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Vehicle specific behaviour in macroscopic traffic modelling through stochastic advection invariant

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Abstract

In this contribution a new model to include stochastic vehicle specific behaviour and interaction in traffic flow modelling is presented. The First Order Model with Stochastic Advection (FOMSA) is presented as a first order macroscopic kinematic wave model in a platoon-based Lagrangian coordinate system. The use of Lagrangian coordinates allows characteristics of specific vehicles or vehicle-groups to propagate along with the traffic flow using a vehicle specific invariant. The invariant reflects how vehicle or platoon specific characteristics propagate with the vehicles and influence the local behaviour of a vehicle or platoon on a macroscopic level and in interaction with other surrounding vehicles. It represents a local vehicle specific adjustment to the critical density and makes use of two parameters: a stochastic boundary parameter and a transition parameter. These parameters indicate the extent of differences between vehicles or platoons. A case study is also presented in which a demonstration of the model is given and the face validity and sensitivity of the parameters are shown. Previously, similar approaches have made use of second order model descriptions. The formulation of this model as a first order model makes use of the advantages of first order models and also applies the improved accuracy of Lagrangian coordinates over the Eulerian coordinate system in time-stepping.

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Keywords: *stochastic; macroscopic modelling; advection invariant; traffic behaviour*

1. Introduction

Traffic is a highly dynamic and complex system, which encompasses human behaviour through the act of driving. Human driving behaviour is complex in itself, but exists of a general core behaviour related to the general rules of driving, i.e. traversing a lane in a certain direction at a certain speed without collision, and of intrinsic behavioural aspects that can be driver specific (Fuller 2005; Toledo 2007). The core behaviour is seen as something

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that is easily understood, observable and reproducible in models. However individual driver behaviour is somewhat harder to capture and reproduce. Efforts to capture and understand stochastic driver behaviour have been successful and have described many aspects of driving behaviour. Driver behaviour has a direct effect on vehicle behaviour, which can be seen as the consequence thereof. In this contribution the focus is on the latter, which is of course influenced by the former. With an increase in microscopic modelling, and especially agent-based models, much stochastic behaviour of individual vehicles and interaction between vehicles has been included in modelling. This easily allows stochastic behaviour in longitudinal and lateral movements to be included by simply adding terms describing this to a vehicles behavioural algorithm (Arasan and Koshy 2005; Mallikarjuna and Rao 2009; Treiber, Kesting, and Helbing 2006). However, in macroscopic traffic modelling each individual vehicle is often considered to adhere to identical or similar behaviour. This is especially the case in deterministic modelling. Although this has a number of advantages and often seems to produce acceptable results, interaction between vehicles is generally ignored. However observations of traffic flows show that considering differences between vehicles and their stochastic behaviour is relevant, especially for constrained or critical traffic states (Kerner 2013; Persaud, Yagar, and Brownlee 1998; Polus and Pollatschek 2002). This is also demonstrated later in this contribution.

Capturing such fluctuations in behaviour between vehicles in macroscopic traffic flow however demands a certain levels of disaggregation of the macroscopic flow, which is not traditionally inherent to such models. In this contribution we aim to overcome this difficulty to allow stochastic behaviour from vehicles and between vehicles to be modelled in a first order macroscopic setting. This is achieved through the use of a Kinematic Wave Model, which considers the movement of vehicles according to first order traffic theory in a platoon-based Lagrangian coordinate system (Leclercq, Laval, and Chevallier 2007). Consideration of the stochastic behaviour of vehicles is included through the application of a vehicle specific invariant term that describes local stochastic characteristics of vehicles and drivers within and between individual vehicles or platoons. These characteristics implicitly describe aspects of driver behaviour such as desired time headway . The use of Lagrangian coordinates allows the vehicle specific invariant term to propagate along with the vehicles for which it is valid and thus avoids numerical diffusion of driver behaviour variables (Leclercq, Laval, and Chevallier 2007; van Wageningen-Kessels et al. 2009). This approach is unique to first order macroscopic models, and is generally found in the more elaborate second order models. Details on this process are described later in the paper.

This contribution offers a unique approach based on proven theories to include vehicle specific behaviour in first order macroscopic modelling, filling a void that has been previously solved for microscopic models, but lacking in macroscopic models. The modelling principles applied in the described approach are first explained in section 2 to give the reader the required knowledge to understand the approach. The developed approach is then described in section 3, including the assumptions made and the limitations. In section 4 an experimental case is given to demonstrate the approach, in which a further comparison is made with a non-stochastic reference case to demonstrate the necessity of considering stochastic driving behaviour in macroscopic modelling. Finally the conclusions are given in section 5.

