Design of a cosine-theta dipole magnet wound with coated conductors considering their deformation at coil ends during winding process

Naoyuki Amemiya*, Hidetoshi Miyahara, Toru Ogitsu, Tsutomu Kurusu

aKyoto University, Kyoto-Daigaku-Katsura, Nishikyo, Kyoto 615-8510, Japan
bHigh Energy Accelerator Research Organization, Oho, Tsukuba 305-0801, Japan
cToshiba Corporation, Horikawacho, Saiwai, Kawasaki 212-8585, Japan

Abstract

By using differential geometry, we modeled the three-dimensional shapes of the coil ends of cosine-theta magnets while considering local edge-wise bend, local flat-wise bend, and torsion of coated conductors. We focus on the feasibility of winding coil ends against the stress caused by bending. We discussed the feasibility of winding based on two assumptions to form coil ends: all turns of coated conductors are free from edge-wise bend; faces of all turns of coated conductors are completely parallel. Using the first assumption, we designed a cosine-theta dipole magnet wound with coated conductors.

Keywords: Coated conductor; accelerator magnet; cosine-theta; deformation

1. Introduction

When using coated conductors in accelerator magnets, one of the major technical issues is their field quality. In order to generate the precise magnetic field, firstly, we must design the shape of the coils carefully and wind them precisely. Secondly, tape magnetizations can affect the field quality. This paper deals with the first one. If we wind a
cosine-theta coil with coated conductors, they must be deformed three-dimensionally to form the coil ends. Their edge-wise (hard-way) bending is difficult, whereas their flat-wise bending or their torsion is easier. The shape of the turn of a coated conductor without edge-wise bend is natural, but, this shape is unique for a specified base curve on a mandrel, and if we assume that each turn follows such a shape, the angle of a coated conductor against the face of the mandrel at a coil end varies turn by turn. On the contrarily, if we assume that a turn is wound so that its face could be completely parallel to that of the inner adjacent turn, that is, the inclinations of coated conductors against the face of the mandrel at a coil end are identical, some edge-wise bend appears in coated conductors.

In this paper, we focus on the feasibility of winding coil ends against the stress caused by bending rather than the degradation of superconducting properties due to strain. First, we design coil ends with various shapes based on the above-mentioned two assumptions and discuss the feasibility of winding. Then, using the first assumption, we design a cosine-theta dipole magnet wound with coated conductors.

2. Theoretical expression of three-dimensional shape of coated conductor using differential geometry [1]

We modeled coated conductors with strips. The generalized Frenet-Serret’s equations for strips are given by

\[
\frac{dT}{ds} = \kappa_n n - \kappa_b b, \quad \frac{dn}{ds} = -\kappa_n T + \kappa_b b, \quad \frac{db}{ds} = -\kappa_n T - \kappa_b n.
\]

where \(\kappa_g\) is the geodesic curvature, \(\kappa_n\) is the normal curvature, and \(\tau\) is the torsion. \((T, n, b)\) is the Frenet frame consists of the tangent vector \(T\), the normal vector \(n\), and the binormal vector \(b\). Considering local edge-wise (hard-way) bend, local flat-wise bend, and torsion, the three-dimensional shape of a coated conductor which is wound following a base curve on a three-dimensional mandrel as shown in Fig. 1 can be represented by using these equations.

It should be noted that a geodesic strip, which is developable to a straight strip on to a plane, can be modeled by the base curve and the vector field \(d\), which is identical to the Darboux vector field of the base curve.

\[
d = \tau T + \kappa_n b,
\]

where binormal vector of the strip \(b\) is identical to that of the base curve. Consequently, the shape of a coated conductor without edge-wise bend is uniquely defined for a specified base curve.

The application of additional twist angle \(\delta\) around the tangent vector \(T\) provides an additional degree of freedom to the shape of a coated conductor for a specified base curve, but \(\kappa_g\) and \(\kappa_n\) vary by the additional twist. If we apply an additional twist to a coated conductor without edge-wise bend, the resulting shape could not be free from edge-wise bend any more. The relations between the curvatures before the application of the twist \((\kappa_g, \kappa_n)\) and those after the application of the twist \((\kappa_g^*, \kappa_n^*)\) are given as

\[
\kappa_g^* = \cos \delta \kappa_g + \sin \delta \kappa_n, \quad \kappa_n^* = \cos \delta \kappa_n - \sin \delta \kappa_g.
\]

![Fig. 1. Vectors in Frenet frame: the tangent vector \(T\); the normal vector \(n\); the binormal vector \(b\). \(d = \tau T + \kappa_n b\).](image_url)
3. Comparison between two assumptions for shape of coil end

Three saddle-shape coils with different angles of straight sections on a mandrel as shown in Fig. 2 were designed by using the theory described in section 2 under the two assumptions: (assumption A) all turns of coated conductors are free from edge-wise bend; (assumption B) faces of all turns of coated conductors are completely parallel. In both cases, the first turn is free from edge-wise bend. In assumption A, because the inclination of the coated conductor decreases with increasing number of turns as shown in Fig. 3(a), the base curve shifts outward, that is, the coated conductor slips outward at the nose, to keep the minimum distance between adjacent turns. In assumption B, the separation between the base curve is kept constant, that is, a turn is wound so that its back is put on the face of the inner adjacent turn, but additional twists are applied in order to keep faces all turns of coated conductors parallel.

