As multiple-precedence is understood precedence to another existing precedence. According to Borje multiple-precedence can be calculated as the power of the precedence matrix. Appropriate level of power indicates how long (what multiples) precedence in the matrix is recorded. The highest power of nonzero matrix gives the longest precedence in the system. Borje defined determination of the existence of precedence between two elements as binary multiplication of binary matrix; in practice it is possible to use a classic multiplication of binary matrix to determine the frequency of precedence of appropriate length (see references Botlík, Botliková).

3.4. Parallel operations

In some cases, we need to find in the learning process deuces in continuity for example between subjects. Continuity can again be direct or mediate. Fig. 7 shows the subjects that follow each other (for passing the subject it is necessary to have graduated another subject) and oriented edges are set directions of bonds (which item is necessary pass earlier). In some cases, it may lead to concurrency continuity, shown in Figure it is shown that the subject F can be taken subsequently after passing subjects B,C and D. Subject D can be taken subsequently after passing the subject C and D subsequently after passing the subject A. At the same time the by subject B it is stated that to complete the course it is required to pass the subject A. Finding such concurrency can be applied to task where we find such a power precedence matrices that have the same index of the non-zero elements (the value "1" occurs in the same rows and columns of the matrix with different power).

The task is easy to solve, for example with the counting the matrices, when in conformity of precedence different lengths will be in element of the resulting matrix values larger than 1.

![Figure 8 coexistence of the first, second and third precedence between nodes A and F](image)

4. Conclusion

The paper demonstrates universal precedence model suitable for the analysis of the learning process. Forte of the model is universality when we define independently a set of elements, a set of attributes and a set of links. This independence allows creating many of independent structures and carrying out time-independent and space-independent analysis. Changing the structure is easily reached selecting the relevant incidence matrix, change the parameters of the system is realized by changing the vector of attribute values and calculating the appropriate precedence.

The model is applicable wherever where we work with disparate values (number of students, study programs, continuity on subjects etc.), furthermore, where we work simultaneously with the state variables (number of students, number on subjects etc.) and with flow values (increase the number of students, duration of study, etc.). The model allows analyzing even flow operation between the surroundings and the system, allows defining more
types surrounding (e.g. unsuccessful students after first year unsuccessful students after the second year, students who transfer to another school, students who come from another school, etc.).

The paper demonstrated on concrete examples a fraction of the model options that exceed the scope of this paper.

References


