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The quark-gluon mixed condensate calculated via field correlators

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Abstract

The quark–gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}F_{\mu\nu}q\rangle$ is calculated in the Gaussian approximation of the field correlator method. In the large N_c limit and for zero mass quarks one obtains a simple result, $m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}F_{\mu\nu}q\rangle/\langle \bar{q}q\rangle = \frac{16\sigma}{\pi}$, where σ is the string tension. For a standard value $\sigma = 0.18 \text{ GeV}^2$ one obtains $m_0^2 = 1 \text{ GeV}^2$ in good agreement with the QCD sum rules estimate $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ and the latest lattice result $m_0^2 \cong 1 \text{ GeV}^2$. © 2004 Published by Elsevier B.V. Open access under CC BY license.

1. Introduction

The mixed quark–gluon condensate (QGC) is an important characteristics of the nonperturbative QCD vacuum, which together with the quark condensate $\langle \bar{q}q \rangle$ signals the chiral symmetry breaking. Moreover, the QGC measures the average interaction of the quark color-magnetic moment with the vacuum fields, which is an important ingredient of the quark dynamics in the vacuum (e.g., it is this term which gives attraction of in quark zero modes).

In the QCD sum rules the QGC plays an important role [1] and the phenomenological analysis suggests the value of m_0^2 in the range $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [1], see [2] for a review. One should stress at this point that for a nonzero quark mass *m* the (diverging) perturbative part should be subtracted.

As will be seen below the resulting nonperturbative dependence of m_0^2 on *m* is very weak in agreement with lattice data. Lattice studies of QGC [3–5] have not yet converged to a definite prediction. A problem there is the extrapolation to zero quark mass and the quenched approximation. In Ref. [4] the simulations are done in the quenched approximation, the condensate is measured by use of staggered quarks, and the result for m_0^2 is definitely larger than the sum-rules value. Ref. [5] uses an optimized version [6] of domain wall fermions, which are better in principle for the chiral limit, again in the quenched approximation. Their result is $m_0^2 = 1 \text{ GeV}^2$, which agrees with QCD sum rules. It is therefore worthwhile to calculate QGC by a different nonperturbative method.

In the framework of the field correlator method (FCM) [7] the color-magnetic quark–gluon interaction term $g\sigma_{\mu\nu}F_{\mu\nu}$ enters essentially in the Fock– Feynman–Schwinger representation (FFSR) of the quark propagator in the vacuum background field [8].

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In particular the quadratic average of this term defines the hyperfine $q\bar{q}$ interaction where the nonperturbative part is proportional to the field correlator $\langle F_{\mu\nu}(x)F_{\rho\sigma}(0)\rangle$ measured on the lattice [9]. Even more important the term $g\sigma_{\mu\nu}F_{\mu\nu}$ is in the contribution to the bound quark self-energy [10], where it is of paramagnetic character, i.e., negative and strongly decreases the masses of hadrons, putting them in accordance with experimental data [11]. Explicit correction to the bound quark mass squared is [10]

$$\Delta m_q^2 = -\frac{4\sigma}{\pi}\eta,\tag{1}$$

where $\eta = \eta(mT_g)$ is a calculable function of the quark current mass m, renormalized at the scale of 1 GeV. The function η is given in [10] and in Appendix A and for zero quark mass is normalized to one: $\eta(0) = 1$. We calculate in the next section the QGC, or rather the parameter m_0^2 in the same way, as it was done in [10] for Δm_q^2 , with the result

$$m_0^2 = -4\Delta m_q^2 = \frac{16\sigma}{\pi}\eta.$$
 (2)

For $\sigma = 0.18 \text{ GeV}^2$ one obtains $m_0^2 = 1 \text{ GeV}^2$ which is in agreement with the lattice data [5], and with the QCD sum rules estimate quoted above.

