ELSEVIER
SCIENCE @ DIRECT®
PHYSICS LETTERS B
Physics Letters B 568 (2003) 261–269
www.elsevier.com/locate/npe

A supersymmetric and smooth compactification of M-theory to AdS_5

S. Cucu [a,1], H. Lü [b,2], J.F. Vázquez-Poritz [c,3]

[a] *Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium*
[b] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843-4242, USA*
[c] *Physique Théorique et Mathématique, Université Libre de Bruxelles, Campus Plaine, C.P. 231, B-1050 Bruxelles, Belgium*

Received 22 April 2003; accepted 30 May 2003

Editor: M. Cvetič

Abstract

We obtain smooth M-theory solutions whose geometry is a warped product of AdS_5 and a compact internal space that can be viewed as an S^4 bundle over S^2. The bundle can be trivial or twisted, depending on the even or odd values of the two diagonal monopole charges. The solution preserves $\mathcal{N}=2$ supersymmetry and is dual to an $\mathcal{N}=1$, $D=4$ superconformal field theory, providing a concrete framework to study the AdS_5/CFT_4 correspondence in M-theory. We construct analogous embeddings of AdS_4, AdS_3 and AdS_2 in massive type IIA, type IIB and M-theory, respectively. The internal spaces have generalized holonomy and can be viewed as S^n bundles over S^2 for $n=4$, 5 and 7. Surprisingly, the dimensions of spaces with generalized holonomy includes $D=9$. We also obtain a large class of solutions of $AdS \times H^2$.

1. Introduction

AdS_5 spacetime arises naturally in type IIB supergravity, which provides a non-trivial and relatively simple framework for examining the holographic principle *via* the AdS_5/CFT_4 correspondence [1–3]. The embedding of AdS_5 spacetime in eleven-dimensional supergravity has also been studied in the past. A smooth but non-supersymmetric compactification of eleven-dimensional supergravity to AdS_5 was obtained in [4] where the internal space is a Kähler manifold. More recently, an internal background of $\mathbb{CP}^2 \times T^2$ was found in [5]. Although the compactification is not supersymmetric at the level of supergravity, it was argued in [5] that it is fully supersymmetric at the level of M-theory, since it is T-dual to the $AdS_5 \times S^5$ of type IIB theory. In the above

E-mail address: jvazquez@ulb.ac.be (J.F. Vázquez-Poritz).
[1] Research supported in part by the Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole P5/27 and the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime.
[2] Research supported in part by DOE grant DE-FG03-95ER40917.
[3] Research supported in part by the Francqui Foundation (Belgium), the Actions de Recherche Concertées of the Direction de la Recherche Scientifique—Communauté Francaise de Belgique, IISN-Belgium (convention 4.4505.86) and by a "Pole d'Attraction Interuniversitaire."

0370-2693
doi:10.1016/j.physletb.2003.05.002

two examples, the AdS spacetime and internal manifold are direct products without warp factors. Smooth but non-supersymmetric M-theory solutions have been constructed in [6], which are warped and twisted products of $AdS_5 \times S^2$ or $AdS_5 \times H^2$ with a squashed four-sphere.

In [7,8], supersymmetric embeddings of AdS_5 in M-theory were found as warped geometries with a compact internal metric. This construction can be understood from that the fact that S^5 can be expressed as a foliation of S^3 and S^1. One can then T-dualize the $AdS_5 \times S^5$ of type IIB theory on the $U(1)$ bundle of the S^3 and obtain a solution in M-theory [9]. However, there is a naked singularity in such a construction, since the $U(1)$ circle of the S^3 can shrink to zero. Supersymmetric and smooth embeddings of AdS_5 in M-theory were obtained in [10]. The eleven-dimensional metric is a warped product of AdS_5 with an internal metric that can be viewed as an S^4 bundle over H^2, a hyperbolic 2-plane. The construction can give rise to both $\mathcal{N}=2$ and $\mathcal{N}=4$ supersymmetry.

In this Letter, we report a supersymmetric and smooth compactification of M-theory to AdS_5, with the internal space being an S^4 bundle over S^2. The construction is only possible for $\mathcal{N}=2$ supersymmetry, and hence it gives rise to the minimum AdS_5 gauged supergravity coupled to matter. This solution provides a supergravity dual to $\mathcal{N}=1$, $D=4$ superconformal field theory. We also obtain supersymmetric and smooth compactifications of M-theory to AdS_2 and type IIB to AdS_3. The internal space is an S^p bundle over S^2, where $p=7$ and 5, respectively. We also construct a supersymmetric compactification of massive IIA to AdS_4, which is singular.

