the correlator drum to sample at a corresponding time from each record, and then summing these signals in an integrating device. Thus the signal records must be passed through the machine once for each sampling point.

A second averaging device, the Average Response Computer, is a digital device which permits on-line recording and processing. It is a transistorized computer with 256 magnetic core storage registers of 18 bits each. The input signal is sampled at as many as 254 points, and the sampled magnitudes are accumulated in the registers. The computer also operates as a histogram compiler. Each register accumulates the number of times a signal occurrence falls within a specified range of values. Display of results of this computer is either by oscilloscope, pen plotter, or punched paper tape.

The last processor mentioned, the Amplitude and Latency Measuring Instrument with Digital Outputs, has a peak detector circuit which emits a pulse at the occurrence of a peak. This pulse stops a timing counter which was started by the stimulus pulse, thus recording the latency. The peak amplitude is also converted to a digital representation.

The fourth appendix lists 49 references which represent the work and experimental philosophy of the Communications Biophysics Group.

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Statistical Independence in Probability, Analysis and Number Theory, Carus Mathematical Monograph No. 12. By MARK KAC. Distributed by John Wiley, New York, 1959. 93 pp. $3.00.

Probability and Related Topics in Physical Sciences, Lectures on Applied Mathematics, Vol. I. By MARK KAC. Distributed by Interscience Publishers, New York, 1958. 266 pp. $5.60.

Recent Advances in Probability Theory, in Some Aspects of Analysis and Probability, Surveys in Applied Mathematics, Vol. IV. By I. KAPLANSKY, E. HEWITT, M. HALL, JR., and R. FORTET. (Paper by ROBERT FORTET, pp. 169–240.) John Wiley, New York, 1958. $9.00.

The two delightful little books of Kac cannot be too highly recommended: they provide the programs of many (overlapping) informal trips through the mathematical landscape, with a guide who succeeds in the task of preserving in print the inimitable flavor of his lectures. From the viewpoint of content, there is, unfortunately, little that is of direct bearing to the professional interests of the presumed readers of this journal; but they can trust that they will gain immensely in sophistication about how to use probability theory; besides, they are bound to enjoy themselves.

"To the author, the main charm of probability theory lies in the enormous variety of its applications." He has indeed succeeded in exhibiting it most convincingly. The *first book* is the more elementary of the two, being based upon lectures given to students of Haverford College. The thread which connects the different chapters is one which has led Kac since some of his first papers (written with his teacher, Hugo Steinhaus, to whom the book is dedicated). Most arguments start by the restatement of some very elementary reasoning of calculus, followed by an invitation to have a second look: This second look eventually leads to points that require much more advanced mathematics; if it is so, Kac states the required background theorems, without ever breaking the thread of the argument by difficulties that are irrelevant to his current point.

Sketch of the table of contents. *Chapter I. From Vieta to the notion of statistical independence:* a formula of Vieta; another look; an accident or something deeper?; heads or tails; independence and "independence." *Chapter II. Borel and after:* "laws of large numbers," . . . what price abstraction? (the answer is: very dear indeed!). *Chapter III. The normal law:* De Moivre; Markoff's method made rigorous (Markoff's method is the physicists' name for the method of characteristic functions, that is, for operational calculus as applied to probability); is the normal law a law of nature of a mathematical theorem? *Chapter IV. Primes play a game of chance:* the statistics of the Euler phi-function; almost every integer m has approximately log log m prime divisors; the normal law in number theory. *Chapter V. From kinetic theory to continued fractions:* paradoxes of kinetic theory; Boltzmann's reply; the abstract formalism; the ergodic theorem and continued fractions (the last two paragraphs are worth paraphrasing: "I could have easily started with the abstract formalism of ergodic theory, and have avoided any mention of dynamics and kinetic theory. But had I done this I would have suppressed the most exciting and, to my mind, most instructive part of the story, for the road from kinetic theory, as conceived by Boltzmann and others, to continued fractions is a superb example of the often forgotten fact that mathematics is not a separate entity but that it owes a great deal of its power and its beauty to other disciplines").

The second book of Kac, first presented to a 1957 summer seminar held in Boulder, Colorado, is far more advanced and is "supposed to furnish an introduction to probability theory to a mature audience with little or no prior knowledge of the subject." The first book, and chapters 1 and 2 of the present one, cover very much the same material. However, since one of Kac's purposes here is to impress ignorant mathematicians with the "respectability" of probability theory, he occasionally indulges in analytic pyrotechnics, and one is not surprised to find statements such as the following: "the reader has, no doubt, already noticed that the probability language is not a convenient way of stating certain mathematical facts about rather complicated sums or integrals."

On the other hand, different results are not played up in proportion to their mathematical difficulty: "This (a theorem just proved) is a cumbersome statement and as such seemingly devoid of any intrinsic interest. Does it become more interesting by being written (in probabilistic language)? Or does it only become less cumbersome? . . . There is no denying that our reaction to a scientific statement depends greatly on the context in which it is made. In one context it may be simply a mildly amusing statement whose proof is quite easy and its truth alone may be a rather feeble excuse for making it. In other context, and in a suitable language, it may be highly revealing and suggestive." A final feature of the book is that there is little emphasis on measure theoretical difficulties; "how much 'fuss' over measure theory is necessary for probability theory is a matter of taste. Personally, (Kac) prefers as little fuss as possible because (he) firmly believes that probability theory is more closely related to analysis, physics, and statistics than to measure theory as such." "That the extremely 'thin' framework of (Kolmogoroff's axiomatic) can support a rich and fruitful theory may appear surprising until one realizes that the richness and fruitfulness are due mainly to special measures and special sets."

Let us now summarize chapters III and IV. Chapter III, the longest by far, treats of "probability in some problems of classical statistical physics." (This chapter is supplemented by an appendix by G. E. Uhlenbeck.) It touches upon a wide variety of topics from the kinetic theory of gases to the theory of stochastic processes. Part of the flavor of the more elementary parts of this development may be inferred from Kac's well-known memoir, reproduced in Wax's "Selected Papers on Noise and Stochastic Processes." The last chapter is devoted to "Integration in function spaces and some applications"; the basic ideas here are due to Wiener and are, by now, fairly widely known in the communication community. The book is followed by the Uhlenbeck appendix, an appendix by Hibbs, and two appendices by van der Pol.

Let us now say a few words about the review paper by R. Fortet. It is also a highly personal collection of topics which are to a large extent taken from the Russian and French literature and do not overlap with the topics treated by Kac. The reader will be particularly interested in the following chapters: III, "General Random Elements"; IV, "Functionals of Random Functions"; V, "The Theorems of Kolmogoroff and of Smirnoff"; and VI, "Tests and Estimations Related to Stochastic Processes." The style is less fluent and less "philosophical" than that of Kac, but the book will be found to be very useful because of the breadth of its coverage, and it will direct the reader to many little known references.

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Errata

Volume 2, Number 3, September 1959, in the article entitled, "On a Problem of Nonlinear Mechanics" by Demetrios G. Magiros, (pages 297–309):

Page 302, line following equation (19) should read, "The reality of r implies $\mu = 0, \lambda > 0$, that is, $\bar{k} = 0, c_1/c_3 = (1 - \bar{c}_1)/\bar{c}_3 > 0$;".

Page 306, line following equation (38b), delete the words, "from positive values".

Page 306, line 10 from bottom, insert, "Fig. 5b" between "(b_1)" and "if".

Page 308, Figure 7(a) ii), the shaded region must be as shown below: