SOME EMPIRICAL RELATIONS BETWEEN TRAVEL SPEED, TRAFFIC VOLUME AND TRAFFIC COMPOSITION IN URBAN ARTERIALS

Eleni I. VLAHOGIANNI, Ph.D.
Institute of Transportation Studies
University of California, Berkeley
California, USA

(Received November 6, 2006)

The effects of traffic mix (the percentage of cars, trucks, buses and so on) are of particular interest in the speed-volume relationship in urban signalized arterials under various geometric and control characteristics. The paper presents some empirical observations on the relation between travel speed, traffic volume and traffic composition in urban signalized arterials. A methodology based on emerging self-organizing structures of neural networks to identify regions in the speed-volume relationship with respect to traffic composition and Bayesian networks to evaluate the effect of different types of motorized vehicles on prevailing traffic conditions is proposed. Results based on data from a large urban network indicate that the variability in traffic conditions can be described by eight regions in speed-volume relationship with respect to traffic composition. Further evaluation of the effect of motorized vehicles in each region separately indicates that the effect of traffic composition decreases with the onset of congestion. Moreover, taxis and motorcycles are the primary affecting parameter of the form of the speed-volume relationship in urban arterials.

Key Words: Speed-volume relationship, Urban signalized arterials, Traffic composition, Emergent self-organizing maps, Bayesian networks

1. INTRODUCTION AND OBJECTIVES

Mathematical relationships between speed and volume measurements are considered fundamental to traffic operations. Their importance lies in speed being an essential indicator of freeway service quality; it is used as a congestion or mobility indicator in freeways and it is a parameter of primary importance in advanced traveler information systems. Moreover, the functional form of the speed-volume relationship provides valuable information regarding the onset of congestion and incidents[1].

Most efforts in constructing mathematical relationships between speed and volume measurements are applicable to uninterrupted flow conditions (particularly in freeways)[2,3]. From a methodological standpoint, starting with the Greenshields model[4], various models and approaches have been developed to approximate the functional form of the speed-volume relationship; among them are the model-based[5-10], macroscopic through curve fitting[11,12], regression[1] and simulation approaches[13,14]. Regardless of the method used or the mathematical function utilized, the literature agrees on certain findings; speed-volume relationship has the free-flow and congested branches and a middle area where sudden speed changes occur[15-17]. Although the discontinuous form of the speed-volume relationship is widely supported, the critical points at which speed changes suddenly or transitions to congestion occur have not been fully resolved[1,7,16]. This is probably due to the difficulty in establishing relationships between speed and volume that can account for traffic's spatiotemporal evolution, geometric and environmental condition variability[11,18].

The above comment is critical in urban arterials; the speed-volume relationship can be described as complex because of the mixed traffic conditions and transitional nature of the functional relationship that governs most traffic variables (volume, speed, occupancy and so on); This is, at least in part due to the effect of traffic composition that is the mix of passenger cars, trucks, buses and so on[19]. Traffic flow in urban signalized arterials is characterized by a high degree of manoeuvrability and lane changing, a result of the interaction of fast-moving and slow moving vehicles[14]. In urban areas, mixed traffic is also affected by substantial pedestrian movement, violation at intersections, taxi circulation, on-street parking and narrow roads. This heterogeneity of traffic flow imposed by traffic's composition is recognized in various fields of urban traffic operations particularly near urban signalized intersections (for example saturation flow estimation, delay at signalized intersections and so on)[14, 20-22]. These approaches are related to specific geometric and control conditions. Moreover, in most of these studies, only the effect of trucks in traffic flow is considered[10,23]. The combined effect of traffic's composition in urban traffic flow has not been treated in the literature.

The present paper utilizes a data-driven approach to evaluate the contribution of different types of motorized vehicles to the various traffic conditions as described by the speed-volume relationship. We present a framework based on emergent self-organization and Bayesian learning to identify regions in the speed-volume relationship in urban arterials under mixed traffic and evaluate the effect of traffic composition in each of the regions separately. The remainder of the paper is organized as follows: in the next section the proposed methodological framework is concisely presented. Then, the implementation concerning data and algorithmic calibration issues and the results are presented. The final section summarizes the findings of the paper and offers some concluding remarks.

2. METHODOLOGY

The proposed methodology has two goals; first to identify prevailing regions on the speed-volume relationship related to traffic composition and, second, to evaluate the influence of traffic composition on each revealed speed-volume region. The first step is achieved through a neural network approach that aims at revealing the emergent structures in complex traffic datasets by self-organization. The final step is based on a probabilistic Bayesian network. The next two sections will present the conceptual and mathematical aspects of the proposed methodological framework.