The width and the thickness of the coated conductor is 4 mm and 0.1 mm, respectively. The radius of the mandrel is 70 mm. The developed base curves of the coil ends are elliptical. The length of the coil end, that is, the semi-major (minor) axis of the ellipse, is determined so as to minimize the edge-wise curvature in the designs based on assumption B, when the angle of straight sections on the mandrel is 40 degrees or 90 degrees.

The designed results are listed in Table 1 together with the specifications: the inclination of the coated conductor at the nose of the coil end is shown for the first turn and the twenty-fifth turn; the maximum edge-wise strain, which is calculated from the edge-wise curvature, is shown for the first turn and the twenty-fifth turn. In the designs based on assumption A, increasing inclination of the coated conductor might result in some voids between coated conductors. However, the estimated dimensions of such voids as well as the slips of coated conductors are the order of micrometer per turn, and they may not be serious to wind coil ends. Meanwhile, in the designs based on assumption B, because remarkable edge-wise strains appear, winding coil ends might be difficult because of the stresses caused by edge-wise bending or spring-back. Both assumptions are extreme cases, and, the shape of a turn may fall between them: some small slip outward allows relaxing the strain to reach more likely shape, even though this shape might not be completely free from edge-wise bend.
Table 1. Specifications and designed results of three saddle-shape coils with different angles of straight sections on mandrel.

<table>
<thead>
<tr>
<th>Angle of straight sections on mandrel</th>
<th>40 degrees</th>
<th>90 degrees</th>
<th>140 degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coated conductor width</td>
<td>4 mm</td>
<td>4 mm</td>
<td>4 mm</td>
</tr>
<tr>
<td>Coated conductor thickness</td>
<td>0.1 mm</td>
<td>0.1 mm</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>Minimum distance between adjacent turns</td>
<td>0.025 mm</td>
<td>0.025 mm</td>
<td>0.025 mm</td>
</tr>
<tr>
<td>Radius of mandrel</td>
<td>70 mm</td>
<td>70 mm</td>
<td>70 mm</td>
</tr>
<tr>
<td>Length of coil end</td>
<td>20 mm</td>
<td>60 mm</td>
<td>100 mm</td>
</tr>
<tr>
<td>Number of turns</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

Assumption A

| Inclination of first turn at nose     | 70.7 degrees | 51.8 degrees | 43.7 degrees |
| Inclination of twenty-fifth turn at nose | 68.7 degrees | 50.8 degrees | 43.2 degrees |
| Edge-wise strain of first turn        | 0%          | 0%          | 0%          |
| Edge-wise strain of twenty-fifth turn at nose | 0%         | 0%          | 0%          |

Assumption B

| Inclination of first turn at nose     | 70.7 degrees | 51.8 degrees | 43.7 degrees |
| Inclination of twenty-fifth turn at nose | 70.7 degrees | 51.8 degrees | 43.7 degrees |
| Edge-wise strain of first turn        | 0%          | 0%          | 0%          |
| Edge-wise strain of twenty-fifth turn | 0.4%        | 0.1%        | 0.08%       |

4. Design of a cosine-theta dipole magnet

We designed a cosine-theta dipole magnet, whose specifications are not completely same but similar to those of a magnet for rotating gantry for carbon cancer therapy [2]. The reference radius, the inner radius of the coil, the inner radius of the return iron yoke are 30 mm, 60 mm, and 120 mm, respectively. The effect of the return iron yoke was considered by image currents. The width and the thickness of the coated conductor were 5 mm and 0.2 mm, respectively. The minimum distance between adjacent coated conductors was assumed to be 0.1 mm.

At first, we carried out the two-dimensional field design. The designed arrangement of coated conductors is shown in Fig. 4. Two coils were paired with 0.5 mm radial separation, and three pairs of coils were arranged with 5 mm radial separation. The number of turns is 2718, and the dipole component of 2.88 T was obtained by the current of 200 A. Higher multi-pole components normalized by the dipole component were less than $10^{-4}$.

Fig. 4. Two-dimensional design of dipole magnet.
Next, we designed the coil ends of the magnet by using assumption A. A design between the two assumptions might be more realistic, but the slips of coated conductors to keep the minimum distance between adjacent turns are very small even if we design based on assumption A. The accuracy of the magnetic field was evaluated using the integrated magnetic field. The two-dimensional design and the coil end design were combined and were carried out iteratively to reduce higher harmonics of the integrated magnetic field. The shapes of the coils and the parameters of the designed magnet are shown in Fig. 5 and Table 2. The number of turns is 2844, and the integrated dipole component of 2.64 Tm was obtained by the current of 200 A, while the length of the entire coil is 1082 mm including the coil ends. Higher multi-pole components normalized by the dipole component were less than $10^{-4}$.

5. Conclusion

If we wound coated conductors into saddle-shape coils putting the back of a coated conductor on the face of the inner adjacent coated conductor, remarkable edge-wise curvatures appear in coated conductors, and winding in this way looks not feasible. However, a small slip of the coated conductor allows to eliminate the edge-wise bend, and winding becomes more feasible. We successfully designed a dipole magnet for a rotating gantry for carbon cancer therapy.

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References