2. Modelling principles

2.1 Kinematic Wave Model

The kinematic wave model (KWM) is one of the most fundamental types of macroscopic traffic flow models. It captures the aggregated propagation of traffic flow described as the propagation of traffic waves and the adhering traffic characteristics. The concept of modelling kinematic waves of traffic was first introduced by Lighthill and Whitham (1955) and by Richards (1956) and is therefore often referred to as the LWR model. Since the introduction of the KWM various extensions have been proposed, however the underlying theory as originally described in the LWR model remains intact. Construction of the kinematic waves is achieved through use of the fundamental relationship of traffic flow which is generally described by the relationship between the density ρ and the flow q of traffic. The model further relies on the conservation equation and initial boundary conditions. The conservation equation and the fundamental relation are denoted by:

$$\partial_t \rho + \partial_x q = 0 \tag{1}$$

$$q = Q(\rho) \tag{2}$$

In which ρ is the traffic density in time t and q is the flow in space x. $Q(\rho)$ denotes the form of the fundamental relation. As the KWM is a macroscopic model, it makes use of aggregation of individual vehicles and describes an aggregated flow. van Wageningen-Kessels et al. (2014) points out that empirical density–flow plots usually show wide scatter, which is not captured by an aggregated flow. Macroscopic models presume some sort of equilibrium, which results in crisp steady state conditions in flow regimes. However van Wageningen-Kessels et al. (2014) go on to point out that the scatter is a consequence of not all data representing such a steady state condition.

2.2 Lagrangian Coordinates

In traditional macroscopic modelling Eulerian coordinates are usually applied which state that for a specific time and location, a flow, such as traffic, will pass with certain characteristics (Helbing and Treiber 1999; van Wageningen-Kessels et al. 2009). In this case it is the flow which moves in relation to the coordinate system. Lagrangian coordinates in contrast are not fixed in space, but are given the freedom to transform with the resulting flow. This can be described such that particles in the flow are explicitly considered in individual consecutive cells. Therefore the coordinates follow the flow rather than the flow following the coordinates. A graphical demonstration is shown in Figure 1. The Eulerian formulation of the KWM was given in Eq. (1)-(2). When describing the KWM in Lagrangian coordinates the same equations are formulated slightly differently. The conservation equation is given as:

$$\partial_t s + \partial_n v = 0 \tag{3}$$

Here s denotes the mean space headway of vehicles in a single cell. v denotes the mean speed of vehicles, while n is the vehicles number, which decreases in the driving direction. The fundamental relation in Lagrangian coordinates makes use of the speed v in relation to the density ρ, which is derived from the mean headway spacing s: $s = 1/\rho$. The fundamental relation is denoted as:

$$v = V(s) \tag{4}$$

The use of Lagrangian coordinates has been proven to lead to more accurate results as a result of a reduction in numerical diffusion that occurs in the transfer of flows between cells in Eulerian coordinates, but is almost non-existent in Lagrangian coordinates (Leclercq, Laval, and Chevallier 2007; van Wageningen-Kessels et al. 2010; van Wageningen-Kessels et al. 2009).

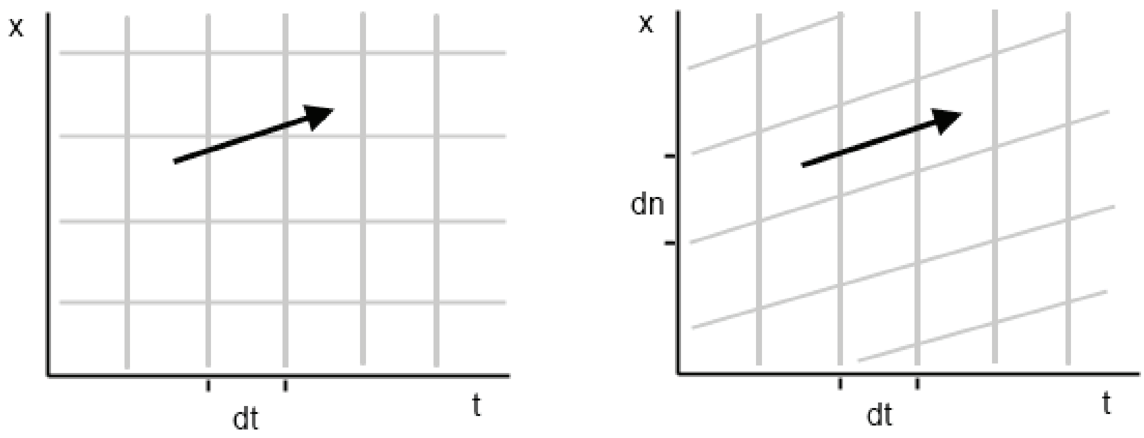

Fig. 1 Comparison of Eulerian (left) and Lagrangian (right) coordinates. The arrow represents the direction of traffic flow.

