A STABILITY MODEL FOR STEAM GENERATORS

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Abstract—A mathematical model to analyze the stability of the two-phase flow in a generalized steam generator is developed. A counter flow heat exchanger in which a high temperature primary fluid heats and vaporizes a lower temperature secondary fluid is considered as the system. The governing equations of this system is obtained by using the transient field equations, constitutive relations, boundary conditions and the initial conditions of both the primary and the secondary fluids. The governing equations of the secondary fluid are decoupled from the equations of the primary fluid by determining a heat flux profile and superimposing it on the wall of the channel of the secondary fluid. With this superimposed heat flux profile an equivalent system is obtained which utilizes the fundamental equations of the secondary fluid to analyze the stability of the flow. To investigate the stability of the system, a relation between the variation of the inlet velocity and the variation of the total channel pressure drop is needed. The Laplace transform of this relation is called the transfer function of this system and is obtained by using a small perturbation technique and linearization. Liapunov's theorem is used to investigate the stability of the nonlinear system from linearized system. The theoretical predictions of this model are observed to be in agreement with experimental results.

INTRODUCTION

Two-phase flow with boiling or condensation in horizontal or vertical conduits occurs in many industrial processes such as power generation, refrigeration and chemical processes. However, experiments have shown that these two-phase flow systems may exhibit a wide variety of sustained flow, pressure, and density oscillations which are generally called two-phase flow instabilities. One of the most commonly observed of these instabilities is described in terms of the propagation of density waves and is called the density wave instability.

The appearance of any of these oscillations is highly undesirable. Sustained oscillations may cause mechanical vibration of the components of a system and the amplitude of these vibrations may become large enough to cause structural failure of these components. Flow oscillations also affect the local heat transfer characteristics and may induce a boiling crisis. Hence, the onset of these instabilities may represent the operating limit of a system.

As a result of these imposed limitations there has been a lot of research done on the subject.
However, some necessary stability analyses are still not available because of the extreme complexity of two-phase flow systems and because most of the analyses used assumptions which are valid only for specific operational and geometrical conditions. For example, most of these models are based on the assumption of a uniform heat flux profile with the exception of a few recent models [1, 2] which considered nonuniform heat flux profiles. Attempts to apply the uniform heat flux profile models to systems with a nonuniform heat flux profile were not always successful [3]. Also, when the above nonuniform heat flux profile models were applied to sinusoidal heat flux profile systems, the results sometimes qualitatively conflicted with each other or with the experimental results [4]. To date, a generalized analytical stability model for any nonuniform heat flux profile system to predict experimental results quantitatively as well as qualitatively is not available in the open literature.

This study develops an appropriate mathematical model for a generalized two-phase flow system to predict the stability of density wave oscillations. The model is applicable to a wide variety of geometrical and operational conditions, to uniform and nonuniform heat flux profiles and to systems with complete or partial evaporation of the working fluid. Theoretical predictions are presented for the stability of three different systems and are compared with the experimental results of these systems.

DEVELOPMENT OF A MODEL

To meet the above objectives a quantitative formulation and solution of the stability problem is presented for a generalized two-phase flow system. This system is a counterflow heat exchanger with complete evaporation of the secondary fluid by a hotter, subcooled primary liquid and is shown in Fig. 1; the secondary flow is divided into four different regions depending on the heat transfer mechanism. The outside wall of the heat exchanger is assumed to be perfectly insulated, but the formulation of the problem can easily be modified to include other boundary conditions. It is also assumed that the pressure drops of both the primary

![Fig. 1. Schematic representation of the boiling channel.](image-url)
and secondary fluids along the channel are sufficiently small so that the thermodynamic properties are functions of the inlet pressure and another independent thermodynamic property. Assuming a one-dimensional, homogeneous flow model and neglecting axial heat conduction, capillary forces and dissipation terms, the appropriate continuity and energy equations for both the primary and secondary fluids, the energy equation for the wall, and the momentum equation for the secondary fluid are obtained as follows.