2. $AdS_5 \times S^2$ in M-theory

We begin by considering the sector of $D=7$ gauged supergravity with two diagonal $U(1)$ vector fields. The relevant Lagrangian is given by

$$\hat{e}^{-1}\mathcal{L}_7 = \widehat{R} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - \widehat{V} - \frac{1}{4}\sum_{i=1}^{2} X_i^{-2}\big(\widehat{F}^i_{(2)}\big)^2, \tag{2.1}$$

where $X_i = \mathrm{e}^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = \left(\sqrt{2}, \sqrt{\frac{2}{5}}\right), \qquad \vec{a}_2 = \left(-\sqrt{2}, \sqrt{\frac{2}{5}}\right). \tag{2.2}$$

The scalar potential $\widehat{V}$ is given by [11]

$$\widehat{V} = g^2\left(-4X_1X_2 - 2X_0X_1 - 2X_0X_2 + \frac{1}{2}X_0^2\right), \tag{2.3}$$

where $X_0 = (X_1X_2)^{-2}$. The potential can be expressed in terms of the superpotential

$$\widehat{W} = \frac{g}{\sqrt{2}}(X_0 + 2X_1 + 2X_2). \tag{2.4}$$

We now consider a 3-brane ansatz

$$ds^2 = \mathrm{e}^{2u}\,dx^\mu\,dx_\mu + \mathrm{e}^{2v}\,\lambda^{-2}\,d\Omega_2^2 + d\rho^2, \qquad F^i_{(2)} = \epsilon m_i \lambda^{-2}\Omega_{(2)}, \tag{2.5}$$

where the constant ϵ takes the values 1, -1 and 0, if $d\Omega_2^2$ is the metric for a unit S^2, hyperbolic H^2 or 2-torus T^2. $\Omega_{(2)}$ is the corresponding volume form. The system admits the following first-order equations

$$\frac{d\vec{\phi}}{d\rho} = \sqrt{2}\left(-\frac{\epsilon}{2\sqrt{2}}\big(m_1\vec{a}_1X_1^{-1} + m_2\vec{a}_2X_2^{-1}\big)\,\mathrm{e}^{-2v} + \frac{d\widehat{W}}{d\vec{\phi}}\right),$$

$$\frac{dv}{d\rho} = -\frac{1}{5\sqrt{2}}\left(2\sqrt{2}\,\epsilon\left(m_1 X_1^{-1} + m_2 X_2^{-1}\right) e^{-2v} + \widehat{W}\right),$$
$$\frac{du}{d\rho} = \frac{1}{5\sqrt{2}}\left(\frac{\epsilon}{\sqrt{2}}\left(m_1 X_1^{-1} + m_2 X_2^{-1}\right) e^{-2v} - \widehat{W}\right), \tag{2.6}$$

provided that the constraint

$$\lambda^2 = (m_1 + m_2) g \tag{2.7}$$

is satisfied. This set of first-order equations were derived for the case of H^2 in [10] by studying the Killing spinor equations of $D = 7$ gauged supergravity. In [12], a different method was used to obtain them for H^2, S^2 and T^2. The equations of (2.6) were analyzed in detail in [12]. Here, we report on only a subclass of solutions where $\vec{\phi}$ and v are constants. In this case, for $\epsilon = 0$, the solution is nothing but AdS_7 written in Poincaré coordinates.

For $\epsilon = \pm 1$, we find that the solution is given by

$$e^{\sqrt{2}\phi_1} = \frac{m_2 - m_1 \pm \sqrt{m_2^2 + m_1^2 - m_1 m_2}}{m_2}, \qquad e^{-\sqrt{\frac{5}{2}}\phi_2} = \frac{4}{3}\cosh(\phi_1/\sqrt{2}),$$
$$e^{-2v} = -\frac{\epsilon g\, e^{-\frac{3}{\sqrt{10}}\phi_2}}{m_1 e^{-\frac{\phi_1}{\sqrt{2}}} + m_2 e^{\frac{\phi_1}{\sqrt{2}}}}, \qquad u = -\frac{1}{2} g\, e^{-\frac{4}{\sqrt{10}}\phi_2}\rho. \tag{2.8}$$