2. Calculation of m_0^2

We proceed in the Euclidean space-time and write

$$\langle \bar{q} g \sigma_{\mu\nu} F_{\mu\nu} q \rangle_{q,A} = \operatorname{tr} \langle g \sigma_{\mu\nu} F_{\mu\nu} (x) S_q (x, x) \rangle_A$$

= $\operatorname{tr} \langle S_q (x, x) g \sigma_{\mu\nu} F_{\mu\nu} (x) \rangle,$ (3)

where $S_q(x, y)$ is the Euclidean quark propagator, for which one can write using the FFSR

$$S_{q}(x, y) = (m + \hat{D})_{x,y}^{-1} = (m - \hat{D})_{x} (m^{2} - \hat{D}^{2})_{x,y}^{-1}$$

$$= (m - \hat{D})_{x} \int_{0}^{\infty} ds \, Dz_{x,y} e^{-K} \Phi_{z}(x, y) P_{F}$$

$$\times \exp \int_{0}^{s} \lambda(z(\tau)) d\tau.$$
(4)

In (4) the following notations are used: $K = m^2 s + \frac{1}{4} \int_0^s \dot{z}_{\mu}^2 d\tau$, $D_{\mu} \equiv \partial_{\mu} - igA_{\mu}$, $(Dz)_{x,y}$ is the pathintegral measure for paths starting at y and ending at the point x,

$$(Dz)_{x,y} = \lim_{N \to \infty} \prod_{n=1}^{N} \left(\frac{d^4 \Delta z(n)}{(4\pi\varepsilon)^2} \right) \frac{d^4k}{(2\pi)^4} \\ \times e^{ik(\sum_{n=1}^{n} \Delta z(n) - (x-y))},$$

while $\Phi_z(x, y)$ is the phase factor (parallel transporter) along the path $z_\mu(\tau) \Phi_z(x, y) = P_A \exp ig \int_y^x A_\mu dz_\mu$, with P_A , P_F —the ordering operators of the matrices $A_\mu(z)$ and $\lambda(z)$, where $\lambda(z)$ is defined to be¹

$$\lambda(z) \equiv g\sigma_{\mu\nu}F_{\mu\nu}(z), \qquad \sigma_{\mu\nu} = \frac{1}{4i}(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu}).$$
(5)

For what follows it will be advantageous to take in (5) $\lambda(z(\tau)) = g(\tau)\sigma_{\mu\nu}F_{\mu\nu}(z(\tau))$, since the functional derivative $\frac{\delta}{\delta g(\tau)}$ at $\tau \to 0$ or $\tau \to s$ inside the FFSR (4) brings down additional factor $\lambda(y)$ or $\lambda(x)$. When one has y = x, as in (3), then both contributions add, which formally is obtained by putting g(0) = g(s). In this way one can rewrite (3) as follows

$$\langle \bar{q}(x)\lambda(x)q(x) \rangle = \operatorname{tr} \langle \lambda(x)S_q(x,x) \rangle$$

$$= 2 \operatorname{tr} \frac{\delta}{\delta g(0)} \langle S_q(x,x) \rangle.$$
(6)

As the next step one can write the average $\langle S_q(x, x) \rangle$ in the form of cluster expansion [7]

$$S_{q}(x,x) \rangle = (m-i\hat{p}) \int_{0}^{\infty} ds \, e^{-K} (Dz)_{xx} \\ \times \exp\left\{-\frac{1}{2} \int dv_{\lambda\rho} \int dv_{\sigma\nu} \, \langle gF_{\lambda\rho}gF_{\sigma\nu} \rangle\right\}, \quad (7)$$

where only the contribution of the lowest cumulant $\langle FF \rangle$ is retained in accordance with estimates [12], and the non-Abelian Stokes theorem is used to express A_{μ} through $F_{\mu\nu}$, with the notation

$$dv_{\lambda\rho} = ds_{\lambda\rho} - i\sigma_{\lambda\rho} d\tau,$$

$$gF_{\lambda\rho}dv_{\lambda\rho} = gF_{\lambda\rho}(u) ds_{\lambda\rho}(u) - ig(\tau)\sigma_{\lambda\rho}F_{\lambda\rho}(z(\tau))$$
(8)

