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Nuclear incompressibility at finite temperature and entropy

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Abstract

Features of the nuclear isothermal incompressibility κ and adiabatic incompressibility κ_Q are investigated. The calculations are done at zero and finite temperatures and non-zero entropy and for several equations of state with a long range attraction and a short range repulsion. It is shown that κ_Q decreases with increasing entropy while the isothermal κ increases with increasing T. A duality is found between the adiabatic κ_Q and the $T = 0$ isothermal κ. The effect of correlations on κ is studied. A peak in κ can occur from attractive scattering correlations in various two-nucleon spin–isospin channels. The second virial coefficient or ρ^2 term in P versus density parallels a result that appears in the theory of superconductivity.

The behavior of nuclear systems at moderately high temperature and density is of current interest for several reasons. Such studies are important for understanding features of current medium energy collisions [1], for future RIA experiments, and for nuclear astrophysics as in supernovae explosions. The equation of state (EOS) of pressure versus density and temperature and its associated incompressibilities, isothermal and adiabatic, are important in understanding flow produced in nuclear collisions as reviewed in Ref. [2] and, in general, these quantities appear in the description of the thermodynamic properties of fermionic systems. For example, strongly correlated fermionic systems are of interest in condensed matter physics and the physics of the quark–gluon plasma. This Letter focuses on an important quantity for understanding properties of these systems which is the nuclear incompressibility. While the nuclear incompressibility at zero temperature has been studied for an extended period [3–5], it is only relatively recently that its temperature dependence has been of concern. See, for example, the quantum Monte Carlo results of Ref. [6]. Ref. [6] shows a peak in the incompressibility coefficient and in the specific heat C_V. This increase in C_V is also seen in Ref. [7] using a totally different approach based on recursive methods to obtain the finite temperature partition function of hadronic

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matter. Here, we will study the effect of two nucleon correlations on the EOS. Our approach is similar in spirit to electron pairing in superconductivity as will be pointed out below.

In this Letter we will study the behavior of the isothermal incompressibility with T and properties of the equation of state (EOS) at higher than normal density. As a baseline, we will begin with a mean field discussion of its behavior with T to see how large the incompressibility can become without correlations.

First, we define a quantity κ, the incompressibility coefficient, as

$$\kappa = k_F^2 \frac{d^2(E/A)}{dk_F^2} = 9\rho^2 \frac{d^2(E/A)}{d\rho^2} = -9V^2 \frac{d^2(E/A)}{dV^2}. \quad (1)$$

This quantity is evaluated at the saturation density ρ_0 where E/A has a minimum. The giant monopole resonance energy is then

$$E_0 = \sqrt{\frac{\hbar^2 \kappa A}{m\langle r^2\rangle}}.$$

If the temperature is kept constant in the above derivatives we have the isothermal incompressibility κ, and if the entropy is held fixed, the result is the adiabatic incompressibility κ_Q. The κ and κ_Q are equal at $T = 0$ only. The quantity κ defined above is not the isothermal compressibility defined in thermal physics as

$$K = -\frac{1}{V}\left(\frac{dV}{dP}\right)_T$$

with T held fixed and here P is the pressure. Since $P = -dF/dV$, we have

$$K = \frac{1}{V}\left(\frac{1}{d^2F/dV^2}\right).$$

At $T = 0$, $F = E - TS = E$, and thus $\kappa = 9/(\rho_0 K)$. This reciprocal connection between K and κ is no longer true at finite T. Besides the isothermal compressibility, an adiabatic compressibility

$$K_Q = -\frac{1}{V}\left(\frac{dV}{dP}\right)_S$$

can be obtained by keeping the entropy constant. The reciprocal is related to κ_Q, the adiabatic incompressibility as $\kappa_Q = 9/(\rho_0 K_Q)$. Since the natural variables for energy are entropy and volume from $dE = T\,dS - P\,dV$ variations of the energy with V at constant S bear a similar relation to variations of the Helmholtz free energy with V at constant T where $dF = -S\,dT - P\,dV$. Thus the adiabatic incompressibility of Eq. (1) can go to zero for a Skyrme interaction as we shall see. The minimum point (also maximum point) in the energy occurs at zero pressure since dE/dV at constant entropy is $-P$. Therefore, E at constant S has the same maximum and minimum points with variations in V or R or density as F at constant T since both derivatives are $-P$ which is set to 0. The behavior of the adiabatic incompressibility is linked to a phase change. As we shall see, the behavior of the isothermal incompressibility may be associated with the appearance of a strongly correlated fermionic system at high density and temperature.

