# The quark-gluon mixed condensate calculated via field correlators 

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#### Abstract

The quark-gluon mixed condensate $g\left\langle\bar{q} \sigma_{\mu \nu} F_{\mu \nu} q\right\rangle$ is calculated in the Gaussian approximation of the field correlator method. In the large $N_{c}$ limit and for zero mass quarks one obtains a simple result, $m_{0}^{2} \equiv g\left\langle\bar{q} \sigma_{\mu \nu} F_{\mu \nu} q\right\rangle /\langle\bar{q} q\rangle=\frac{16 \sigma}{\pi}$, where $\sigma$ is the string tension. For a standard value $\sigma=0.18 \mathrm{GeV}^{2}$ one obtains $m_{0}^{2}=1 \mathrm{GeV}^{2}$ in good agreement with the QCD sum rules estimate $m_{0}^{2}=(0.8 \pm 0.2) \mathrm{GeV}^{2}$ and the latest lattice result $m_{0}^{2} \cong 1 \mathrm{GeV}^{2}$. © 2004 Published by Elsevier B.V. Open access under CC BY license.


## 1. Introduction

The mixed quark-gluon condensate (QGC) is an important characteristics of the nonperturbative QCD vacuum, which together with the quark condensate $\langle\bar{q} q\rangle$ signals the chiral symmetry breaking. Moreover, the QGC measures the average interaction of the quark color-magnetic moment with the vacuum fields, which is an important ingredient of the quark dynamics in the vacuum (e.g., it is this term which gives attraction of in quark zero modes).

In the QCD sum rules the QGC plays an important role [1] and the phenomenological analysis suggests the value of $m_{0}^{2}$ in the range $m_{0}^{2}=(0.8 \pm 0.2) \mathrm{GeV}^{2}$ [1], see [2] for a review. One should stress at this point that for a nonzero quark mass $m$ the (diverging) perturbative part should be subtracted.

[^0]As will be seen below the resulting nonperturbative dependence of $m_{0}^{2}$ on $m$ is very weak in agreement with lattice data. Lattice studies of QGC [3-5] have not yet converged to a definite prediction. A problem there is the extrapolation to zero quark mass and the quenched approximation. In Ref. [4] the simulations are done in the quenched approximation, the condensate is measured by use of staggered quarks, and the result for $m_{0}^{2}$ is definitely larger than the sum-rules value. Ref. [5] uses an optimized version [6] of domain wall fermions, which are better in principle for the chiral limit, again in the quenched approximation. Their result is $m_{0}^{2}=1 \mathrm{GeV}^{2}$, which agrees with QCD sum rules. It is therefore worthwhile to calculate QGC by a different nonperturbative method.

In the framework of the field correlator method (FCM) [7] the color-magnetic quark-gluon interaction term $g \sigma_{\mu \nu} F_{\mu \nu}$ enters essentially in the Fock-Feynman-Schwinger representation (FFSR) of the quark propagator in the vacuum background field [8].

In particular the quadratic average of this term defines the hyperfine $q \bar{q}$ interaction where the nonperturbative part is proportional to the field correlator $\left\langle F_{\mu \nu}(x) F_{\rho \sigma}(0)\right\rangle$ measured on the lattice [9]. Even more important the term $g \sigma_{\mu \nu} F_{\mu \nu}$ is in the contribution to the bound quark self-energy [10], where it is of paramagnetic character, i.e., negative and strongly decreases the masses of hadrons, putting them in accordance with experimental data [11]. Explicit correction to the bound quark mass squared is [10]
$\Delta m_{q}^{2}=-\frac{4 \sigma}{\pi} \eta$,
where $\eta=\eta\left(m T_{g}\right)$ is a calculable function of the quark current mass $m$, renormalized at the scale of 1 GeV . The function $\eta$ is given in [10] and in Appendix A and for zero quark mass is normalized to one: $\eta(0)=1$. We calculate in the next section the QGC, or rather the parameter $m_{0}^{2}$ in the same way, as it was done in [10] for $\Delta m_{q}^{2}$, with the result
$m_{0}^{2}=-4 \Delta m_{q}^{2}=\frac{16 \sigma}{\pi} \eta$.
For $\sigma=0.18 \mathrm{GeV}^{2}$ one obtains $m_{0}^{2}=1 \mathrm{GeV}^{2}$ which is in agreement with the lattice data [5], and with the QCD sum rules estimate quoted above.