¹ The definition of $\sigma_{\mu\nu}$ in (5) (as well as in [7,8]) differs from the standard definition in QCD sum rules, where enters $\frac{1}{2}$ instead of $\frac{1}{4}$ in (5). Therefore one obtains additional factor 2 in the definition of m_0^2 in (17).

and $ds_{\lambda\rho}$ is the element of the area of the surface enclosed by the contour $z_{\mu}(\tau), z_{\mu}(0) = z_{\mu}(s) = x_{\mu}$. Performing differentiation in (6) one gets

$$\begin{split} \langle \bar{q}\lambda q \rangle &= 2g^2 \sigma_{\mu\nu} \sigma_{\lambda\rho} \int\limits_{0}^{\infty} ds \, (Dz)_{xx} e^{-K} (m-i\,\hat{p}) \\ &\times \int\limits_{0}^{s} d\tau \left\langle F_{\lambda\rho} \big(u(\tau) \big) F_{\mu\nu}(x) \right\rangle \\ &\times \exp\left\{ -\frac{g^2}{2} \int dv_{\lambda\rho} \int dv_{\sigma\nu} \left\langle F_{\lambda\rho} F_{\sigma\nu} \right\rangle \right\}. \end{split}$$

Using the identities [8]

$$(Dz)_{xx} = (Dz)_{xu} d^4 u (Dz)_{ux},$$

$$\int_0^{\infty} ds \int_0^s d\tau f(s,\tau) = \int_0^{\infty} ds \int_0^{\infty} d\tau f(s+\tau,\tau), \quad (10)$$

where $f(s, \tau)$ is an arbitrary function, one has

$$\langle \bar{q}\lambda q \rangle = 2\sigma_{\mu\nu}\sigma_{\lambda\rho} \int \langle G(x,u)S_q(u,x) \rangle$$
$$\times D^{(2)}_{\lambda\rho,\mu\nu}(u-x) d^4(u-x). \tag{11}$$

Here we have defined as in [7]

$$D^{(2)}_{\lambda\rho,\mu\nu}(z) \equiv (\delta_{\lambda\mu}\delta_{\rho\nu} - \delta_{\lambda\nu}\delta_{\rho\mu})D(z) + \frac{1}{2}(\partial_{\lambda}z_{\mu}\delta_{\rho\nu} + \partial_{\rho}z_{\nu}\delta_{\lambda\mu} - \partial_{\lambda}z_{\nu}\delta_{\rho\mu} - \partial_{\rho}z_{\mu}\delta_{\lambda\nu})D_{1}(z)$$
(12)

and

$$G(x, u) = \int_{0}^{\infty} d\tau \, e^{-K} (Dz)_{xu} \\ \times \exp\left\{-\frac{1}{2} \int dv_{\lambda\rho} \int dv_{\sigma\nu} \, \langle gF_{\lambda\rho} gF_{\sigma\nu} \rangle\right\}.$$
(13)

Note that $G_0(x, u)$ and $S_q(u, x)$ share common factors depending on a piece of common *z* between u_μ and x_μ and in general do not factorize.

At this point we shall use the properties of the correlators D(z), $D_1(z)$ found on the lattice [9], in the quenched case one has

$$D(z) \cong 3D_1(z) = D(0) \exp(-|z|\delta),$$

$$\delta \equiv 1/T_g \approx 1 \text{ GeV}.$$
(14)

Analytic calculations based on the gluelump spectrum [13] suggest even larger value, $\delta \approx 1.4-1.5$ GeV. The string tension σ can be expressed through D(z)(the correction due to higher correlators is limited by the Casimir scaling arguments to a few percent [12])

$$\sigma = \frac{1}{2} \int D(z) d^2 z. \tag{15}$$

Since the distance |u - x| is of the order of T_g , we can now use the argument of the small T_g limit (large δ) for the constant σ to factorize the product $\langle G(x, u)S_q(u, x) \rangle$ as follows

$$\lim_{T_g \to 0} \langle G(x, u) S_q(u, x) \rangle \cong G_0(x - u) \langle S_q(x, x) \rangle.$$
(16)