Continuity:

$$\frac{\partial \rho'}{\partial t} - \frac{\partial (\rho' v')}{\partial v} = 0$$  \hspace{1cm} (1)

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial z} = 0.$$  \hspace{1cm} (2)

Energy equations:

$$\rho' \frac{\partial i'}{\partial t} - \rho' v' \frac{\partial i'}{\partial z} = -\frac{2\pi r \rho' h'}{A'} (T' - T_w)$$  \hspace{1cm} (3)

$$2\pi r \rho \omega C_p \frac{\partial T_w}{\partial t} = k_w \frac{\partial}{\partial r} \left( 2\pi r \frac{\partial T_w}{\partial r} \right)$$  \hspace{1cm} (4)

$$-k_w \left( \frac{\partial T_w}{\partial r} \right)_{r=r_1} = h'(T' - T_w)$$  \hspace{1cm} (5)

$$k_w \left( \frac{\partial T_w}{\partial r} \right)_{r=r_2} = h(T_{w_2} - T)$$  \hspace{1cm} (6)

$$\rho \frac{\partial i}{\partial t} + \rho v \frac{\partial i}{\partial z} = \frac{2\pi \rho h}{A} (T_{w_2} - T).$$  \hspace{1cm} (7)

Momentum equation:

$$-\frac{\partial P}{\partial z} = \rho \left[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} + g + \frac{f}{2D_h} v^2 \right].$$  \hspace{1cm} (8)

Here the variables of the primary fluid are denoted with a superscript prime, the variables of the wall with a subscript \(w\) and the variables of the secondary fluid have neither a subscript nor a superscript. The heat capacitance of the wall is assumed negligible.

The above equations are greatly simplified by assuming that the time variations of the heat flux profile are not an essential feature of the instability mechanism. This assumption is supported by many investigators [1, 5, 6] and permits the heat flux profile to be evaluated from the steady-state equations. These steady-state equations for the heat flux profile and the appropriate boundary conditions are given below for the system shown in Fig. 1.

Energy equations:

$$\frac{dT_0'}{dz} = \frac{2\pi r h'}{m c_p} (T_0' - T_{w_1})$$  \hspace{1cm} (9)

$$r_h (T_0' - T_{w_1}) = k_w \left[ \frac{f}{\ln r_2 \over r_1} (T_{w_1} - T_{w_2}) \right]$$  \hspace{1cm} (10)
Boundary conditions:

\[ i_0 = i_0(z_0) \text{ at } z = z_0 \]  

\[ T'_6 = T'_6(z_4) \text{ at } z = z_4. \]

These equations consist of two differential equations, two algebraic equations, and the seven unknowns, \( T_0, T'_6, T''_6, i_0, h, \) and \( h' \). Two of the required three additional equations are the two constitutive equations for the heat transfer coefficients \( h \) and \( h' \) which are determined by using the Dittus Boelter equation \([7]\) and Chen’s \([8]\) correlation, respectively. The third equation is obtained by introducing the void fraction \( \alpha \) of the secondary fluid in the following two equations:

\[ \rho = \alpha \rho_s + (1 - \alpha) \rho_f \]  

\[ i = \frac{\alpha \rho_i i_s + (1 - \alpha) \rho_j j_f}{\rho}, \]  

which is valid for both steady-state operation and transient operation. Equations 9 through 16 are used to find the primary and the secondary steady-state temperature profiles which are in turn then used to determine the steady state heat flux profile as

\[ q_0 = q_0(z), \]  

which may be different in each region and are specified as \( q_{a0}, q_{b0}, q_{c0}, \) and \( q_{d0}, \) respectively, for the regions. This heat flux profile is then superimposed on the secondary fluid to give an equivalent system shown in Fig. 2. This equivalent system utilizes only the fundamental transient equations of the secondary fluid to analyze the density wave instability. These transient continuity, energy and momentum equations of the secondary fluid are as follows.

Continuity equation:

\[ \frac{\partial \rho}{\partial t} + \rho u \frac{\partial v}{\partial z} = 0. \]  

Energy equation:

\[ \rho \frac{\partial i}{\partial t} + \rho v \frac{\partial i}{\partial z} = \frac{q_0 \zeta}{A}. \]  

Momentum equation:

\[ -\frac{\partial P}{\partial z} = \rho g \left[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} + g + \frac{f}{2D_n} v^2 \right]. \]  

The corresponding initial and boundary conditions are the following.
Fig. 2. The equivalent system with an arbitrary heat flux profile and possible variations of the secondary fluid and inside wall temperatures.