Lagrangian coordinates were introduced to traffic flow from the domain of hydrodynamics (Makigami, Newell, and Rothery 1971; Moskowitz 1965). Jin, Han, and Ran (2014) state that Lagrangian coordinates can be incorporated into continuum traffic flow modelling by either establishing moving boundary conditions for Euler formulations (Claudel and Bayen 2010; Herrera and Bayen 2010) or by application of hydrodynamic flow (Leclercq, Laval, and Chevallier 2007). The latter has been the more forthcoming in relation to advancement and application and was shown by Leclercq, Laval, and Chevallier (2007) to be able to be derived by using a space function based on variational theory (Daganzo 2005). More recently Laval and Leclercq (2013) further applied the theory of Hamilton-Jacobi to KWM in which the theory is applied to three two-dimensional coordinate systems, which included Eulerian and Lagrangian systems and is continuing to be extended by a number of researchers (Jin, Han, and Ran 2014). This however goes beyond the scope of this contribution in which the Lagrangian description as given in Eq. (3)-(4) is applied and which was also previously described (van Wageningen-Kessels et al. 2010; van Wageningen-Kessels et al. 2009).

2.3 Advection

The difficulty following from problems in the aggregated representation of flows in macroscopic models has previously been widely acknowledged. These difficulties have not only been identified in the lack of stochastic behaviour and it's consequence, but even more in the detailed introduction of vehicle behaviour and driver attributes, such as anticipation of drivers or the consideration of vehicle diversity. The main effort to describe such driver behaviour in a macroscopic model has been performed in second order macroscopic models. Application of between-vehicle stochastics has generally not been applied in first order macroscopic models. While first order models describe the conservation of vehicles according to Eq. (1), second order models also consist a second differential equation that describes the velocity dynamics. There are different formulations present, Aw and Rascle (2000) formulate it as:

$$\partial_t(v + p(\rho)) + v\partial_x(v + p(\rho)) = 0 \qquad (5)$$

in which $p(\rho)$ is a pressure term. Originally second order models were criticised for resulting in some inacceptable behaviour, such as vehicles being able to move backwards (Daganzo 1995). However further developments solved this. Aw and Rascle (2000) proposed adjustments to the original definition by replacing the space derivative with a convective derivative. Zhang (2002) described this similarly and explicitly state that traffic flow moves with the velocity along the trajectory and therefore it becomes a Lagrangian quantity.

Lebacque, Mammar, and Salem (2007) applied the same rationale to generalise the ARZ models (Aw and Rascle 2000; Zhang 2002). The ARZ models apply an invariant term to represent the relative speed of vehicles which is connected to these vehicles. Lebacque, Mammar, and Salem (2007) define this term as a general invariant that can also be related to global flow properties and therefore represent other characteristics of microscopic flow. The model is described as a generic second order model (GSOM) after the flexibility one has to define an invariant that can take on many different purposes. The conservation of vehicles is as Eq.(5), while the conservation of the invariant term and description of the fundamental relation with invariant are given by:

$$\partial_t(\rho I) + \partial_x(\rho v I) = \rho\varphi(I) \qquad \text{Dynamics of a driver attribute} \qquad (6)$$

$$v = V(\rho, I) \qquad \text{Fundamental relation} \qquad (7)$$

In which x is the position, I is the invariant term, and the V is the fundamental relation.

This approach has been applied in a number of consequential publications (Costeseque and Lebacque 2014; Costeseque and Lebacque 2015; Lebacque and Khoshyaran 2013). One such application describes the invariant term as a stochastic driver attribute describing the random driver interactions of a driver with other drivers (Lebacque and Khoshyaran 2013). Their Stochastic Generic 2nd Order Model describes the stochastic behaviour as a Brownian process and white noise process and if further defined in Lagrangian coordinates. While the GSOM also allows a first order description to be formulated (Lebacque, Mammar, and Salem 2007; Lebacque and Khoshyaran 2013), applications of the GSOM are generally not found in first order formulations. First order models on the other hand have advantages due to their relative computational efficiency in practice.