This approximation is equivalent to the expansion in the parameter $\xi \equiv \sigma T_g^2 \ll 1$. As the result one obtains the following representation for the ratio

$$m_0^2 \equiv 2 \frac{\langle q \lambda q \rangle}{\langle \bar{q} q \rangle} = 4 \sigma_{\mu\nu} \sigma_{\lambda\rho} \int G_0(z) D_{\lambda\rho,\mu\nu}^{(2)}(z) d^4 z, \qquad (17)$$

 $G_0(z)$ is easily calculated using (13) to be the free propagator of the scalar quark with mass *m*,

$$G_0(z) = \frac{mK_1(m|z|)}{4\pi^2|z|},$$
(18)

where K_1 is the McDonald function, and *m* is the current (pole) quark mass normalized at 1 GeV.

Taking into account that²

$$\sigma_{\mu\nu}\sigma_{\lambda\rho}D^{(2)}_{\lambda\rho,\mu\nu}(z) = 6(D(z) + D_1(z))$$
(19)

one obtains for m_0^2

$$m_0^2 = 12m \int_0^\infty z^2 dz \, K_1(mz) \big(D(z) + D_1(z) \big) \tag{20}$$

or, with the help of (14),

$$m_0^2 \cong 16m \int_0^\infty z^2 dz K_1(mz) D(z) = \frac{16\sigma}{\pi} \varphi(m/\delta),$$
(21)

² Note the misprint in Eq. (15) of [10], where coefficients of D, D_1 differ from those in (19). Nevertheless the final result in Eq. (29) of [10] is the same as in our Eq. (1) due to the relation $D_1 \approx \frac{1}{3}D$ [9] valid for the quenched case, considered here, whereas in the unquenched case one obtains instead of (1): $\Delta m_q^2 (m \to 0) = -3 \int_0^\infty z \, dz \, (D+D_1) \cong -\frac{3}{\pi} \sigma$.

where we have defined

$$\varphi(m/\delta) \equiv m\delta^2 \int_0^\infty z^2 dz \, K_1(mz) \exp(-\delta z),$$

$$\varphi(0) = 1.$$
(22)

It is easy to see with the help of (15) that in the limit of small quark mass, $m \rightarrow 0$, one obtains for $\sigma = 0.18 \text{ GeV}^2$ (in the quenched case)

$$m_0^2(m \to 0) = \frac{16}{\pi}\sigma = 0.92 \text{ GeV}^2.$$
 (23)

It is appropriate at this point to discuss the accuracy of our result (23). The main uncertainty appears in expressions (14)–(16) and we consider the accuracy of the corresponding approximations point by point.

The lattice calculations [9] of D(z) and $D_1(z)$ define the amplitudes A, A_1 and slopes δ, δ_1 ; the first ones are reabsorbed in the value of σ , while the latter are equal with accuracy of few (1-2) percent to the value given in (14). The approximation of (15) reduces to the neglect of higher correlators, contributing to the observed string tension σ . This accuracy was tested in [12] using the Casimir scaling and is also of the order of few percent. The largest possible error may come from the replacement (16), where one can use the fact that the integral over $d^4(u-x)$ in (11) is taken with the weight $D^{(\bar{2})}(u-x)$. The latter is exponentially decreasing at the distance $1/\delta$, while the range of G(x, u) is defined by the confining exponent in (13), which produces the effective quark mass, computed through σ and equal to 0.35 GeV for the lowest state (see [11] for references and explicit calculations). Introducing this mass instead of m in (18), (20), (21) one obtains $\varphi \approx 0.75$ –0.8, and using (21) one comes to the conclusion that m_0^2 is in the range $0.7 \text{ GeV}^2 \lesssim m_0^2 \lesssim 1 \text{ GeV}^2$. This range lies very close to the limits predicted in the QCD sum rules.

The explicit analytic form of $\varphi(x)$ was obtained in [10] and is given here in Appendix A. For $\delta =$ 1 GeV, and m = 0.175, 1.7 and 5 GeV one obtains respectively $\varphi = 0.88$, 0.234 and 0.052.