2.1 Relationships between traffic variables by emergence

The problem of relating speed-volume relationship to traffic composition is complex due, first, to speed and volume's spatial and temporal variability and, second, because traffic composition is time dependent and inhomogeneously distributed in an urban network[17,20,24-26]. One of the most robust approaches to revealing complex structures in multidimentional data is the Kohonen Self-Organizing Map (SOM). Given an input dataset $\{x_1, x_2, \ldots, x_d\}$ with $x_i \in D$, the SOM training generates a set of prototype vectors (output neurons) $M = \{n_1, n_2, \ldots, x_k\}$ (with associated weigh vectors $W = \{w_1, w_2, \ldots, w_k\}$) representative of the original data and carries out a topology preserving projection of the prototypes from the high-dimensional input space usually onto a two-dimensional grid[27]. Each data point x_i is mapped to its best-match neuron $bm(x_i) = n_b \in M$ such that $d(x, w_b) \leq d(x, w_j) \forall w_j \in$ W, where d is the distance on the data set. Each output neuron n_i has a set N_i of neighboring neurons on the map. This ordered grid can be used as a convenient visualization surface for showing different features of the SOM (and thus of the data), for example, the cluster structure[27]. Apart from the common approach to utilizing a number of output neurons that is significantly smaller that the original data, the present paper implements the Emergent SOM (ESOM) that utilizes a larger number of neurons than original data to replicate the emergent behavior of traffic flow. Such use of ESOM has been proven to disentangle cluster structures that are linear not separable[28].

The emergent structures characterizing speed, volume and traffic composition can be observed by accounting for the distance of each output neuron from its intermediate neighbors in a map called U-matrix[29]. The U-matrix is the graphical representation of the average distance $uh(i)$ of n_i's weight vectors to the weight vectors of its immediate neighbors N_i[30]:

$$uh(i) = \frac{1}{n}\sum_j d(w_i, w_j),\ j \in N(i),\ n = |N(i)| \quad (1)$$

The resulting map is a "landscape" where weight vectors of neurons with large $uh(i)$ are distant from other vectors in the data space and weight vectors of neurons with small $uh(i)$ are surrounded by other vectors in the data space. The resulting topology can be used to visualize the data structure such as best-matching neurons (placed at depressions), outliers (at funnels), cluster centers (valleys) end so on. The above graphical representation of the emergent structures observed in multi-dimensional datasets are visualizations of clusters that include specific knowledge regarding the relationships that govern the data variables under study. Based on these structures, algorithms have been developed that produce the desired clusters; a simple and efficient way to discover clusters in an emergent SOM is to segment the resulting map by utilizing image processing techniques such as a area-filling algorithm in the U-matrix as discussed[31,32].

2.2 Augmented Naïve Bayesian Network Classifier

By acquiring a set of clusters/ regions of speed-volume relationship that are influenced by traffic composition, it is possible to construct a *classifier*, meaning a function that, given a set of features (for example speed, volume, traffic composition), can assign data to a specific *class*. This paper implements a probabilistic classifier with demonstrated simplicity and predictive performance that belongs to the category of Naïve Bayesian Classifiers. A *Naïve Bayesian Classifier* learns the conditional probability of each feature A_i given the class C; Classification is conducted by predicting the class c_k that maximizes $P(C = c_k | \mathbf{A})$, for a data vector $\mathbf{A}$, under the restrictive assumption that each feature A_i is conditionally independent of every other feature given the class membership[33]:

$$P\left(\mathbf{A} \middle| C = c_k\right) = \prod_i P\left(A_i \middle| C = c_k\right) \quad \text{..............................(2)}$$

In order to overcome this unfeasible assumption, the naïve bayes classifier is implemented in a Bayesian network framework; A Bayesian network $B = \langle N, A, \boldsymbol{\theta} \rangle$ is a directed acyclic graph $\langle N, A \rangle$ where each node $n \in N$ represents a random variable (a dataset feature for example), and each arc $a \in A$ between nodes represents a probabilistic dependency, quantified using a conditional probability distribution $\theta_i \in \boldsymbol{\theta}$ for each node n_i[34]. The lack of an arc between random variables denotes independency while a node in the network for a variable A_i represents the probability of A_i conditioned on the variables that are immediate parents of A_i, denoted $\prod(A_i)$. Nodes with no parents represent the prior probability for that variable. In a network of the above type the independence assumption can be overcome by allowing the network to develop relations between features; an arc from A_i to A_j implies that the influence of A_i on the assessment of the class variable also depends on the value of A_j[35]. These structures called *Augmented Naïve Bayesian Networks* have increased computational time regarding training, but, provide more feasible probabilistic structure among data features.

To evaluate the contribution of each feature A_i to the class membership the conditional mutual information is used that is defined by[35]:

$$I(\mathbf{X}; \mathbf{Y}) = \sum_{A_i, A_j} P(x, y) \log \frac{P(x, y)}{P(x) P(y)} \quad \text{........................(3)}$$

The Equation (3) accounts for the information that $\mathbf{Y}$ provide to $\mathbf{X}$.

3. IMPLEMENTATION AND EMPIRICAL FINDINGS

The field data inputs required for the model are: (a) traffic speed, (b) traffic volume, (c) traffic composition. In order to collect such data, an extensive survey was conducted in the center of Patras, the second major port in Greece (Fig. 1). The road network under study consists of 1900 arterial links of various geometric and control characteristics. Floating car runs were used to collect travel speed measurements and counters were utilized to collect volume and traffic composition data. The resulting dataset consists of volume (pcu/h) and travel speed (km/h) measurements, as well as information on the percent of different types of motorized vehicles that empirically are said to affect urban traffic flow. These are: passenger cars, trucks, buses, motorcycles and taxis.