Our mean field discussion is based on a Skyrme interaction which shares some features with a van der Walls interaction with a long range attraction and a short range repulsion. To keep the discussion simple, we consider uncharged symmetric nuclear matter with no surface energy terms. The Skyrme interaction energy is then

$$\frac{U}{A} = -a_0\rho + a_\alpha \rho^{1+\alpha}. \quad (2)$$

The a_0 term gives a medium range attraction while the a_α term is a short range repulsion.

At $T = 0$, the kinetic energy $E_K/A = (3/5)E_F(\rho)$ with the Fermi energy

$$E_F = \frac{\hbar^2}{2m}\left(\frac{6\pi^2}{4}\rho\right)^{2/3}.$$

The coefficients a_0 and a_α are fixed to give a binding energy per particle $E_B/A = 16$ MeV at density $\rho = \rho_0 = 0.15\ \mathrm{fm}^{-3}$, which gives $b_0 = a_0\rho_0 = 37 + 23/\alpha$ and $b_\alpha = a_\alpha \rho_0^{1+\alpha} = 23/\alpha$ in MeV. The incompressibility coefficient κ at $T = 0$ is then

$$\kappa = -2\frac{E_K}{A} + 9(1+\alpha)\alpha a_\alpha \rho_0^{1+\alpha} = 165 + 207\alpha. \quad (3)$$

For $\alpha = 1/3$, $\kappa = 234$ MeV. Smaller values of α lead to softer equations of state and lower κ. In the limit $\alpha \to 0$, logarithmic terms appear in Eq. (2) coming from the presence of a factor

$$\frac{x}{\alpha}\left(1 - x^\alpha\right) \to -x\log(x).$$

The $x = \rho/\rho_0 = (R_0/R)^3$ with $R_0^3 = A/(\rho_0 4\pi/3)$. The $\alpha \to 0$ limit is the softest EOS allowed by Eq. (2), and this limit gives from Eq. (3) a value of $\kappa = 165$ MeV. A stiff EOS has $\alpha = 1$ and $\kappa = 372$ MeV. Recent calculations done at $T = 0$ [8–10] have a value of $\kappa = 210$–270 MeV and suggest a value of $\alpha = 1/3$ in a Skyrme type approach. A larger range of values of κ, from 211 to 350 MeV, were reported in Ref. [11]. Because of these uncertainties in κ, from more realistic forces, we will present results for various values of α, from $\alpha \approx 0$ to $\alpha = 1$.

At non-zero $T \ll E_F$, the kinetic energy is [12]

$$\frac{E_K}{A} = \frac{3}{5}E_F + \frac{\pi^2}{4}\frac{T^2}{E_F}.$$

The energy per particle is

$$\frac{E}{A} = 21x^{2/3} + \frac{\pi^2}{140}\frac{T^2}{x^{2/3}} - b_0 x + b_\alpha x^{1+\alpha}. \tag{4}$$

The value of x that minimizes E/A is x_m and satisfies the equation:

$$14\left(x_m^{2/3} - x_m^{1+\alpha}\right) - b_0\left(x_m - x_m^{1+\alpha}\right) = \frac{\pi^2}{210}\frac{T^2}{x_m^{2/3}}.$$

Then the $\kappa = \kappa(T)$ is given by

$$\kappa(T) = -42x_m^{2/3} + 0.705T^2/x_m^{2/3} + 9\alpha(1+\alpha)b_\alpha x_m^{1+\alpha}. \tag{5}$$

At $T = 2.5$, 5, and 7.5 MeV, and for $\alpha = 1/3$, the values of κ are 242, 265 and 302 MeV, respectively. The corresponding values of x_m are: 1.011, 1.043, 1.091. When T is replaced with entropy per particle S/A then this T dependent term becomes

$$\frac{E_F}{\pi^2}\left(\frac{S}{A}\right)^2 = \frac{35}{\pi^2}\left(\frac{S}{A}\right)^2 x^{2/3}$$

since $S = (\pi^2/2)TA/E_F$ at low T. This S/A term can simply be added to the first term on the right side of Eq. (4) since both have the same $x^{2/3}$ dependence.