This solution is invariant under the simultaneous interchanges of $m_1 \leftrightarrow m_2$ and $\phi_1 \leftrightarrow -\phi_1$. The reality conditions of the solution constrain the constants m_i and g, as well as the choice of $\pm$ in the solution. Let us first consider the case $\epsilon = -1$, corresponding to $d\Omega_2^2$ as the metric of a unit (non-compact) hyperbolic 2-plane. In this case, the reality of the solution implies that $m_1 m_2 \geqslant 0$. This includes the choice of $m_1 = 0$ (or $m_2 = 0$) and $m_1 = m_2$, which were discussed in [10]. The first case gives rise to $\mathcal{N} = 4$ supersymmetry in $D = 5$, whilst the second case gives rise to $\mathcal{N} = 2$ supersymmetry.

We are particularly interested in a compact internal space. Thus, we now turn to the choice of $\epsilon = +1$, corresponding to $d\Omega_2^2$ as the metric of S^2. In this case, the reality conditions for (2.8) imply that $m_1 m_2 < 0$. The condition (2.7) implies further that $m_1 \neq -m_2$. Therefore, the $AdS_5 \times S^2$ solution can only have $\mathcal{N} = 2$ supersymmetry, but cannot arise from the pure $D = 7$ minimal gauged supergravity.

If we define a charge parameter $q = 2m_1/(m_1 + m_2)$, then the condition for having S^2 versus H^2 can be summarized as

$$q \in [0, 2] \Longrightarrow H^2, \qquad q \in (-\infty, 0) \text{ or } (2, \infty) \Longrightarrow S^2. \tag{2.9}$$

It is straightforward to lift this solution to $D = 11$ by using the ansatz obtained in [11]. Since the solutions for general m_i are rather complicated to present, we only consider a representative example with $m_1 = 5g$ and $m_2 = -3g$. The M-theory metric is given by

$$ds_{11}^2 = \Delta^{\frac{1}{3}}\left[ds_{AdS_5}^2 + \frac{1}{g^2 c}\left\{\frac{1}{4c}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) + \frac{1}{\Delta}\left(\frac{1}{4}d\mu_0^2 + \frac{1}{5}\left(d\mu_1^2 + \mu_1^2\left(d\phi_1 - \frac{5}{2}\cos\theta\, d\varphi\right)^2\right)\right.\right.\right.$$
$$\left.\left.\left. + d\mu_2^2 + \mu_2^2\left(d\phi_2 + \frac{3}{2}\cos\theta\, d\varphi\right)^2\right)\right\}\right], \tag{2.10}$$

where $c = 10^{-2/5}$ and μ_i are spherical coordinates which satisfy $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. The warp factor Δ is given by

$$\Delta = c\left(4\mu_0^2 + 5\mu_1^2 + \mu_2^2\right) > 0. \tag{2.11}$$

The AdS_5 metric is given by

$$ds_{AdS_5}^2 = e^{-\frac{2\rho}{R}}\, dx^\mu\, dx_\mu + d\rho^2, \tag{2.12}$$

where the AdS radius is given by $R = \frac{1}{2cg}$. The 4-form field strength in $D = 11$ can also be obtained using the reduction ansatz in [11]. It is given by

$$\begin{aligned}
*F_{(4)} &= -(2g)^{-1}\big(8\mu_0^2 + 15\mu_1^2 + 7\mu_2^2\big)\epsilon_{(5)} \wedge \sin\theta\, d\theta \wedge d\varphi \\
&\quad + g^{-1}\left(\frac{1}{5}d\big(\mu_1^2\big) \wedge \left(d\phi_1 - \frac{5}{2}\cos\theta\, d\varphi\right) - 3d\big(\mu_2^2\big) \wedge \left(d\phi_2 + \frac{3}{2}\cos\theta\, d\varphi\right)\right) \wedge \epsilon_{(5)},
\end{aligned} \tag{2.13}$$

where $\epsilon_{(5)}$ is the volume form for the AdS_5 metric.

Thus, the internal space of the $D = 11$ metric can be viewed as an S^4 bundle over S^2, with two diagonal $U(1)$ bundles. In general, the internal metric can be labeled by $(q_1, q_2) = (\frac{2m_1}{m_1+m_2}, \frac{2m_2}{m_1+m2})$. In the specific example above, $(q_1, q_2) = (5, -3)$ and the solution is smooth everywhere. For general (m_1, m_2), the metric does not have a power-law singularity. However, it could have a conical orbifold singularity, which is absent only if (q_1, q_2) are integers. Since the q_i satisfy the constraint $q_1 + q_2 = 2$, it follows that they are either both even or both odd integers. In the even case, the bundle is topologically trivial, whilst it is twisted for the odd case.