3. First order model with stochastic advection

3.1 Model formulation

The first order model with stochastic advection (FOMSA) is a first order macroscopic model based on the conservation of vehicles and adhering to the fundamental relation according to Lighthill and Whitham (1955) and Richards (1956) and given in Eq. (1)-(2). However contrary to the KWM, the FOMSA makes use of a different definition of the fundamental relation defining it in terms of traffic velocity, which is more in line with Eq. (7). The main difference is the inclusion of an additional invariant term I, which describes the stochastic nature of traffic. This term is also conserved in space and time. The model is therefore described by:

$\partial_t \rho + \partial_x(\rho v) = 0$	Conservation of vehicles	(8)
$\partial_t \rho I + \partial_x(\rho v I) = 0$	Conservation of invariant	(9)
$v = V(\rho, I)$	Fundamental relation	(10)

Here the invariant, I, is the vehicle specific invariant, a term that denotes a vehicle dependant adjustment factor that directly influences the density p for each vehicle or group of vehicles depending on the level of discretisation. The vehicle specific invariant acts as a descriptive term that describes driving-style in relation to other vehicles. This is explained in more detail in section 3.3.

3.2 Model discretisation

The Godunov scheme is a commonly applied approach for the discretisation of macroscopic models (Lebacque 1996). The FOMSA is defined in Lagrangian coordinates rather than the traditional Eulerian coordinates. van Wageningen-Kessels et al. (2009) previously described how the Godunov scheme in Lagrangian coordinates is reduced to an upwind scheme, independent of traffic state with conservation equation:

$$\partial_t s + \frac{v\left(s^j(t)\right) - v\left(s^{j-1}(t)\right)}{\Delta n} = 0 \qquad (11)$$

Where Δn is the vehicle group size and $s^j(t) = s(j\Delta n, t)$ is the space headway of the $j\Delta n$-th vehicle at time t. van Wageningen-Kessels et al. (2009) and van Wageningen-Kessels et al. (2013) then define the Lagrangian formulation in time as an explicit semi-discretised scheme. As explicit time stepping is used, the semi-discretised scheme from Eq.(11) is made explicit for application and is given by (van Wageningen-Kessels et al. 2009):

$$\frac{s^{j,k+1} - s^{j,k}}{\Delta t} + \frac{v\left(s^{j,k}\right) - v\left(s^{j-1,k}\right)}{\Delta n} = 0 \qquad (12)$$

Where Δt is the time step and $s^{j,k}$ is the space headway a the position of the $j\Delta n$-th vehicle group at time $t = k\Delta t$. (van Wageningen-Kessels et al. 2013) also describe the scheme implicitly, that has an added advantage that it is relatively easy to solve as it only relies on traffic states in one direction and does not need to consider the propagation of traffic state changes with the flow as these are implicitly considered with the movement of vehicles, which follow the traffic flow. It is however the explicit scheme that is applied in this contribution as this is consistent with the applied extension of invariant advection.

3.3 Vehicle specific invariant

A main contribution of the FOMSA is the inclusion of vehicle behaviour in relation to inter-vehicular interaction and behaviour. This is achieved through the vehicle specific invariant term. This term is derived from previous work by Lebacque, Mammar, and Salem (2007), who introduced a generic invariant term which allows numerous descriptive variables to be propagated with traffic flow in a second order macroscopic model. In the FOMSA an invariant term is introduced as a first order Lagrangian model, which retains the relatively simplicity of first order modelling approaches. The vehicle specific invariant is a term that influences the density of traffic and is vehicle (group) specific and is applied in the fundamental equation. In traffic different drivers harbour different driving behaviour and levels of aggressiveness. This can often be described by the desired headways maintained, which is what the influence of the density directly describes, as:

$$s = \frac{1}{\rho} \tag{13}$$

Here s is the space headway and ρ is the density of traffic. As adjustment of the density would directly lead to inconsistencies in traffic conservation, the invariant is applied to the deterministic critical density $\rho_{crit.0}$ and jam density $\rho_{max.0}$ in the fundamental relation $v = V(\rho, I)$:

$$\rho_{crit} = I\rho_{crit.0} \tag{14}$$

$$\rho_{max} = I\rho_{max.0} \tag{15}$$

Empirical analysis has shown that driver behaviour and therefore also vehicle behaviour is also influenced by the traffic state, i.e. a driver may be less aggressive in congestion as this may have little advantage. This is accounted for by a traffic state term f applied to the Eq. (14)-(15) that is dependent on an adjusted ratio of the current density and the critical density, such that the formulation becomes:

$$\rho_{crit} = I^{f\,(\rho(t),\rho_{crit},\rho_{max})}\rho_{crit.0} \tag{16}$$

$$\rho_{max} = I^{f\,(\rho(t),\rho_{crit},\rho_{max})}\rho_{max.0} \tag{17}$$

Differences between behaviour of vehicles and vehicle-groups may be presumed to be randomly distributed in space. For example, it is not likely to have all aggressive drivers followed by all conservative drivers. However we hypothesise that drivers also influence other drivers in the direct vicinity and that some clustering may occur. For this reason it should not be presumed that the distribution of driver types (indicated by their vehicle specific invariant value) is perfectly random. This is considered in the model through the addition of a transition term β, that describes how the vehicle specific invariant is distributed over vehicles or vehicle groups in space:

$$I(n) = I(n-1)\begin{cases} -\min(I(n-1) - X, \beta) & for\, I(n-1) > X \\ +\min(X - I(n-1), \beta) & for\, I(n-1) < X \end{cases} \tag{18}$$

$$X \sim U([1-\alpha, 1+\alpha]) \tag{19}$$

Where $I(n)$ is the value of the vehicle specific invariant, which is dependent on the value $I(n-1)$ of the previous vehicle group n (note that a vehicle group may contain one single vehicle or multiple vehicles as a platoon). X is a random number between $[1-\alpha, 1+\alpha]$ in which α is the stochastic boundary parameter which indicates the maximum extent of the stochastic influence. Parameter β is the transition parameter that indicates the maximum change in I between consecutive vehicle groups. Parameter β is in itself also dependant on the size of vehicle group sizes, if a vehicle group n is not equal to a single vehicle. The vehicle specific invariant, I, is assigned to each vehicle or platoon at the entrance of a network according to Eq. (18)-(19). In this contribution perfect values for α and β are not analysed. This is recommended for later research.

4. Experimental case

4.1 Setup and results

The first order model with stochastic advection is demonstrated in an experimental case. The experimental case is setup for a single highway corridor of 11 kilometres on which two bottleneck locations are present. The first bottleneck is less severe and has a reduced capacity of 8%, while the second bottleneck further downstream has a capacity reduction of 15%. Traffic flow into the corridor is maintained at a constant flow of 2000 veh/hr, which is sufficient to lead to congestion in deterministic traffic flow equivalent to a bottleneck with an 11% capacity reduction. Therefore, in the deterministic case the first bottleneck will not be activated, while the second will always be activated. The applied time step Δt is 5 seconds, while the values for the stochastic boundary parameter α and the transition parameter β are randomly assigned to vehicles to show their effects, but remain static in time for a single vehicle. α is varied in the range [0.1, 0.4], leading to a I value of [0.95:1.05, 0.6:1.4], and β is varied in the range [0.1, 0.3]. The occurrence of a traffic state influence, f, is ignored in this experimental case and will be examined in later research. The formulation of $f(\rho(t), \rho_{crit}, \rho_{max})$ is not explicitly given in this contribution and should be derived from empirical analysis.

Two simulation runs with different random values for the vehicle specific invariant are shown in Figure 2. From this it is clear that the effect of the behavioural term has a marked influence on the occurrence of congestion, as in both cases the same traffic demand is applied, however the characteristics of each vehicle groups is different. In Figure 2a both bottlenecks are activated while the case in Figure 2b shows that only the second more severe bottleneck is activated, which would indicate that in the second case the vehicles have a higher invariant value and therefore vehicle at closer proximity to each other. Note that for a deterministic simulation run, only the second bottleneck would have been activated. This therefore demonstrates that consideration of stochastic variations in vehicle behaviour between vehicles can have a detrimental effect of traffic flow, as is also the case in real life.