If the corrections to the nuclear matter incompressibility at $T = 0$ from finite temperature terms are small, then these corrections can be obtained by using the following method. Let $E_0(R)$ be the nuclear matter energy per particle EOS and which has a minimum at R_0 and an incompressibility κ_0. If we add to this a term $E_x(R)$, so that $E(R) = E_0(R) + E_x(R)$ then the minimum shifts to a new point $R_m = R_0 + \Delta R_x$. The new minimum and κ can be found by making a Taylor expansions around of R_0. The new κ is

$$\kappa = \kappa_0 + R^2\left(\frac{E_x}{A}\right)'' - 2R\left(\frac{E_x}{A}\right)' - R\left(\frac{E_x}{A}\right)'\frac{Sk}{\kappa_0}. \tag{6}$$

The various quantities are evaluated at R_0 and each $'$ represents one derivative wrt R. Corrections to κ involving the skewness $Sk = R^3(E_0(R)/A)'''$ or third derivative of the energy were pointed out in Refs. [4,5]. Ellis et al. [13] used the correlation between compression modulus and skewness coefficient to examine the implications in a relativistic Hartree–Fock approximation where the E_x is the Coulomb interaction. The above expression is a modified version of their result. Eq. (3) gives an expression for the incompressibility at $T = 0$. This will be $\kappa_0 = \kappa_0(\alpha)$ in Eq. (6). The skewness is

$$Sk(\alpha) = -3\left(509 + 828\alpha + 207\alpha^2\right)$$

in MeV. Comparing Sk with κ of Eq. (3) we see that the ratio of Sk/κ_0 is of the order of 10 and somewhat insensitive to α.

At low T, taking $E_x(R, T) = 0.0517\,T^2R^2/A^{2/3}$, we obtain the $\kappa(\alpha \to 0) = 165 + 1.16T^2$, $\kappa(\alpha = 1/3) = 234 + 1.32T^2$ and $\kappa(\alpha = 1) = 373 + 1.61T^2$. Thus we see that the first term is very sensitive to α but the finite temperature correction is somewhat insensitive to α. At fixed entropy, the second derivative of $E(R)/A$ has a very different behavior than at fixed T. Namely, it decreases with S/A. This can easily be seen by noting that $E_x(R, S) = 4.836(S/A)^2A^{2/3}/R^2$ compared to $E_x(R, T) = 0.0517\,T^2R^2/A^{2/3}$. We have the following final results (in MeV):

$$\kappa_Q(\alpha \to 0) = 165 - 30(S/A)^2,$$
$$\kappa_Q(\alpha = 1/3) = 234 - 38(S/A)^2,$$
$$\kappa_Q(\alpha = 1) = 373 - 53(S/A)^2.$$

At higher T, the nearly degenerate Fermi gas kinetic energy term is replaced by a virial expansion in $\rho\lambda^3$, where

$$\lambda = \sqrt{\frac{2\pi\hbar}{mT}}$$

is the quantum wavelength. Namely,

$$\frac{E_K}{A} = \frac{3}{2}T\left(1 + \sum_n c_n\left(\rho\lambda^3/4\right)^n\right)$$

with coefficients arising from antisymmetrization that are

$$c_1 = \frac{1}{2^{5/2}} = 0.177,$$
$$c_2 = \frac{1}{8} - \frac{2}{3^{5/2}} = -3.3 \times 10^{-3},$$
$$c_3 = 1.11 \times 10^{-4}, \quad \ldots$$