3.1 Regions of travel speed-volume relationship

The ESOM architecture implemented to reveal regions in speed-volume relationship relating to traffic composition is sensitive both to its structural characteristics (for example the extent of the map), as well as to the learning process. Regarding structural issues, a boundless toroid grid with 4,100 neurons (with a ratio of rows and columns different from unity in order to avoid border

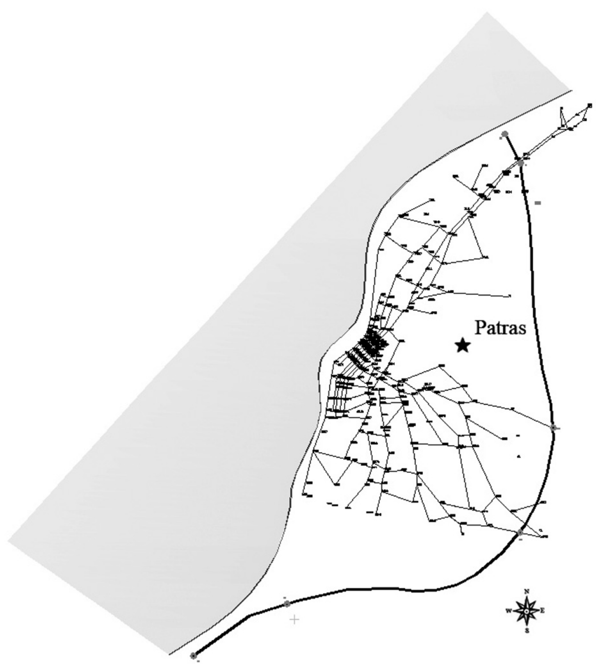

Fig. 1 A graphical representation of the arterial network under study

effects and topology errors) is implemented; literature indicates that the above structural specifications are adequate for the ESOM to generate valid results[30]. Moreover, the learning algorithm implemented is a combination of an online and a batch learning algorithm: online learning exhibits better performance with respect to data representation than batch learning, but is time consuming. For this, the *k*-batch learning algorithm that provides the same quality of representation as online learning but maintains several important properties of batch learning is implemented[36]. The concept is to update the map structure after processing $k < n$, where n is the number of data points, instead of only once per learning cycle as done in batch learning (literature indicates the use of 15% of learning data as a good choice for k). Figure 2 depicts the U-matrix of the trained ESOM that is further clustered to produce the characteristic regions of speed-volume relationship with regards to traffic composition.

Results, presented in Table 1 and Figure 3, indicate the existence of eight different speed-volume regions in the urban arterial network under study. Interestingly, there are three major traffic patterns regarding the manner in which travel speed and volume evolve; the first pattern reflects free-flow conditions where travel speed exhibits a weak oscillating behavior around a mean value, but volume exhibits stronger variability. This pattern is described by four different regions in the speed-volume relationship where volume is kept below 250 pcu/h for a mean speed of 30 km/h and 40 km/h (Regions I and II) and below 1,850 pcu/h for mean speeds of 50 km/h and 70 km/h (Regions III and IV). The second pattern suggests the opposite traffic behavior; volume oscillates less around mean values while speed varies greatly. This behavior is described by three regions with volume exhibiting stability at 400 pcu/h, 500 pcu/h and 650 pcu/h respectively (Regions V to VII). The second pattern prob-

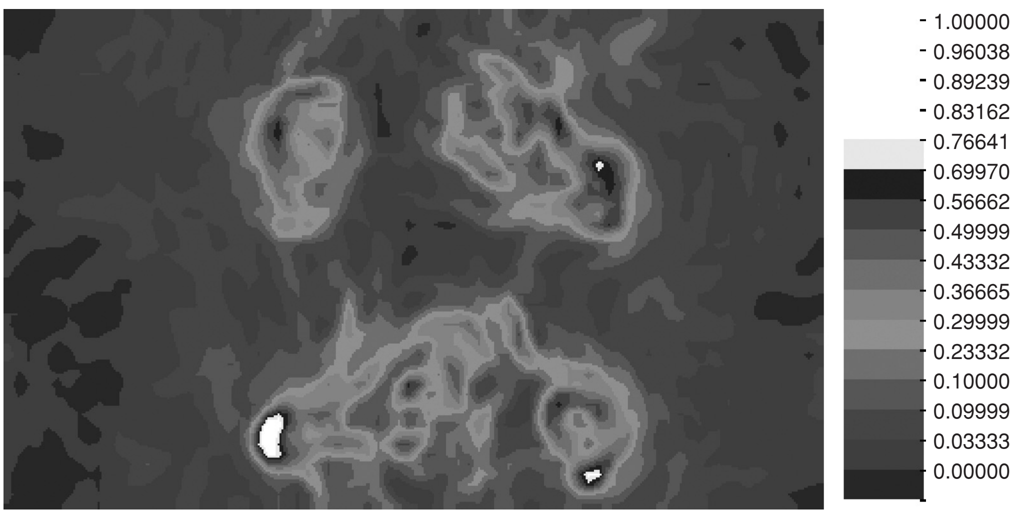

Fig. 2 U-matrix of ESOM based on speed, volume and traffic composition measurements. The scale of distances between the prototypes in the map is seen to the right: low to high distances is equivalent to dark to light color.