## 2. Calculation of $\boldsymbol{m}_{0}^{2}$

We proceed in the Euclidean space-time and write

$$
\begin{align*}
\left\langle\bar{q} g \sigma_{\mu \nu} F_{\mu \nu} q\right\rangle_{q, A} & =\operatorname{tr}\left\langle g \sigma_{\mu \nu} F_{\mu \nu}(x) S_{q}(x, x)\right\rangle_{A} \\
& =\operatorname{tr}\left\langle S_{q}(x, x) g \sigma_{\mu \nu} F_{\mu \nu}(x)\right\rangle, \tag{3}
\end{align*}
$$

where $S_{q}(x, y)$ is the Euclidean quark propagator, for which one can write using the FFSR

$$
\begin{align*}
S_{q}(x, y)= & (m+\hat{D})_{x, y}^{-1}=(m-\hat{D})_{x}\left(m^{2}-\hat{D}^{2}\right)_{x, y}^{-1} \\
= & (m-\hat{D})_{x} \int_{0}^{\infty} d s D z_{x, y} e^{-K} \Phi_{z}(x, y) P_{F} \\
& \times \exp \int_{0}^{s} \lambda(z(\tau)) d \tau . \tag{4}
\end{align*}
$$

In (4) the following notations are used: $K=m^{2} s+$ $\frac{1}{4} \int_{0}^{s} \dot{z}_{\mu}^{2} d \tau, D_{\mu} \equiv \partial_{\mu}-i g A_{\mu},(D z)_{x, y}$ is the pathintegral measure for paths starting at $y$ and ending at
the point $x$,

$$
\begin{aligned}
(D z)_{x, y}= & \lim _{N \rightarrow \infty} \prod_{n=1}^{N}\left(\frac{d^{4} \Delta z(n)}{(4 \pi \varepsilon)^{2}}\right) \frac{d^{4} k}{(2 \pi)^{4}} \\
& \times e^{i k\left(\sum_{n=1}^{n} \Delta z(n)-(x-y)\right)},
\end{aligned}
$$

while $\Phi_{z}(x, y)$ is the phase factor (parallel transporter) along the path $z_{\mu}(\tau) \Phi_{z}(x, y)=P_{A} \exp i g \int_{y}^{x} A_{\mu} d z_{\mu}$, with $P_{A}, P_{F}$-the ordering operators of the matrices $A_{\mu}(z)$ and $\lambda(z)$, where $\lambda(z)$ is defined to be ${ }^{1}$

$$
\begin{equation*}
\lambda(z) \equiv g \sigma_{\mu \nu} F_{\mu \nu}(z), \quad \sigma_{\mu \nu}=\frac{1}{4 i}\left(\gamma_{\mu} \gamma_{\nu}-\gamma_{\nu} \gamma_{\mu}\right) . \tag{5}
\end{equation*}
$$

For what follows it will be advantageous to take in (5) $\lambda(z(\tau))=g(\tau) \sigma_{\mu \nu} F_{\mu \nu}(z(\tau))$, since the functional derivative $\frac{\delta}{\delta g(\tau)}$ at $\tau \rightarrow 0$ or $\tau \rightarrow s$ inside the FFSR (4) brings down additional factor $\lambda(y)$ or $\lambda(x)$. When one has $y=x$, as in (3), then both contributions add, which formally is obtained by putting $g(0)=g(s)$. In this way one can rewrite (3) as follows

$$
\begin{align*}
\langle\bar{q}(x) \lambda(x) q(x)\rangle & =\operatorname{tr}\left\langle\lambda(x) S_{q}(x, x)\right\rangle \\
& =2 \operatorname{tr} \frac{\delta}{\delta g(0)}\left\langle S_{q}(x, x)\right\rangle . \tag{6}
\end{align*}
$$