[12,14]. Since the c_n's become small rapidly, we will keep terms up to c_2. Then κ is given by

$$\kappa(T) = -27T\left(\lambda^3\rho_0/4\right)^2 c_2 x_m^2 + 9\alpha(1+\alpha)b_\alpha x_m^{1+\alpha}. \tag{7}$$

The x_m is again the minimum of E/A, but now evaluated with the new kinetic energy. The x_m is affected by both the c_1 and c_2 terms at temperatures where c_1 dominants. A limiting value of κ can be obtained by taking T very large where c_1 term leads a minimum x_m given by

$$x_m^\alpha = \frac{b_0}{(1+\alpha)b_\alpha}\left(1 - \frac{3}{2}T\frac{c_1}{b_0}\frac{\rho_0\lambda^3}{4}\right).$$

In this high T limit κ is given by the second term on the right side of Eq. (7) and is

$$\kappa(T \gtrsim 10\ \text{MeV}) = \kappa_{\text{sat}}\left(1 - \frac{3}{2}T\frac{c_1}{b_0}\frac{\rho_0\lambda^3}{4}\right)^{1+1/\alpha}, \tag{8}$$

and goes to its saturation value

$$\kappa_{\text{sat}} = 9\alpha b_0\left(\frac{b_0}{(1+\alpha)b_\alpha}\right)^{1/\alpha}$$

with a T dependence of $1/\sqrt{T}$. We note that the sign of c_1 determines whether it approaches from above or below. For purely antisymmetric correlations c_1 is positive because of the statistical repulsion of fermions. If c_1 becomes negative as will be discussed below it would approach from above. At infinite T for $\alpha = 1$, $\kappa = 704$ MeV with $x_m = 1.304$ or a minimum density $\rho_m = 1.304\rho_0$ and for $\alpha = 1/3$, $\kappa = 468$ MeV with $x_m = 1.53$ or $\rho_m = 1.53\rho_0$. In the limit $\alpha \to 0$, $\kappa = 380$ MeV with $x_m = e^{14/23} = 1.84$ or $\rho_m = 1.84\rho_0$. These are the limiting values for κ and x_m.

We can also include an effective mass in our results. If we parameterize it as

$$\frac{m^*}{m} = \frac{1}{1 + r(\rho/\rho_0)^\gamma},$$

then at zero temperature the E/A will simply read

$$\frac{E}{A} = \frac{E_K}{A}x^{2/3} + r\frac{E_K}{A}x^{2/3+\gamma} - b_0 x + b_\alpha x^{1+\alpha}.$$

For general α and m^*/m:

$$\kappa(\alpha, r) = 3\bigl(55 + 69\alpha + 7r[3\gamma - 1]\bigl([3\gamma - 1] - 3\alpha\bigr)\bigr). \tag{9}$$

For $\gamma = 1/3$ or for $\alpha = \gamma - 1/3$ this κ is r independent. When we parametrize m^*/m with $\gamma = 1$, for $\alpha = 1/3$, $\kappa = 234 + 42r$ and for $r = 1/2$, $\kappa = 255$ MeV. For $\alpha = 1$, $\kappa = 372 - 42r \to 351$ MeV at $r = 1/2$. The details and other T dependences will be given in a future paper [15].

Before discussing how a peak in κ may arise in our approach, we briefly investigate the case of constant entropy in the ideal gas limit using the Sackur–Tetrode law [16]:

$$\frac{S}{A} = \frac{5}{2} - \ln\bigl(\lambda^3\rho/4\bigr).$$

This law connects T to ρ or V as $T = C_S\rho^{2/3}$. Here

$$C_S = \frac{2\pi(\hbar c)^2}{(mc^2)}\exp\left[\frac{2}{3}\frac{S}{A} - \frac{5}{3}\right].$$

The resulting $E(R)/A$ has a structure similar to the result for a degenerate Fermi gas since both have a $\rho^{2/3}$ dependence for the kinetic energy term but with different coefficients. This feature and a similar result at lower T suggests a duality in the energy per particle EOS at constant entropy and its associated κ_Q and the $T = 0$ EOS and its associated constant T κ. We also note a parallel between F as a function of T and V and E as a function of S and V.