Table 1 Regions of characteristic traffic behavior in the speed-volume relationship

	Characteristic	Travel Speed (km/h)	Volume (pcu/h)
Region I	*Stability in Travel Speed*	30	<250
Region II		40	<250
Region III		50	<1,818
Region IV		70	<1,830
Region V	*Stability in Volume*	<35	400
Region VI		<45	500
Region VII		<60	650
Region VIII	*Low Speed and Volume*	<15	<250

ably describes the areas of abrupt speed changes[1,5]. The last pattern is when speed takes on low values and probably depicts the congested branch of speed-volume relationship (Region VIII).

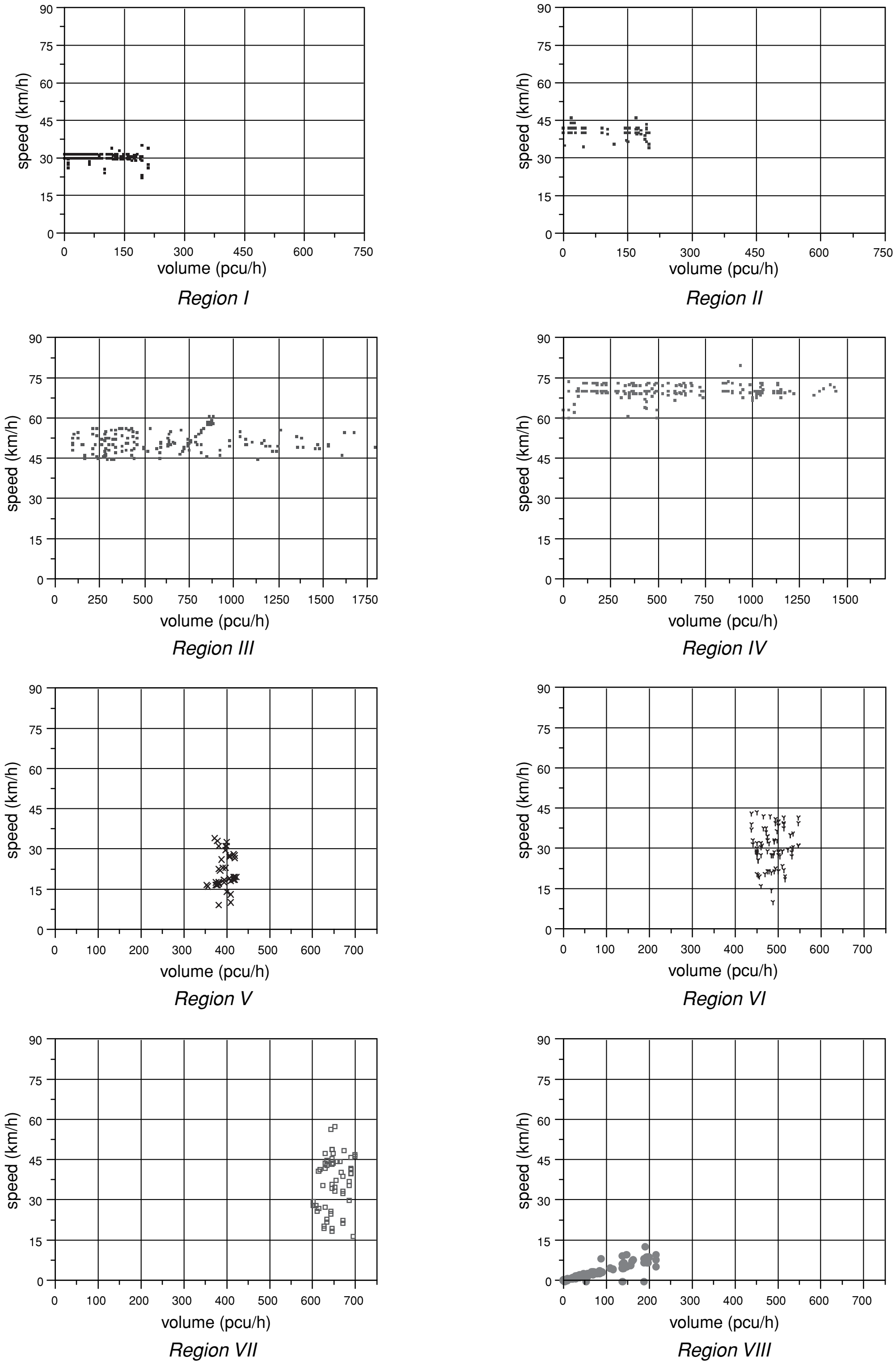

Fig. 3 Graphical representation of regions of speed-volume relationship

3.2 Effect of traffic composition on the travel speed and volume relationship

In order to assess on the contribution of traffic composition to each resulting region in speed-volume relationship, an augmented naïve Bayesian network is developed. The seven available variables describing traffic conditions (Travel Speed, Volume, Passenger Cars(%), Motorcycles(%), Buses(%), Trucks(%) and Taxis(%)), as well as the produced regions (Classes) are depicted as nodes; the scope is to produce a set of links between the nodes having as target value the Classes. The above are graphically presented in Figure 4(a). The network is then trained. Results can be seen in detail in Table 2; rows reflect the actual and columns the predicted classes. The number in cells is the percentage of the correct predictions of the target variable (class). The Bayesian classifier developed can use the associated data for the prediction of the Classes with high precision (function of the number of correct predictions of the target variable) of 97.5%.