As the next step one can write the average $\left\langle S_{q}(x, x)\right\rangle$ in the form of cluster expansion [7]

$$
\begin{align*}
& \left\langle S_{q}(x, x)\right\rangle \\
& \quad=(m-i \hat{p}) \int_{0}^{\infty} d s e^{-K}(D z)_{x x} \\
& \quad \times \exp \left\{-\frac{1}{2} \int d v_{\lambda \rho} \int d v_{\sigma \nu}\left\langle g F_{\lambda \rho} g F_{\sigma \nu}\right\rangle\right\}, \tag{7}
\end{align*}
$$

where only the contribution of the lowest cumulant $\langle F F\rangle$ is retained in accordance with estimates [12], and the non-Abelian Stokes theorem is used to express $A_{\mu}$ through $F_{\mu \nu}$, with the notation

$$
\begin{align*}
& d v_{\lambda \rho}=d s_{\lambda \rho}-i \sigma_{\lambda \rho} d \tau \\
& g F_{\lambda \rho} d v_{\lambda \rho}=g F_{\lambda \rho}(u) d s_{\lambda \rho}(u)-i g(\tau) \sigma_{\lambda \rho} F_{\lambda \rho}(z(\tau)) \tag{8}
\end{align*}
$$

[^1]and $d s_{\lambda \rho}$ is the element of the area of the surface enclosed by the contour $z_{\mu}(\tau), z_{\mu}(0)=z_{\mu}(s)=x_{\mu}$. Performing differentiation in (6) one gets
\[

$$
\begin{align*}
\langle\bar{q} \lambda q\rangle= & 2 g^{2} \sigma_{\mu \nu} \sigma_{\lambda \rho} \int_{0}^{\infty} d s(D z)_{x x} e^{-K}(m-i \hat{p}) \\
& \times \int_{0}^{s} d \tau\left\langle F_{\lambda \rho}(u(\tau)) F_{\mu \nu}(x)\right\rangle \\
& \times \exp \left\{-\frac{g^{2}}{2} \int d v_{\lambda \rho} \int d v_{\sigma \nu}\left\langle F_{\lambda \rho} F_{\sigma \nu}\right\rangle\right\} . \tag{9}
\end{align*}
$$
\]

Using the identities [8]
$(D z)_{x x}=(D z)_{x u} d^{4} u(D z)_{u x}$,
$\int_{0}^{\infty} d s \int_{0}^{s} d \tau f(s, \tau)=\int_{0}^{\infty} d s \int_{0}^{\infty} d \tau f(s+\tau, \tau)$,
where $f(s, \tau)$ is an arbitrary function, one has

$$
\begin{align*}
\langle\bar{q} \lambda q\rangle= & 2 \sigma_{\mu \nu} \sigma_{\lambda \rho} \int\left\langle G(x, u) S_{q}(u, x)\right\rangle \\
& \times D_{\lambda \rho, \mu \nu}^{(2)}(u-x) d^{4}(u-x) . \tag{11}
\end{align*}
$$