We now turn to the issue of clusters or more precisely correlations at moderately high T and high $\rho > \rho_0$. We study the corrections to the ideal gas law using the virial expansion

$$P = \rho T\bigl(1 + c_1(\rho\lambda^3/4) + c_2(\rho\lambda^3/4)^2 + \cdots\bigr).$$

Our results are thus restricted to values of ρ and T where this expansion is valid and the gas is nearly non-degenerate. This also insures that the fermions have not "quenched out" the dynamical correlations, i.e., the N–N scattering cannot occur into final states that are already occupied. If antisymmetry effects are the only corrections, the coefficients can be calculated by following a procedure in Refs. [17,18] and are the coefficients already given before Eq. (7). This procedure is an extension of our fragmentation model by simply noting that a cycle of length k is analogous to a cluster of size k with a weight function x_k. The grand canonical partition function is $\log Z = \sum x_k e^{\beta\mu k}$ and the mean number of cycles is $\langle n_k\rangle = x_k e^{\beta\mu k}$ [19]. The pressure is

$$\frac{PV}{T} = \log Z = \sum x_k e^{\beta\mu k}.$$

A constraint exists, $\langle A\rangle = \sum k\langle n_k\rangle = A$ which determines the fugacity $z = e^{\beta\mu}$ in a power series in A by inverting the series. Then we arrive at

$$\frac{PV}{T} = A + \frac{-x_2}{x_1^2}A^2 + \frac{4x_2^2 - 2x_1x_3}{x_1^4}A^3 + \frac{-20x_2^3 + 18x_1x_2x_3 - 3x_1^2x_4}{x_1^6}A^4 + \cdots. \quad (10)$$

Substituting

$$x_k = (-1)^{k+1}\frac{V\lambda^3}{k^{5/2}}$$

gives the desired power series in $(A/V)\lambda^3/4$ for fermions [17,18]. The factor of 4 is spin and isospin degeneracy. The same procedure applies for bosons with $x_k = (V/\lambda^3)/k^{5/2}$. For fragmentation in the Boltzmann limit the

$$x_k = \frac{V}{\lambda^3(k)}Z_{\mathrm{int}}(k) = \frac{V}{\lambda^3(k)}e^{F_k/T},$$

where F_k is the internal free energy of a cluster of size k and $\lambda(k) = \lambda/k^{1/2}$. The effect of antisymmetry for odd k clusters and symmetry for even k clusters can be included. The grand canonical ensemble represents a system of fermions (odd cluster sizes) obeying FD statistics and bosons (even cluster sizes) obeying BE statistics. The constraint of chemical equilibrium $\mu_k = k\mu_1$ or $\mu_k = z\mu_p + n\mu_n$ is imposed which determines the fugacity from the constraint. In the x_k model of Refs. [19–22] this amounts to having various terms in x_k that represent both cycles and clusters. For example,

$$x_1 = \frac{V}{\lambda^3},$$
$$x_2 = -\frac{1}{2^{5/2}}\frac{V}{\lambda^3} + 2^{3/2}\frac{V}{\lambda^3}\frac{Z_{\mathrm{int}}(2)}{4},$$
$$x_3 = \frac{1}{3^{5/2}}\frac{V}{\lambda^3} + 3^{3/2}\frac{V}{\lambda^3}\frac{Z_{\mathrm{int}}(3)}{4}.$$

x_4 will have terms from the antisymmetry of monomers from cycles of length 4, from symmetrization of dimers and from clusters of size 4. Once the x_k's are given the canonical partition function can be generated by a recurrence relation [19,23]. A factor 1/4 appears from spin–isospin degeneracy which has been included. The internal partition

$$Z_{\mathrm{int}}(k) = \sum g(E_j)e^{\beta E_j(k)} + \frac{1}{\pi}\sum_{J,T}\frac{(2J+1)(2T+1)}{\pi} \times \int \frac{d\delta_{J,T}}{dE}e^{-\beta E}\,dE.$$