3. $AdS_4 \times S^2$ in massive type IIA

The scalar potential in gauged supergravity with two $U(1)$ isometries was obtained in [13]. From this, we deduce that the relevant Lagrangian involving the two $U(1)$ vector fields is given by

$$\hat{e}^{-1}\mathcal{L}_6 = \widehat{R} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - \widehat{V} - \frac{1}{4}\sum_{i=1}^{2} X_i^{-2}\big(\widehat{F}^i_{(2)}\big)^2, \tag{3.1}$$

where $X_i = \mathrm{e}^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = \left(\sqrt{2}, \frac{1}{\sqrt{2}}\right), \qquad \vec{a}_2 = \left(-\sqrt{2}, \frac{1}{\sqrt{2}}\right). \tag{3.2}$$

The scalar potential is given by

$$\widehat{V} = \frac{4}{9}g^2\big(X_0^2 - 9X_1X_2 - 6X_0X_1 - 6X_0X_2\big), \tag{3.3}$$

where $X_0 = (X_1X_2)^{-3/2}$. As in the previous case, the scalar potential can be expressed in terms of the superpotential

$$\widehat{W} = \frac{g}{\sqrt{2}}\left(\frac{4}{3}X_0 + 2X_1 + 2X_2\right). \tag{3.4}$$

We consider a membrane solution of the type given by (2.5). The system admits the following first-order equations

$$\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\left(-\frac{\epsilon}{2\sqrt{2}}\big(m_1\vec{a}_1X_1^{-1} + m_2\vec{a}_2X_2^{-1}\big)\,\mathrm{e}^{-2v} + \frac{d\widehat{W}}{d\vec{\phi}}\right), \\
\frac{dv}{d\rho} &= -\frac{1}{4\sqrt{2}}\left(\frac{3}{\sqrt{2}}\epsilon\big(m_1X_1^{-1} + m_2X_2^{-1}\big)\,\mathrm{e}^{-2v} + \widehat{W}\right), \\
\frac{du}{d\rho} &= \frac{1}{4\sqrt{2}}\left(\frac{\epsilon}{\sqrt{2}}\big(m_1X_1^{-1} + m_2X_2^{-1}\big)\,\mathrm{e}^{-2v} - \widehat{W}\right),
\end{aligned} \tag{3.5}$$

provided that the constraint $\lambda^2 = (m_1 + m_2)g$ is satisfied. The solutions were analyzed in detail in [12]. Here, we shall only consider the subset of solutions with constant scalars. For $\epsilon = 0$, one just reproduces the AdS_6 metric in

Poincaré coordinates. For $\epsilon = \pm 1$, we have

$$e^{\sqrt{2}\phi_1} = \frac{3}{2}\frac{m_2 - m_1 \pm \sqrt{(m_2-m_1)^2 + \frac{4}{9}m_1 m_2}}{m_2}, \qquad e^{-\sqrt{2}\phi_2} = \frac{3}{2}\cosh\left(\frac{\phi_1}{\sqrt{2}}\right),$$
$$e^{-2v} = -\frac{4\epsilon g\, e^{-\frac{\phi_2}{\sqrt{2}}}}{m_1\, e^{-\frac{\phi_1}{\sqrt{2}}} + m_2\, e^{\frac{\phi_1}{\sqrt{2}}}}, \qquad u = -\frac{g}{3}\, e^{-\frac{3}{\sqrt{8}}\phi_2}\rho. \tag{3.6}$$

As in the $D = 7$ result, we can define a charge parameter $q = \frac{2m_1}{m_1+m_2}$. We have H^2 or S^2 depending on the following condition:

$$q \in [0,2] \Longrightarrow H^2, \qquad q \in (-\infty, 0) \text{ or } (2,\infty) \Longrightarrow S^2. \tag{3.7}$$

When $q = 0$ or $q = 2$, the system has $\mathcal{N} = 4$ supersymmetry. Otherwise, we have $\mathcal{N} = 2$ supersymmetry.

Using the reduction ansatz in [13,14], it is straightforward to lift the solution back to $D = 10$, giving rise to a solution of massive type IIA supergravity. The metric is given by

$$\begin{aligned} ds_{10}^2 = {} & \mu_0^{\frac{1}{12}} X_0^{\frac{1}{8}} (X_1 X_2)^{\frac{1}{4}} \Delta^{\frac{3}{8}} \\ & \times \big[ds_6^2 + g^{-2}\Delta^{-1}\big(X_0^{-1} d\mu_0^2 + X_1^{-1}\big(d\mu_1^2 + \mu_1^2\big(d\varphi_1 + gA_{(1)}^2\big)^2\big) \\ & \qquad + X_2^{-1}\big(d\mu_2^2 + \mu_2^2\big(d\varphi_2 + gA_{(2)}^1\big)^2\big)\big)\big], \end{aligned} \tag{3.8}$$

where $\Delta = \sum_{\alpha=0}^2 X_\alpha \mu_\alpha^2 > 0$ and $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. Thus, the $D = 10$ metric is a warped product of AdS_4 with an internal six-metric, which is an S^4 bundle over S^2 or H^2, depending on the charge parameter p, according to the rule (3.7).

As an example of a supersymmetric, though singular, compactification of AdS_4 from massive IIA, we can take $m_1 = 7g$ and $m_2 = -5g$, and a choice of negative sign in (3.6). This gives $X_0 = 6c$, $X_1 = 7c$ and $X_2 = c$, where $c = 6^{-1/4}7^{-3/8}$. Also, $A_{(1)}^1 = -\frac{7}{2g}\cos\theta\, d\varphi$ and $A_{(1)}^2 = \frac{5}{2g}\cos\theta\, d\varphi$, and the radius of AdS_4 is given by $R = 1/(2cg)$.

4. $AdS_3 \times S^2$ in type IIB

Let us now consider the $D = 5$ minimal gauged supergravity coupled to two vector multiplets. The Lagrangian is given by

$$e^{-1}\mathcal{L}_5 = \widehat{R} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - \frac{1}{4}\sum_{i=1}^3 X_i^{-2}\big(\widehat{F}_{(2)}^i\big)^2 - \widehat{V} + e^{-1}\frac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}\widehat{F}_{\mu\nu}^1 \widehat{F}_{\rho\sigma}^2 \hat{A}_\lambda^3, \tag{4.1}$$

where $X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = \left(\sqrt{2}, \frac{2}{\sqrt{6}}\right), \qquad \vec{a}_2 = \left(-\sqrt{2}, \frac{2}{\sqrt{6}}\right), \qquad \vec{a}_3 = \left(0, -\frac{4}{\sqrt{6}}\right). \tag{4.2}$$

The scalar potential is given by

$$\widehat{V} = -4g^2 \sum_{i=1}^3 X_i^{-1}. \tag{4.3}$$

The scalar potential $\widehat{V}$ can also be expressed in terms of the superpotential

$$\widehat{W} = \sqrt{2}\, g \sum_i X_i . \tag{4.4}$$

We find that the string solution of the type given by (2.5) admits the following first-order equations

$$\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\left(-\frac{\epsilon}{2\sqrt{2}}\left(m_1\vec{a}_1 X_1^{-1} + m_2\vec{a}_2 X_2^{-1} + m_3\vec{a}_3 X_3^{-1}\right) \mathrm{e}^{-2v} + \frac{d\widehat{W}}{d\vec{\phi}}\right),\\
\frac{dv}{d\rho} &= -\frac{1}{3\sqrt{2}}\left(\sqrt{2}\,\epsilon\left(m_1 X_1^{-1} + m_2 X_2^{-1} + m_3 X_3^{-1}\right) \mathrm{e}^{-2v} + \widehat{W}\right),\\
\frac{du}{d\rho} &= \frac{1}{3\sqrt{2}}\left(\frac{\epsilon}{\sqrt{2}}\left(m_1 X_1^{-1} + m_2 X_2^{-1} + m_3 X_3^{-1}\right) \mathrm{e}^{-2v} - \widehat{W}\right),
\end{aligned} \tag{4.5}$$

provided that the constraint $\lambda^2 = (m_1 + m_2 + m_3)g$ is satisfied. The general solution for this system was analyzed in [12]. Here, we consider only the solutions with constant scalar fields. For $\epsilon = 0$, the solution is AdS_5 in Poincaré coordinates. For $\epsilon = \pm 1$, fixed-point solutions exist only for non-vanishing m_i. The solution is given by

$$\begin{aligned}
&\mathrm{e}^{\sqrt{2}\phi_1} = \frac{m_1}{m_2}\left(\frac{m_3 + m_2 - m_1}{m_3 - m_2 + m_1}\right), \qquad \mathrm{e}^{\sqrt{6}\phi_2} = \frac{m_1 m_2 (m_3^2 - (m_1 - m_2)^2)}{m_3^2 (m_1 + m_2 - m_3)^2},\\
&\mathrm{e}^{-2v} = -\epsilon g\left(\frac{(m_1 + m_2 - m_3)(m_3^2 - (m_1 - m_2)^2)}{m_1^2 m_2^2 m_3^2}\right)^{1/3},\\
&u = -g\, \mathrm{e}^{\frac{\phi_2}{\sqrt{6}}}\left(\cosh(\phi_1/\sqrt{2}) + \frac{1}{2}\mathrm{e}^{-\sqrt{\frac{3}{2}}\phi_2}\right)\rho.
\end{aligned} \tag{4.6}$$

The reality condition of the solution implies that when three vectors with the magnitudes $|m_i|$ can form a triangle, $d\Omega_2^2$ should be the H^2 metric. On the other hand, when they cannot form a triangle, the metric should be that of S^2.[4] If any of the m_i vanish, there is no fixed-point solution, except when one vanishes with the remaining two equal [12].

Using the reduction ansatz obtained in [11], we can easily lift the solution back to $D = 10$. Since the solution with general m_i is complicated to present, we consider a simpler case with $m_2 = m_1$. The $D = 10$ type IIB metric is

$$\begin{aligned}
ds_{10}^2 = \sqrt{\Delta}\Bigg\{ & ds_{AdS_3}^2 + \epsilon g^{-2}\left(\frac{m_1}{m_3 - 2m_1}\right)^{1/3}\left(\frac{1}{2}q_1 d\Omega_2^2 + d\theta^2\right)\\
& + g^{-2}\Delta^{-1}\Bigg[c^{-1/3}\cos^2\theta\left(d\psi^2 + \sin^2\psi\left(d\varphi_1 + \frac{1}{2}q_1 A_{(1)}\right)^2 + \cos^2\psi\left(d\varphi_2 + \frac{1}{2}q_1 A_{(1)}\right)^2\right)\\
& + c^{2/3}\sin^2\theta\left(d\varphi_3 + \frac{1}{2}q_3 A_{(1)}\right)^2\Bigg]\Bigg\},
\end{aligned} \tag{4.7}$$

where

$$\begin{aligned}
&c = \left|\frac{m_1}{2m_1 - m_3}\right|, \qquad \Delta = c^{1/3}\cos^2\theta + c^{-2/3}\sin^2\theta > 0, \qquad dA_{(1)} = \Omega_{(2)},\\
&ds_{AdS_3}^2 = \mathrm{e}^{-\frac{2\rho}{R}}\left(-dt^2 + dx^2\right) + d\rho^2, \qquad R = \left|\frac{2m_1}{g(4m_1 - m_3)c^{1/3}}\right|.
\end{aligned} \tag{4.8}$$

[4] $AdS_3 \times S^2$ solutions were also recently found in [15] in a different construction.

We have introduced the charge parameters $q_i = 2m_i/(m_1+m_2+m_3)$, and hence they satisfy the constraint $q_1+q_2+q_3=2$. In the above solution, if $|m_3| < 2|m_1|$, we should have $\epsilon=-1$, corresponding to H^2; if $|m_3| > 2|m_1|$, we should have $\epsilon=1$, corresponding to S^2. In general, the internal metric is an S^5 bundle over S^2 or H^2, depending the values of the q_i according to the above rules.

5. $AdS_2 \times S^2$ in M-theory

Let us now consider the $U(1)^4$ gauged $N=2$ supergravity in four dimensions. The Lagrangian is given by

$$e^{-1}\mathcal{L}_4 = \widehat{R} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - \frac{1}{2}(\partial\phi_3)^2 - \frac{1}{4}\sum_{i=1}^{4} X_i^{-2}\big(\widehat{F}^i_{(2)}\big)^2 - \widehat{V}, \tag{5.1}$$

where $X_i = \mathrm{e}^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = (1,1,1), \qquad \vec{a}_2 = (1,-1,-1), \qquad \vec{a}_3 = (-1,1,-1), \qquad \vec{a}_4 = (-1,-1,1). \tag{5.2}$$