Figure 4(b) shows the graphical representation of the resulting relationships among speed, volume and traffic composition after training. In a preliminary analysis, the mutual information is computed for each link of the network between the parent (start of link) and the child (end of link) in order to evaluate the contribution of each variable used to the one that is related. The mutual information, as expressed by the Equation 3, provides the amount of information or else the contribution of each node (in our case travel speed, volume or percent of motorized vehicles) to the knowledge of traffic flow regime. Table 3 summarizes the revealed relations with respect to their importance, in decreasing order. As can be observed, the strongest relations are those of speed and volume with the classes (regions of speed-volume relationship). Interestingly, the effect of trucks on the motorcycle movements seems to be significant. Moreover, taxis seem to have more effect to truck movements than the passenger cars do.

Furthermore, relations between the targeted value

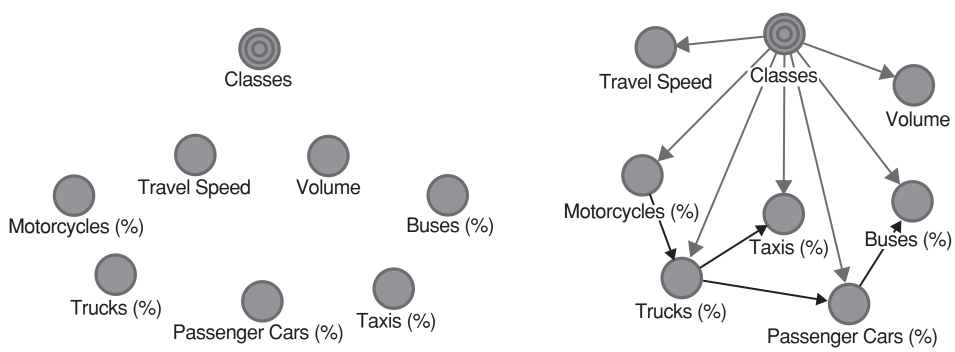

Fig. 4 (a) Bayesian Network before training: Traffic variables and types of motorized vehicles depicted as nodes and classes or else the resulting regions in speed-volume relationship depicted as target node in the analysis.
(b) Bayesian Network after training: Links indicate relations between the variables, the types of motorized vehicles and the classes.

Table 2 Results (predicted versus actual class) of augmented Naïve Bayesian Network

Predicted (%) / Actual (%)	I	II	III	IV	V	VI	VII	VIII
I	99	0	0	0	0	0	0	0
II	1	100	2	0	0	0	0	0
III	0	0	91	0	0	0	0	10
IV	0	0	1	100	0	0	0	0
V	0	0	0	0	100	0	0	0
VI	0	0	0	0	0	100	0	0
VII	0	0	0	0	0	0	100	0
VIII	0	0	6	0	0	0	0	90

(Classes) and the remaining variables are evaluated on average, but also for each class separately. Table 4 shows each node's relative significance ($\frac{\text{mutual information}_i}{\max\{\text{mutual information}_i\}}$, $i = 1, 2, \ldots, n$, where n is the number of variables describing traffic conditions) on average with respect to the information gain brought by each node to the knowledge of the target node (Classes) (sorted by descending order). Results indicate that traffic conditions in urban arterials are typically affected by the movements of taxis and motorcycles and less affected by trucks and passenger cars. Buses appear to have small contribution to traffic conditions; this result is probably due to the small-scale transit system in the city of Patras.

In Tables 5-7 the above relative significance is further examined separately for regions describing stable

Table 3 Relations between the traffic variables with respect to their importance, in a decreasing order. The mutual information is computed for each link of the network between the parent (start of link) and the child (end of link).

Parent	Child	Mutual Information
Classes	Travel Speed	2.133
Classes	Volume	1.322
Motorcycles (%)	Trucks (%)	1.082
Trucks (%)	Taxis (%)	1.078
Trucks (%)	Passenger Cars (%)	0.848
Passenger Cars (%)	Buses (%)	0.548
Classes	Motorcycles (%)	0.518
Classes	Buses (%)	0.198
Classes	Passenger Cars (%)	0.171
Classes	Taxis (%)	0.136
Classes	Trucks (%)	0.086

Table 4 Information gain, on average, brought by each variable to the knowledge of the speed-volume relationship

Node	Relative Significance
Travel Speed (km/h)	1.000
Volume (pcu/h)	0.620
Taxis (%)	0.252
Motorcycles (%)	0.243
Trucks (%)	0.216
Passenger Cars (%)	0.162
Buses (%)	0.121

Table 5 Information gain brought by each variable to the knowledge of the speed-volume regions for stable speed traffic conditions