Here we have defined as in [7]

$$
\begin{align*}
D_{\lambda \rho, \mu \nu}^{(2)}(z) \equiv & \left(\delta_{\lambda \mu} \delta_{\rho \nu}-\delta_{\lambda \nu} \delta_{\rho \mu}\right) D(z) \\
& +\frac{1}{2}\left(\partial_{\lambda} z_{\mu} \delta_{\rho \nu}+\partial_{\rho} z_{\nu} \delta_{\lambda \mu}-\partial_{\lambda} z_{\nu} \delta_{\rho \mu}\right. \\
& \left.\quad-\partial_{\rho} z_{\mu} \delta_{\lambda \nu}\right) D_{1}(z) \tag{12}
\end{align*}
$$

and

$$
\begin{align*}
G(x, u)= & \int_{0}^{\infty} d \tau e^{-K}(D z)_{x u} \\
& \times \exp \left\{-\frac{1}{2} \int d v_{\lambda \rho} \int d v_{\sigma v}\left\langle g F_{\lambda \rho} g F_{\sigma \nu}\right\rangle\right\} . \tag{13}
\end{align*}
$$

Note that $G_{0}(x, u)$ and $S_{q}(u, x)$ share common factors depending on a piece of common $z$ between $u_{\mu}$ and $x_{\mu}$ and in general do not factorize.

At this point we shall use the properties of the correlators $D(z), D_{1}(z)$ found on the lattice [9], in the quenched case one has
$D(z) \cong 3 D_{1}(z)=D(0) \exp (-|z| \delta)$,
$\delta \equiv 1 / T_{g} \approx 1 \mathrm{GeV}$.

Analytic calculations based on the gluelump spectrum [13] suggest even larger value, $\delta \approx 1.4-1.5 \mathrm{GeV}$. The string tension $\sigma$ can be expressed through $D(z)$ (the correction due to higher correlators is limited by the Casimir scaling arguments to a few percent [12])
$\sigma=\frac{1}{2} \int D(z) d^{2} z$.
Since the distance $|u-x|$ is of the order of $T_{g}$, we can now use the argument of the small $T_{g}$ limit (large $\delta$ ) for the constant $\sigma$ to factorize the product $\left\langle G(x, u) S_{q}(u, x)\right\rangle$ as follows
$\lim _{T_{g} \rightarrow 0}\left\langle G(x, u) S_{q}(u, x)\right\rangle \cong G_{0}(x-u)\left\langle S_{q}(x, x)\right\rangle$.
This approximation is equivalent to the expansion in the parameter $\xi \equiv \sigma T_{g}^{2} \ll 1$. As the result one obtains the following representation for the ratio

$$
\begin{align*}
m_{0}^{2} & \equiv 2 \frac{\langle\bar{q} \lambda q\rangle}{\langle\bar{q} q\rangle} \\
& =4 \sigma_{\mu \nu} \sigma_{\lambda \rho} \int G_{0}(z) D_{\lambda \rho, \mu \nu}^{(2)}(z) d^{4} z \tag{17}
\end{align*}
$$

$G_{0}(z)$ is easily calculated using (13) to be the free propagator of the scalar quark with mass $m$,
$G_{0}(z)=\frac{m K_{1}(m|z|)}{4 \pi^{2}|z|}$,
where $K_{1}$ is the McDonald function, and $m$ is the current (pole) quark mass normalized at 1 GeV .

Taking into account that ${ }^{2}$
$\sigma_{\mu \nu} \sigma_{\lambda \rho} D_{\lambda \rho, \mu \nu}^{(2)}(z)=6\left(D(z)+D_{1}(z)\right)$
one obtains for $m_{0}^{2}$
$m_{0}^{2}=12 m \int_{0}^{\infty} z^{2} d z K_{1}(m z)\left(D(z)+D_{1}(z)\right)$
or, with the help of (14),
$m_{0}^{2} \cong 16 m \int_{0}^{\infty} z^{2} d z K_{1}(m z) D(z)=\frac{16 \sigma}{\pi} \varphi(m / \delta)$,

[^2]where we have defined
$\varphi(m / \delta) \equiv m \delta^{2} \int_{0}^{\infty} z^{2} d z K_{1}(m z) \exp (-\delta z)$,
$\varphi(0)=1$.
It is easy to see with the help of (15) that in the limit of small quark mass, $m \rightarrow 0$, one obtains for $\sigma=0.18 \mathrm{GeV}^{2}$ (in the quenched case)
$m_{0}^{2}(m \rightarrow 0)=\frac{16}{\pi} \sigma=0.92 \mathrm{GeV}^{2}$.
It is appropriate at this point to discuss the accuracy of our result (23). The main uncertainty appears in expressions (14)-(16) and we consider the accuracy of the corresponding approximations point by point.