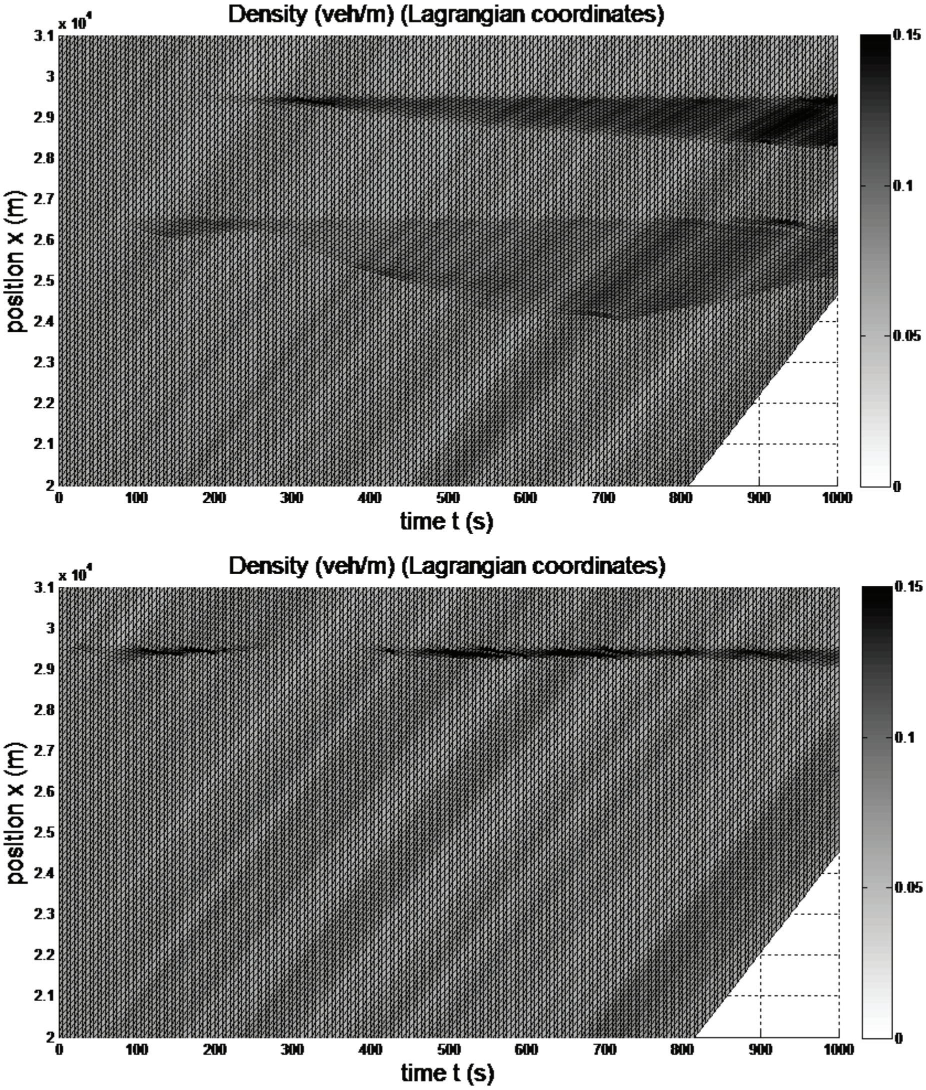

Fig. 2 (a)(b) Simulation results of the First order model with stochastic advection (FOMSA) for a dual bottleneck case for two different random procedures with settings α=0.2 and β=0.1

It is not the goal of this contribution to fine-tune the applied parameters. However further simulations are performed to demonstrate the effects of changes to parameter values. In Figure 3 the value of β is held at 0.1, while the boundary parameter α is given a value of 0.05 and 0.4 for Figure 3a and 3b respectively.

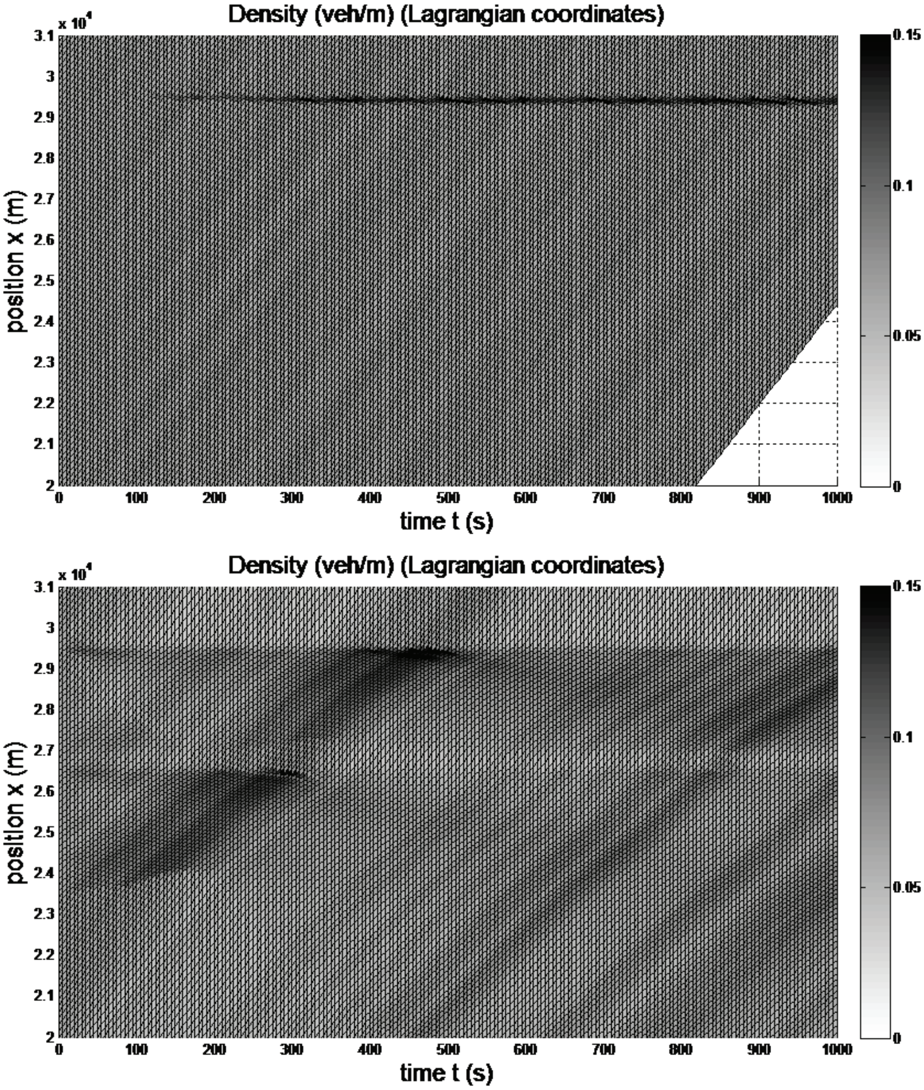

Fig. 3 Simulation results of the First order model with stochastic advection (FOMSA) for a dual bottleneck case for two different random procedures with settings α=0.05 (a) / 0.40 (b) and β=0.1

A low value of α means a low level of stochasticity in the assignment of vehicle specific invariant values and therefore means that the simulation result will be close to the deterministic case (Figure 2a). Performing multiple simulations with these values, showed very little difference between the outcomes. Each simulation also resulted in the second bottleneck always being activated, while the first bottleneck was never activated. This is due to the boundary values for the vehicle specific invariant being outwith of the required deviation to trigger the less severe bottleneck or to prevent the second bottleneck from being activated. The figure also shows little variation in the free flow densities. A high value for α on the other hand means a high level of stochasticity as seen in Figure 3b. As may be expected this leads to extensive congestion in comparison to a lower level of stochasticity. Both bottlenecks are activated and congestion is widespread and propagates quickly. This was found for the majority of the simulations, while a lower number of simulations for α=0.4 also resulted in both bottlenecks not being triggered, which us possible when the random values for the vehicle specific invariant are consistently low. However as the probability of the value remaining low is small, the case in which congestion occurs is greater.

In Figure 4 a demonstration is given of the effect of changes to the transition parameter β. β is given a value of 0.3, while α retains the same value as in Figure 1 of 0.2. From Figure 4 it is clear that an increased boundary for the transition of the vehicle specific invariant value between vehicle groups leads to greater changes between consecutive vehicles. This increases the randomness of traffic flow and reduces homogeneity. However as the effect of a high invariant value at a moment in time can immediately be counteracted by an equally strong low value from the following vehicle, when congestion occurs at the first bottleneck it is often of limited severity and does not last for a long time. Therefore the effect seen from multiple simulations is that congestion occurs more readily compared with the deterministic case, however the severity is similar to other values of β.