Region I			Region II		
Node	Relative Significance	Modal Value	Node	Relative Significance	Modal Value
Travel Speed (km/h)	1.00	<=32	Travel Speed (km/h)	1.00	<=44
Volume (pcu/h)	0.49	<=74	Volume (pcu/h)	0.36	<=218
Motorcycles (%)	0.32	<=22.32	%buses	0.23	<=1
Taxis (%)	0.31	<=11.55	Taxis (%)	0.22	<=11.525
Trucks (%)	0.28	<=3.5	Motorcycles (%)	0.17	<=21.44
Passenger Cars (%)	0.21	<=62.63	Trucks (%)	0.13	<=4.6
Buses (%)	0.12	<=1	Passenger Cars (%)	0.06	<=63.94
Region III			**Region IV**		
Node	Relative Significance	Modal Value	Node	Relative Significance	Modal Value
Travel Speed (km/h)	1.00	<=59	Travel Speed (km/h)	1.00	>59
Volume (pcu/h)	0.32	>700	Taxis (%)	0.22	<=8.52
Motorcycles (%)	0.10	<=20.53	Trucks (%)	0.21	>4.6
Taxis (%)	0.09	<=8.52	Motorcycles (%)	0.20	<=20.53
Trucks (%)	0.09	>4.6	Volume (pcu/h)	0.17	>700
Passenger Cars (%)	0.09	<=62.42	Passenger Cars (%)	0.11	>65.46
Buses (%)	0.07	>2.915	Buses (%)	0.09	<=2.915

Table 6 Information gain brought by each variable to the knowledge of the speed-volume regions of speed change occurrence

Region V			Region VI		
Node	Relative Significance	Modal Value	Node	Relative Significance	Modal Value
Volume (pcu/h)	1.00	<=438	Volume (pcu/h)	1.00	<=551
Travel Speed (km/h)	0.62	<=29	Travel Speed (km/h)	0.43	<=29
Taxis (%)	0.10	<=11.525	Taxis (%)	0.14	<=8.52
Passenger Cars (%)	0.10	<=63.94	Passenger Cars (%)	0.13	<=63.94
Motorcycles (%)	0.09	<=21.44	Motorcycles (%)	0.13	<=20.53
Trucks (%)	0.07	<=4.6	Trucks (%)	0.09	>4.6
Buses (%)	0.03	<=1	Buses (%)	0.07	>2.915

Region VII		
Node	Relative Significance	Modal Value
Volume (pcu/h)	1.00	<=700
Travel Speed (km/h)	0.35	<=44
Taxis (%)	0.22	<=8.52
Motorcycles (%)	0.22	<=20.53
Trucks (%)	0.17	>4.6
Passenger Cars (%)	0.16	<=62.42
Buses (%)	0.05	<=1

speed conditions (Regions I to IV), for those describing speed changes (Regions V to VII), as well as for low values of speed and volume (Region VIII). Moreover, the modal value (most frequent) of the variables under study is also depicted in tables. Some interesting findings emerge from these results: in stable speed conditions, traffic is strongly related to speed and is highly affected by taxis and motorcycles, and less affected by passenger cars and buses. This does not apply to speeds of 40 km/h or above (Region II), where buses seem to be the primary affecting parameter, as well as in the case of 70 km/h (Regions IV) where the effect of trucks is critical. In the case of speed changes, traffic conditions are strongly related to volume; when speed changes occur in low volumes (Region V), motorcycles are the primary affecting parameter with second affecting parameter being the passenger cars. In medium and higher volumes (Regions VI and VII), the effect of taxis is greater than the one of motorcycles. Moreover, speed changes are less affected by buses. Finally, in low values of volume and speed (Region VII), taxis and motorcycles are the primary affecting parameters. Finally, an in depth look at the relations can be obtained by Figures 5 and 6. Figure 5 shows the variability of the relative significance of travel speed and volume with respect to the regions. Moreover, Figure 6 gives the graphical representation of the relative significance of the percent of each motorized vehicle to the regions. As can be observed, the evolution of the effect of passenger

Table 7 Information gain brought by each variable to the knowledge of the speed-volume regions at congestion

Region VIII		
Node	Relative Significance	Modal Value
Travel Speed (km/h)	1.00	<=13
Volume (pcu/h)	0.31	<=74
Motorcycles (%)	0.16	<=22.32
Taxis (%)	0.16	<=11.55
Trucks (%)	0.14	<=3.5
Passenger Cars (%)	0.09	<=62.63
Buses (%)	0.08	<=1

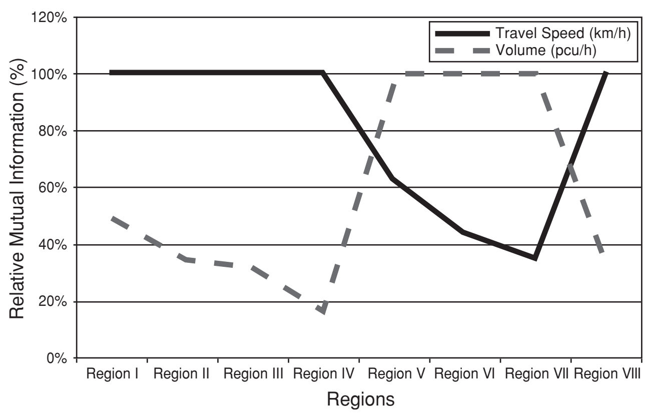

Fig. 5 The graphical representation of the relative significance of travel speed and volume with respect to regions