The sum is over bound state E_j which have degeneracy $g(E_j)$ and $\delta_{J,T}$ is the phase shift in channel of spin J and isospin T. A similar result, excluding the isospin index T, appears in the theory of superconductivity in Ref. [24]. There the electrons are correlated into pairs, while here, because of the two types particles, neutrons and protons, more possibilities exist in spin and isospin. These phase shifts include effects from both attractive and repulsive interactions. Using nucleon–nucleon phase shifts the continuum contributions [25] reduces the bound state contribution by about 50% for moderate temperature ($T \sim 20$ MeV) and less for low temperatures because of the Boltzmann weight factor in the integral. At infinite T, $Z_{\mathrm{int}} \to 0$ since the continuum exactly cancels that bound states by Levinson's theorem [25]. As an initial example for $Z_{\mathrm{int}}(2)2^{3/2}$ we will consider

$$\frac{1}{2}\frac{3}{4}2^{3/2}e^{|E_B|/T}$$

to see how it compare with $1/2^{5/2}$; 1/2 is for the continuum reduction, 3 is for the spin degeneracy of the $S=1$, $T=0$ channel. The $S=1$, $T=0$ channel in free space has a bound state, which is the deuteron. In a medium, the $S=1$, $T=0$ channel may appear as a metastable resonance or attractive correlation. If we neglect the Boltzmann factor in the binding or resonance energy, then we have 1.06. To reduce 1.06 to $1/2^{5/2}$ we would need a reduction factor of 1/6. Also other spin–isospin channels increase Z_{int}. Thus c_1 can easily become minus. For a negative c_1, the κ is above its saturating value and approaches it from above as $T^{-1/2}$. At low T, κ is below its saturating value and initially increases as T^2 because the Fermi sea blocks excitations or “quenches out” dynamical scattering correlations. This behavior automatically implies a peak in κ. The Monte Carlo result [6] has a peak behavior in κ with a peak of $\kappa = 1500$ MeV at $T \sim 14$ MeV. Higher order $k = 3, 4, 5, \ldots$ terms represent higher order correlations of fermions in various J, T channels. For example, $k = 3$ can represent $J = 1/2$, $T = 1/2$ correlations.

In this Letter we investigated the behavior of the infinite nuclear matter incompressibility at finite temperature and entropy using a mean field theory and also considering the role of correlations. Various forms of the EOS are studied using a Skyrme parametrization. Both the isothermal (constant temperature) and adiabatic (constant entropy) incompressibilities are found to be sensitive to the choice of the Skyrme repulsive parameter α which gives the power of the density involved in the repulsive term. These two incompressibilities have very different behaviors. The isothermal incompressibility increases with T initially as T^2 until a saturation value is reached while the adiabatic incompressibility decreases with increasing entropy and eventually goes to zero. In a mean field approximation, the isothermal incompressibility approaches its saturation value as $-T^{-1/2}$ with the minus sign reflecting an approach from below the saturating value with increasing T. This behavior arises from the statistical repulsive correlations that represent the Pauli exclusion principle. The adiabatic incompressibility is shown to arise from an equation of state (EOS) or energy per particle that has a structure that is similar to a $T = 0$ Fermi gas.

We then discussed how a peak can appear in the isothermal incompressibility by looking at coefficients in the virial expansion, and in particular, we investigated the second virial coefficient called c_1 here. Our results only apply in this nearly non-degenerate limit. An expression similar to our expression for our c_1 appears in the theory of superconductivity, where correlations arise from the pairing of electrons [24]. Here, because of the presence of two types of particles, protons and neutrons, correlations are present in various spin isospin channels. The approach of κ to its saturation value was shown to be related to the sign of c_1, with c_1 positive having an approach from below and c_1 negative having an approach to the saturating value from above it. The role of attractive correlations between nucleons was then studied in the various spin isospin channels and compared to the statistical repulsion term at high T. We noted that a strong two nucleon correlation of paired fermions in a high density, but also high temperature, medium can account for the existence of a peak. The detailed structure of the peak is related to the presence of strongly correlated fermions in pairs, triplets, and higher order correlations. Questions related to strongly correlated fermions are of interest in other areas of physics such as condensed matter physics and in quark–gluon plasma, the latter occurring at a much higher density and or temperature than the ρ and T considered here.

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