The resulting value of m_0^2 (23) is in agreement with the QCD sum rule estimates [2], and with the lattice evaluation of m_0^2 , namely $m_0^2 \approx 1 \text{ GeV}^2$ in [5]. One should note, that there is a large perturbative contribution to m_0^2 for nonzero quark mass *m* proportional to $m\Lambda_{\rm UV}^2 \sim m/a^2$, which should be subtracted to get agreement with purely nonperturbative result (23).

On the other hand the purely nonperturbative behavior of m_0^2 as a function of the quark mass m, or rather the ratio $t = m/\delta$ is given in Appendix A, Eq. (A.6),

$$m_0^2(t) = \frac{16\sigma}{\pi} \left(1 + t^2 \left(4 - 3\ln\frac{2}{t} \right) + O\left(t^4\right) \right).$$
(24)

The values $m_0^2(t)$ obtained from (24) agree well with the lattice measured values in [5] for ma > 0. Indeed for three values of ma, ma = 0.05; 0, 1 and 0.15 one obtains from (24) taking $\sigma = 0.18 \text{ GeV}^2$, and $a^{-1} =$ 1.979 GeV [5], $m_0^2 = 0.434, 0.393$ and 0.342 GeV², respectively. This should be compared with the values $m_0^2(ma)$ measured in [5] and equal to 0.371, 0.311 and 0.290 GeV^2 . At the same time the limiting extrapolated value $m_0^2(ma = 0) \approx 1 \text{ GeV}^2$ obtained in [5], agrees with the theoretical one, given by Eq. (24), $m_0^2(ma = 0, \text{ theory}) = 1 \text{ GeV}^2$. One should have in mind, that chiral quark mass corrections present in both the quark condensate and the OGC are canceled in the ratio m_0^2 to the leading order in σT_g^2 , so the remnant ma dependence in m_0^2 comes from quadratic terms in (24) and linear perturbative terms mentioned above.

Recently a study of thermal dependence of $m_0^2(T)$ has been reported in [14], where m_0^2 was found almost independent of T up to $T = T_c$. This is in general agreement with our expression (23), since σ is roughly constant in that region, but more detailed check of behavior near T_c is desirable.

Summarizing, we have obtained a simple nonperturbative estimate for the ratio of condensates, which is in a reasonable agreement with the QCD sum rule results, and lattice results in [5] for nonzero *ma* and zero *ma* limit.

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Appendix A

The function $\varphi(t)$, $t \equiv m/\delta$, defined in Eq. (22) can be written as (note the difference in definition here and in [10])

$$\varphi(t) = t \int_{0}^{\infty} z^2 dz \, K_1(tz) e^{-z}$$
(A.1)

where K_1 is the McDonald function, $K_1(x)(x \to 0) \approx \frac{1}{x}$, so that for t = 0 one obtains

$$\varphi(0) = 1. \tag{A.2}$$

For t > 0 the integration in (A.1) yields two different forms; e.g., for t < 1,

$$\varphi(t) = -\frac{3t^2}{(1-t^2)^{5/2}} \ln \frac{1+\sqrt{1-t^2}}{t} + \frac{1+2t^2}{(1-t^2)^2}$$
(A.3)

while for t > 1 one has instead,

$$\varphi(t) = -\frac{3t^2}{(t^2 - 1)^{5/2}} \arctan\left(\sqrt{t^2 - 1}\right) + \frac{1 + 2t^2}{(1 - t^2)^2}.$$
(A.4)

For large *t* one has the following limiting behavior,

$$\varphi(t) = \frac{2}{t^2} - \frac{3\pi}{2t^3} + O\left(\frac{1}{t^4}\right).$$
 (A.5)

For small t one obtains expanding the r.h.s. of (A.3)

$$\varphi(t) = 1 + t^2 \left(4 - 3\ln\frac{2}{t} \right) + t^4 \left(\frac{7}{4} - \frac{15}{2}\ln\frac{2}{t} \right) + O\left(t^6\right).$$
(A.6)

Some numerical values are useful in applications.

$$\varphi(0.175) \cong 0.88, \qquad \varphi(1.7) \cong 0.234,$$

 $\varphi(5) \cong 0.052.$

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