The scalar potential is given by

$$\widehat{V} = -4g^2 \sum_{i<j} X_i X_j, \tag{5.3}$$

which can also be expressed in terms of the superpotential

$$\widehat{W} = \sqrt{2}\, g \sum_{i=1}^{4} X_i. \tag{5.4}$$

The magnetic black hole solution of the type given by (2.5) admits the following first-order equations

$$\frac{d\vec{\phi}}{d\rho} = \sqrt{2}\left(-\frac{\epsilon}{2\sqrt{2}}\sum_{i=1}^{4} m_i \vec{a}_i X_i^{-1}\,\mathrm{e}^{-2v} + \frac{d\widehat{W}}{d\vec{\phi}}\right), \qquad \frac{dv}{d\rho} = -\frac{1}{2\sqrt{2}}\left(\frac{\epsilon}{\sqrt{2}}\sum_{i=1}^{4} m_i X_i^{-1}\,\mathrm{e}^{-2v} + \widehat{W}\right),$$
$$\frac{du}{d\rho} = \frac{1}{2\sqrt{2}}\left(\frac{\epsilon}{\sqrt{2}}\sum_{i=1}^{4} m_i X_i^{-1}\,\mathrm{e}^{-2v} - \widehat{W}\right), \tag{5.5}$$

provided that the constraint $\lambda^2 = g\sum_{i=1}^{4} m_i$ is satisfied. The general solution for this system was analyzed in [12]. Here, we consider only solutions with constant scalar fields. For $\epsilon=0$, the solution is AdS_4 in Poincaré coordinates. For $\epsilon=\pm1$, we have not obtained the general solution for arbitrary m_i, although we have found many examples of specific solutions. We do find a class of special solutions by setting $m_2=m_3=m_4$. This enables us to consistently set $\phi_1=\phi_2=\phi_3\equiv\phi$. For this truncation, fixed-point solutions for $\epsilon=\pm1$ are given by

$$\mathrm{e}^{2\phi} = \frac{3m_2 - m_1 \pm \sqrt{(m_1-m_2)(m_1-9m_2)}}{2m_2}, \qquad \mathrm{e}^{-2v} = 4g\epsilon\frac{\sinh\phi}{m_1\,\mathrm{e}^{-2\phi} - m_2},$$
$$u = -\frac{1}{2}g\left(\frac{2(m_1\,\mathrm{e}^{-\frac{3}{2}\phi} + 3m_2\,\mathrm{e}^{\frac{1}{2}\phi})}{m_2 - m_1\,\mathrm{e}^{-2\phi}}\sinh\phi + \mathrm{e}^{\frac{3}{2}\phi} + 3\,\mathrm{e}^{-\frac{1}{2}\phi}\right)\rho. \tag{5.6}$$

The reality condition of the solution implies that for $\epsilon=-1$, corresponding to H^2, we must have either $m_2>0$ and $0<m_1\leqslant m_2$ or $m_2<0$ and $m_2\leqslant m_1<-3m_2$. For $\epsilon=1$, corresponding to S^2, we must have $m_2\leqslant 0$ and $m_1>-3m_2$. The $AdS_2\times H^2$ has also been found in [16].

Using the reduction ansatz obtained in [11], we can easily lift the solution back to $D = 11$ with the metric

$$ds_{11}^2 = \Delta^{2/3}\left\{ds_{AdS_2}^2 + \frac{e^{2v}}{(m_1+3m_2)g}\,d\Omega_2^2 + \frac{1}{g^2\Delta}\left[e^{-\frac{3}{2}\phi}\left(d\mu_1^2 + \mu_1^2\left(d\phi_1 + \frac{\epsilon m_1}{m_1+3m_2}A_{(1)}\right)^2\right)\right.\right.$$
$$\left.\left. + e^{\frac{1}{2}\phi}\sum_{i=2}^{4}\left(d\mu_i^2 + \mu_i^2\left(d\phi_i + \frac{\epsilon m_2}{m_1+3m_2}A_{(1)}\right)^2\right)\right]\right\}, \tag{5.7}$$

where

$$\Delta = \left(e^{\frac{3}{2}\phi} - e^{-\frac{1}{2}\phi}\right)\mu_1^2 + e^{-\frac{1}{2}\phi} > 0, \qquad dA_{(1)} = \Omega_{(2)}, \qquad ds_{AdS_2}^2 = -e^{-\frac{2\rho}{R}}\,dt^2 + d\rho^2,$$
$$R = \frac{2}{g}\left[\frac{2(m_1\,e^{-\frac{3}{2}\phi} + 3m_2\,e^{\frac{1}{2}\phi})}{m_2 - m_1\,e^{-2\phi}}\sinh\phi + e^{\frac{3}{2}\phi} + 3\,e^{-\frac{1}{2}\phi}\right]^{-1}. \tag{5.8}$$

In general, the internal metric is an S^7 bundle over S^2 or H^2, depending the values of the m_i.