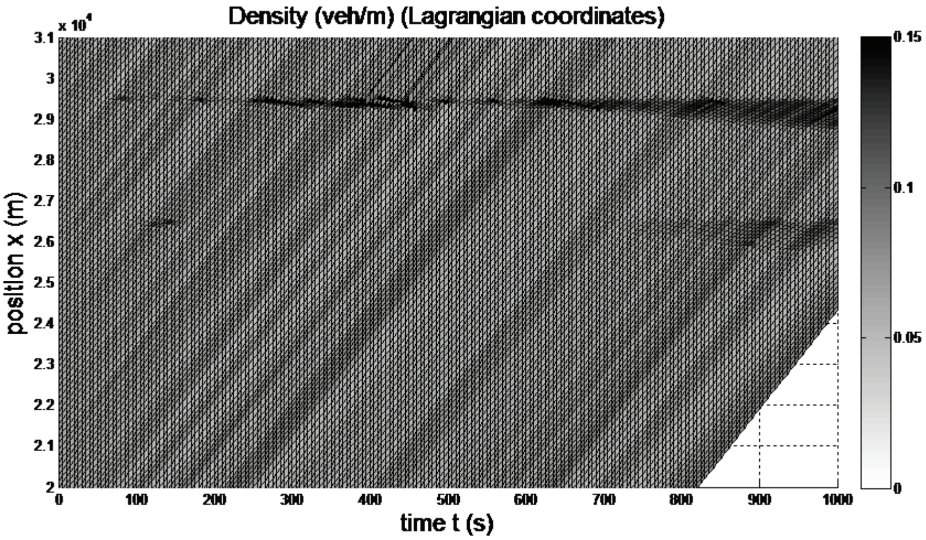

Fig. 4 Simulation results of the First order model with stochastic advection (FOMSA) for a dual bottleneck case for two different random procedures with settings α=0.2 and β=0.3

4.2 Discussion

The values applied in these simulations for α and β are estimates of realistic values, however are not explicitly based on empirical observations. Further research is recommended to determine which values are most suited for these parameters and also to confirm the hypothesis that these parameters are of influence to traffic flow in way described.

The test case has demonstrated the face validity of the model and has further shown that vehicle specific behaviour can lead to situations in which a bottleneck sometimes will be activated and at other times will not be activated under identical traffic flow. The difference is in the characteristics of individual vehicles or platoons, which leads to local anomalies in traffic flow and a local reduction of the critical density, which increases the chance of traffic breakdown. These effects are also seen in real life on roads and confirms the face validity of the approach. The value for the boundary parameter, α, is found to be important for the probability of traffic breakdown and the level of congestion severity. This is not surprising as a large reduction in the critical density leads by definition to a higher probability of traffic breakdown. Even with the probability of higher density values, once congestion occurs, capacity is reduced through the capacity drop and therefore has a greater detrimental effect on traffic flow. The value for transition parameter, β, on the other hand indicates regimes in traffic flow from behaviour and gives a quantity for the interaction between vehicles. A higher value indicates independent vehicle behaviour, while a low value increase the presumed interaction effects. It further shows that a better distribution of vehicle and driver types

(aggressive and conservative drivers) can lead to a reduction in congestion severity. However there is some uncertainty of the validity of such a parameter. The term is included as a hypothetical effect that can explain some characteristics of traffic flow, but has still to be validated against empirical data. This is also a recommended.

5. Conclusions

Capturing micro-stochastic driving behaviour in a macroscopic model is important to accurately describe traffic flow phenomena on a macroscopic level. A first order stochastic macroscopic model formulation is introduced in this contribution that makes use of first order traffic flow theory in conjunction with an additional invariant term, the vehicle specific invariant, that describes the effect of vehicle behaviour and the level of aggressiveness of drivers and represents the vehicle specific change to a deterministic density value. This is performed in the Lagrangian system, which allows the invariant term to propagate along with the vehicles for which it is valid and thus avoids numerical diffusion of vehicle behaviour variables. The use of Lagrangian coordinates have previously been shown to lead also to more accurate results. The vehicle specific invariant is defined as an adaptation of a deterministic density as a function of two further parameters: α, which represent a

stochastic boundary parameter that describes the limitations in variance between vehicles, and a transition parameter β that describes the interaction between vehicle behaviour and gives a quantity of the change in the vehicle specific invariant in time. The described model offers the advantages of including vehicle behaviour with an increased accuracy due to reduced diffusion effects, while doing this in a first order setting and therefore avoiding some of the complexity involved in second order model that are often applied to incorporate vehicle behaviour in macroscopic modelling.

The model is demonstrated in an experimental case on a corridor with two bottlenecks present. The case demonstrates the face validity of the model and offers insight into the effects of different values for the model parameters. A further calibration of the model parameters based on empirical data is recommended as further research, as well as investigating the effects of other types of bottlenecks.

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