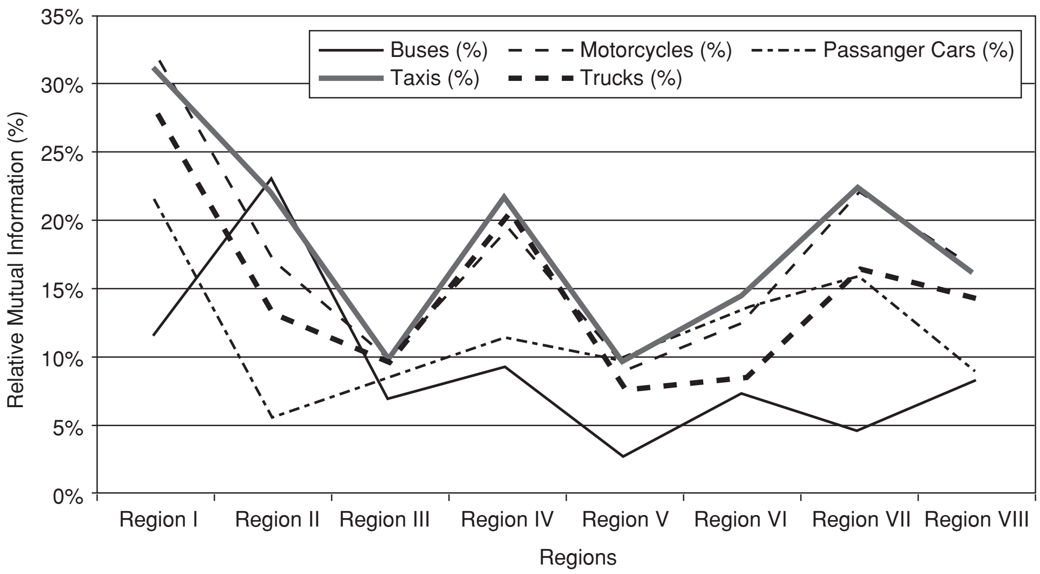

Fig. 6 The graphical representation of the relative significance of the percent of motorized vehicles with respect to regions

cars and buses is different from the one of the effect of taxis, motorcycles and trucks to the knowledge of regions.

4. CONCLUSIONS

Developing speed-volume relationships in urban arterials is a topic of considerable interest and importance in practice. Most approaches for evaluating the functional form of the speed-volume relationship are based on modeling and simulation and are applicable to uninterrupted homogeneous traffic flow and cannot accommodate traffic conditions imposed by the interaction of fast and slow moving vehicles in urban signalized arterials. The present paper treats the effect of traffic composition on the form of the speed-volume relationship; instead of proposing a specific functional form (be it linear or nonlinear and so on), the present paper adopts an advanced data-oriented methodological framework for revealing regions in the speed-volume relationship and assessing on the contribution of different types of motorized vehicles on each region separately.

Results indicate that in the basic three patterns of the speed-volume relationship (free-flow, speed changes and congestion) there exist eight different regions of speed-volume behavior that can be characterized by either stability in travel speed (at different value levels) or stability in volume (at different value levels). It was also determined that the contribution of traffic composition varies in each pattern and in each region. The role of taxis and motorcycles is found to be critical in the operation of most arterials examined and in most regions of speed-volume relationship, with the exception of high speed values where trucks affect significantly the speed-volume relationship.

The proposed methodology evaluates the arterial operation based on the consideration of the effect of traffic composition to the speed-volume relationship without providing an a priori functional assumption. This provides a flexible framework for integrating empirical knowledge for traffic conditions based on real-time traffic information into intelligent transportation systems. Moreover, it provides a data-driven and traffic-oriented framework that can be extended to the study of other relationships between traffic variables (such as the volume-density relationship) that can be adaptable and applicable to urban arterials regardless of geometric and control issues.

REFERENCES

1. Hall, F.L., Pushkar, A., Shi, Y. Some Observations on Speed-Flow and Flow-Occupancy Relationships under Congested Conditions. Transportation Research Record, 1398, pp.24-30. (1993).
2. May, A.D. Traffic Management From Theory to Practice: Past, Present, Future. Transportation Research Record, 1457, pp.5-14. (1994).
3. Jain, M., Coifman, B. Improved Speed Estimates from Freeway Traffic Detectors. "Journal of Transportation Engineering" 131(7): pp.483-495. (2005).
4. Greenshields, B.D. A Study of Traffic Capacity. HRB Proc.,

14th Annual Meeting, Washington, D.C., 14, 1934, pp.448–477. (1934).

5. Pushkar, A., Hall, F.L., Acha-Daza, J.A. Estimation of Speeds from Single-Loop Freeway Flow and Occupancy Data Using Cusp Catastrophe Theory Model. Transportation Research Record, 1457, pp.149-157. (1994).
6. Olszewski, P., Fan, H.S.L., Tan, Y.-W. Area-wide traffic speed-flow model for the Singapore CBD. "Transportation Research A" 29(4): pp.273-281. (1995).
7. Smith, W.S., Hall, F.L., Montgomery, F.O. Comparing the speed-flow relationship for motorways with new data from the M6. "Transportation Research A" 30(2): pp.89-101. (1996).
8. Skabardonis, A., Dowling, R. Improved Speed-Flow Relationships for Planning Applications. Transportation Research Record, 1572, pp.18-23. (1997).
9. Dailey, D.J. A statistical algorithm for estimating speed from single loop volume and occupancy measurements. "Transportation Research Part B" 33: pp.313-322. (1999).
10. Wu, N. A new approach for modeling of fundamental diagrams. "Transportation Research A" 36: pp.867–884. (2002).