The lattice calculations [9] of $D(z)$ and $D_{1}(z)$ define the amplitudes $A, A_{1}$ and slopes $\delta, \delta_{1}$; the first ones are reabsorbed in the value of $\sigma$, while the latter are equal with accuracy of few (1-2) percent to the value given in (14). The approximation of (15) reduces to the neglect of higher correlators, contributing to the observed string tension $\sigma$. This accuracy was tested in [12] using the Casimir scaling and is also of the order of few percent. The largest possible error may come from the replacement (16), where one can use the fact that the integral over $d^{4}(u-x)$ in (11) is taken with the weight $D^{(2)}(u-x)$. The latter is exponentially decreasing at the distance $1 / \delta$, while the range of $G(x, u)$ is defined by the confining exponent in (13), which produces the effective quark mass, computed through $\sigma$ and equal to 0.35 GeV for the lowest state (see [11] for references and explicit calculations). Introducing this mass instead of $m$ in (18), (20), (21) one obtains $\varphi \approx 0.75-0.8$, and using (21) one comes to the conclusion that $m_{0}^{2}$ is in the range $0.7 \mathrm{GeV}^{2} \lesssim m_{0}^{2} \lesssim 1 \mathrm{GeV}^{2}$. This range lies very close to the limits predicted in the QCD sum rules.

The explicit analytic form of $\varphi(x)$ was obtained in [10] and is given here in Appendix A. For $\delta=$ 1 GeV , and $m=0.175,1.7$ and 5 GeV one obtains respectively $\varphi=0.88,0.234$ and 0.052 .

The resulting value of $m_{0}^{2}(23)$ is in agreement with the QCD sum rule estimates [2], and with the lattice evaluation of $m_{0}^{2}$, namely $m_{0}^{2} \approx 1 \mathrm{GeV}^{2}$ in [5]. One should note, that there is a large perturbative contribution to $m_{0}^{2}$ for nonzero quark mass $m$ proportional
to $m \Lambda_{\mathrm{UV}}^{2} \sim m / a^{2}$, which should be subtracted to get agreement with purely nonperturbative result (23).

On the other hand the purely nonperturbative behavior of $m_{0}^{2}$ as a function of the quark mass $m$, or rather the ratio $t=m / \delta$ is given in Appendix A, Eq. (A.6),
$m_{0}^{2}(t)=\frac{16 \sigma}{\pi}\left(1+t^{2}\left(4-3 \ln \frac{2}{t}\right)+O\left(t^{4}\right)\right)$.
The values $m_{0}^{2}(t)$ obtained from (24) agree well with the lattice measured values in [5] for $m a>0$. Indeed for three values of $m a, m a=0.05 ; 0,1$ and 0.15 one obtains from (24) taking $\sigma=0.18 \mathrm{GeV}^{2}$, and $a^{-1}=$ 1.979 GeV [5], $m_{0}^{2}=0.434,0.393$ and $0.342 \mathrm{GeV}^{2}$, respectively. This should be compared with the values $m_{0}^{2}(m a)$ measured in [5] and equal to $0.371,0.311$ and $0.290 \mathrm{GeV}^{2}$. At the same time the limiting extrapolated value $m_{0}^{2}(m a=0) \approx 1 \mathrm{GeV}^{2}$ obtained in [5], agrees with the theoretical one, given by Eq. (24), $m_{0}^{2}(m a=0$, theory $)=1 \mathrm{GeV}^{2}$. One should have in mind, that chiral quark mass corrections present in both the quark condensate and the QGC are canceled in the ratio $m_{0}^{2}$ to the leading order in $\sigma T_{g}^{2}$, so the remnant $m a$ dependence in $m_{0}^{2}$ comes from quadratic terms in (24) and linear perturbative terms mentioned above.