6. Conclusions

We have obtained a large class of supersymmetric embeddings of AdS spacetime in M-theory, as well as type IIB and massive type IIA theories. The internal spaces can be viewed as S^n bundles over S^2 or H^2. Similar solutions have been discussed in [17–19]. In particular, we have found a smooth embedding of AdS_5 in M-theory, with a compact internal space of an S^4 bundle over S^2. The bundle can be trivial or twisted, depending on the two diagonal monopole charges. The solution preserves $\mathcal{N} = 2$ supersymmetry; it is a supergravity dual to an $\mathcal{N} = 1$, $D = 4$ superconformal field theory on the boundary of AdS_5. This provides a concrete framework to study AdS_5/CFT_4 from the point of view of M-theory.

The internal spaces of these embeddings may be regarded as concrete realizations of spaces with generalized holonomy groups advocated in [20], since they are not Ricci flat and involve a form field. An especially interesting example is the S^7 bundle over S^2 or H^2, which is nine-dimensional. While nine-dimensional Ricci-flat manifolds do not have an irreducible special holonomy group, our aforementioned solutions are explicit examples of nine-dimensional spaces which have generalized special holonomy.

The embeddings of AdS spacetimes in M-theory and string theories discussed in this Letter all involve warp factors. We expect that there are many further examples of such solutions. It is of interest to explore them, both from the AdS/CFT perspective as well as for a more concrete understanding and classification of spaces with generalized special holonomy.

Acknowledgements

We are grateful to Gary Gibbons, Chris Pope and Ergin Sezgin for useful discussions.

References

[1] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231;
J.M. Maldacena, Int. J. Theor. Phys. 38 (1999) 1113, hep-th/9711200.
[2] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105, hep-th/9802109.
[3] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.

[4] C.N. Pope, P. van Nieuwenhuizen, Commun. Math. Phys. 122 (1989) 281.
[5] M.J. Duff, H. Lü, C.N. Pope, Phys. Lett. B 409 (1997) 136, hep-th/9704186.
[6] J.P. Gauntlett, N. Kim, S. Pakis, D. Waldram, Class. Quantum Grav. 19 (2002) 3927, hep-th/0202184.
[7] M. Alishahiha, Y. Oz, Phys. Lett. B 465 (1999) 136, hep-th/9907206.
[8] Y. Oz, hep-th/0004009.
[9] M. Cvetič, H. Lü, C.N. Pope, J.F. Vázquez-Poritz, Phys. Rev. D 62 (2000) 122003, hep-th/0005246.
[10] J.M. Maldacena, C. Nunez, Int. J. Mod. Phys. A 16 (2001) 822, hep-th/0007018.
[11] M. Cvetič, M.J. Duff, P. Hoxha, J.T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati, T.A. Tran, Nucl. Phys. B 558 (1999) 96, hep-th/9903214.
[12] S. Cucu, H. Lü, J.F. Vázquez-Poritz, in preparation.
[13] M. Cvetič, S.S. Gubser, H. Lü, C.N. Pope, Phys. Rev. D 62 (2000) 086003, hep-th/9909121.
[14] M. Cvetič, H. Lü, C.N. Pope, Phys. Rev. Lett. 83 (1999) 5226, hep-th/9906221.
[15] S.L. Cacciatori, D. Klemm, W.A. Sabra, hep-th/0302218.
[16] J.P. Gauntlett, N. Kim, S. Pakis, D. Waldram, Phys. Rev. D 65 (2002) 026003, hep-th/0105250.
[17] B.S. Acharya, J.P. Gauntlett, N. Kim, Phys. Rev. D 63 (2001) 106003, hep-th/0011190.
[18] J.P. Gauntlett, N. Kim, D. Waldram, Phys. Rev. D 63 (2001) 126001, hep-th/0012195.
[19] J.P. Gauntlett, N. Kim, Phys. Rev. D 65 (2002) 086003, hep-th/0109039.
[20] M.J. Duff, J.T. Liu, hep-th/0303140.