11 Stewart, J.A., Rakha, H., Van Aerde, M. Analysis of Temporal and Spatial Variability of Free Speed Along a Freeway Segment. Transportation Research Record, 1494, pp.1-10. (1995).

12 Hurdle, V.F., Merlo, M.I., Robertson, D. Study of Speed-Flow Relationships on Individual Freeway Lanes. Transportation Research Record, 1591, pp.7-13. (1997).

13 Mahmassani, H.S., Jayakrishnan, R., Herman, R. Network traffic flow theory: Microscopic simulation experiments on supercomputers. "Transportation Research A" 24: pp.149-162. (1990).

14 Arasan, V.T., Koshy, R.Z. Methodology for Modeling Highly Heterogeneous Traffic Flow. "Journal of Transportation Engineering" 131(7): pp.544-551. (2005).

15 Duncan, N.C. A Further Look at Speed/ Flow/ Concentration. "Traffic Engineering and Control" pp.482-483. (1979).

16 Persaud, B.N., V.F. Hurdle. Some New Data That Challenge Some Old Ideas about Speed-Flow Relationships. Transportation Research Record, 1194, pp.191-198. (1988).

17 Kerner, B.S. Three-phase traffic theory and highway capacity. "Physica A" 333: pp.379-440. (2004).

18 Brilon, W., Ponzlet, M. Variability of Speed-Flow Relationships on German Autobahns. Transportation Research Record, 1555, pp.91-98. (1996).

19 Khan, S.I., Maini, P. Modeling Heterogeneous Traffic Flow. Transportation Research Record, 1678, pp.234-241. (1998).

20 Kimber, R.M., McDonald, M., Hounsell, N. Passenger car units in saturation flows: Concept, definition, derivation. "Transportation Research Part B" 19(1): pp.39-61. (1985).

21 Fisk, C.S. Effects of heavy traffic on network congestion. "Transportation Research B" 24(5): pp.391-404. (1990).

22 Arasan, V.T., Jagadeesh, K. Effect of Heterogeneity of Traffic on Delay at Signalized Intersections. "Journal of Transportation Engineering" 121(5), September/October: pp.397-404. (1995).

23 Highway Capacity Manual (HCM) 2000. TRB, National Research Council, Washington, D.C. (2000).

24 Lum, K.M., Fan, H.S.L., Lam, S.H., Olszewski, P. Speed-Flow Modeling of Arterial Roads in Singapore. "Journal of Transportation Engineering" 124(6): pp.213-222. (1998).

25 Rengaraju, V.R., Rao, V. Vehicle arrival characteristics at urban uncontrolled intersections. "Journal of Transportation Engineering" 121: pp.317-323. (1995).

26 Rao, V., Rengaraju, V.R. Modeling Conflicts of Heterogeneous Traffic at Urban Uncontrolled Intersections. "Journal of Transportation Engineering" 124: pp.23-34. (1998).

27 Vesanto, J., Alhoniemi, E. Clustering of the Self-Organizing Map. "IEEE Transactions on Neural Networks" 11(3): pp.586-600. (2000).

28 Ultsch, A., Moerchen, F. ESOM-Maps: tools for clustering, visualization, and classification with Emergent SOM. Technical Report No.46, Dept. of Mathematics and Computer Science, University of Marburg, Germany. (2005).

29 Kupas, K., Klebe, G., Ultsch, A. Comparison of substructural epitopes in enzyme active sites using self-organizing maps. "Journal of Computer-Aided Molecular Design" 18: pp.697-708. (2004).

30 Ultsch, A. (2005). Clustering with SOM: U*C, In Proceedings Workshop on Self-Organizing Maps, Paris, France, 75-82.

31 Opolon, D., Moutarde, F. Fast semi-automatic segmentation algorithm for Self-Organizing Maps. Proceedings of 12th European Symposium on Artificial Neural Networks (ESANN2004), Bruges, Belgium, 28-30 April, pp.507-512. (2004).

32 Ultsch, A., Moutarde, F. U*F Clustering: a new performant Cluster-mining method based on segmentation of Self-Organizing Maps. Proceedings Workshop on Self-Organizing Maps, Paris, France, pp.25-32. (2005).

33 Sahami, M. Learning limited dependence Bayesian classifiers. Proceedings of the Second International Conference on Knowledge Discovery and Data Mining, Portland, OR: AAAI Press. pp.335-338. (1996).

34 Cheng J., Greiner R. Learning Bayesian Belief Network Classifiers: Algorithms and System. AI 2001, LNAI 2056, pp.141-151. (2001).

35 Friedman, N., Geiger, D., Goldszmidt, M. Bayesian Network Classifiers. "Machine Learning" 29: pp.131-161. (1997).

36 Nöcker, M., Mörchen, F., Ultsch, A. An algorithm for fast and reliable ESOM learning. European Symposium on Artificial Neural Networks, Bruges (Belgium) 26-28 April. (2006).