Recently a study of thermal dependence of $m_{0}^{2}(T)$ has been reported in [14], where $m_{0}^{2}$ was found almost independent of $T$ up to $T=T_{c}$. This is in general agreement with our expression (23), since $\sigma$ is roughly constant in that region, but more detailed check of behavior near $T_{c}$ is desirable.

Summarizing, we have obtained a simple nonperturbative estimate for the ratio of condensates, which is in a reasonable agreement with the QCD sum rule results, and lattice results in [5] for nonzero ma and zero ma limit.

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## Appendix A

The function $\varphi(t), t \equiv m / \delta$, defined in Eq. (22) can be written as (note the difference in definition here and in [10])
$\varphi(t)=t \int_{0}^{\infty} z^{2} d z K_{1}(t z) e^{-z}$
where $K_{1}$ is the McDonald function, $K_{1}(x)(x \rightarrow 0) \approx$ $\frac{1}{x}$, so that for $t=0$ one obtains
$\varphi(0)=1$.
For $t>0$ the integration in (A.1) yields two different forms; e.g., for $t<1$,

$$
\begin{align*}
\varphi(t)= & -\frac{3 t^{2}}{\left(1-t^{2}\right)^{5 / 2}} \ln \frac{1+\sqrt{1-t^{2}}}{t} \\
& +\frac{1+2 t^{2}}{\left(1-t^{2}\right)^{2}} \tag{A.3}
\end{align*}
$$

while for $t>1$ one has instead,

$$
\begin{align*}
\varphi(t)= & -\frac{3 t^{2}}{\left(t^{2}-1\right)^{5 / 2}} \arctan \left(\sqrt{t^{2}-1}\right) \\
& +\frac{1+2 t^{2}}{\left(1-t^{2}\right)^{2}} . \tag{A.4}
\end{align*}
$$

For large $t$ one has the following limiting behavior,
$\varphi(t)=\frac{2}{t^{2}}-\frac{3 \pi}{2 t^{3}}+O\left(\frac{1}{t^{4}}\right)$.
For small $t$ one obtains expanding the r.h.s. of (A.3)

$$
\begin{align*}
\varphi(t)= & 1+t^{2}\left(4-3 \ln \frac{2}{t}\right)+t^{4}\left(\frac{7}{4}-\frac{15}{2} \ln \frac{2}{t}\right) \\
& +O\left(t^{6}\right) . \tag{A.6}
\end{align*}
$$

Some numerical values are useful in applications.
$\varphi(0.175) \cong 0.88, \quad \varphi(1.7) \cong 0.234$,
$\varphi(5) \cong 0.052$.

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[^1]:    ${ }^{1}$ The definition of $\sigma_{\mu \nu}$ in (5) (as well as in [7,8]) differs from the standard definition in QCD sum rules, where enters $\frac{1}{2}$ instead of $\frac{1}{4}$ in (5). Therefore one obtains additional factor 2 in the definition of $m_{0}^{2}$ in (17).

[^2]:    ${ }^{2}$ Note the misprint in Eq. (15) of [10], where coefficients of $D, D_{1}$ differ from those in (19). Nevertheless the final result in Eq. (29) of [10] is the same as in our Eq. (1) due to the relation $D_{1} \approx \frac{1}{3} D[9]$ valid for the quenched case, considered here, whereas in the unquenched case one obtains instead of (1): $\Delta m_{q}^{2}(m \rightarrow 0)=$ $-3 \int_{0}^{\infty} z d z\left(D+D_{1}\right) \cong-\frac{3}{\pi} \sigma$.