Initial conditions:

\[ v = v_0(z) \text{ at } t = 0 \quad (21) \]
\[ \rho = \rho_0(z) \text{ at } t = 0 \quad (22) \]
\[ i = i_0(z) \text{ at } t = 0. \quad (23) \]

Boundary conditions:

\[ \Delta P = \int_{z_0}^{z_t} \frac{\partial P}{\partial z} \, dz = F_l(t) \quad (24) \]
\[ \rho = \rho_i \text{ at } z = z_0 \quad (25) \]
\[ i = i_i \text{ at } z = z_0, \quad (26) \]

where \( F_l(t) \) is a specified forcing function. The physical description of the initial and boundary conditions is as follows. The system is initially operating at the steady-state condition, and, all of a sudden, a perturbation in the total pressure drop of the channel occurs. The response of the inlet velocity to the forcing function \( F_l(t) \) must be obtained by the integration of the momentum equation, Eq. 20, while the values of the density and enthalpy of the secondary fluid at the inlet are constant and are determined from the state equations.
The above boundary conditions are given only for the subcooled liquid region. The inlet boundary condition for each region must be evaluated from the solutions for its preceding region. During steady-state operation, the boundaries of each region can clearly be identified using the energy equation. However, during the transient operation these boundaries move. The movement of these boundaries are taken into account by evaluating the time variations of the variables at the steady-state locations of the boundaries.

METHOD OF SOLUTION

The objective of this work is to analyze the stability of a generalized two-phase flow system. One way of analyzing the stability is to obtain a relation between the variation of the inlet velocity of the fluid for a variation of the total pressure drop of the channel. The Laplace transform of this relation is called the transfer function and can be used to analyze the stability of the system. However, the governing equations derived in the previous section are nonlinear, partial differential equations. An exact solution for the transfer function of these equations by analytical means is most likely inaccessible. Fortunately, a linear analysis can be used to determine the stability of nonlinear systems if the time variations of the variables from the steady state values are small. According to the Liapunov theorem presented in Willems [9], the stability of a nonlinear system corresponds to the stability of the linearized system when the deviations of the nonlinear system from the equilibrium state are sufficiently small. Since this is the case often observed during the density wave instability, this theorem is utilized for the stability analysis.

To obtain the transfer function a small perturbation technique is used where each variable is expressed as a steady-state value plus a perturbation which varies both spatially and time-wise as stated below:

\[ u(z, t) = u^0(z) + \delta u(z, t) \]  
\[ \rho(z, t) = \rho^0(z) + \delta \rho(z, t) \]  
\[ i(z, t) = i^0(z) + \delta i(z, t) \]  
\[ P(z, t) = P^0(z) + \delta P(z, t). \]

The above equations are substituted into Eqs. 18 through 28 and linearized. The resulting steady state and perturbed transient equations are given below.

**Steady-state equations**

Continuity:

\[ \frac{d(\rho^0 \mathbf{u}^0)}{dz} = 0 \]

or

\[ G_0 = \rho^0 \mathbf{u}^0 = \text{constant}. \]

Energy:

\[ G_0 \frac{d\mathbf{i}^0}{dz} = \frac{q_0 \delta}{A}. \]
A stability model for steam generators

Momentum:

\[- \frac{dP_0}{dz} = \rho_0 \left[ v_0 \frac{dV_0}{dz} + g + \frac{fV_0^2}{2D_h} \right]. \tag{34}\]

\[\text{Transient equations}\]

Continuity:

\[\frac{\partial \delta \rho}{\partial t} + \frac{\partial \delta \rho}{\partial z} + \frac{\partial \delta \rho}{\partial z} + \frac{\partial \delta V}{\partial z} + \frac{\partial \delta V}{\partial z} = 0. \tag{35}\]

Energy:

\[\rho_0 \frac{\partial \delta i}{\partial t} + G_0 \frac{\partial \delta i}{\partial z} + v_0 \frac{\partial \delta i}{\partial z} \delta V + \rho_0 \frac{\partial \delta i}{\partial z} \delta V = 0. \tag{36}\]

Momentum:

\[\frac{\partial \delta P}{\partial z} = \rho_0 \left[ \frac{\partial \delta V}{\partial t} + \frac{\partial \delta V}{\partial z} + \frac{\partial \delta V}{\partial z} + \frac{fV_0}{D_h} \delta V \right] - \frac{\delta P_0}{g, \rho_0, \frac{dz}{dz}}. \tag{37}\]

To obtain the transfer function the steady-state values of the variables are obtained; then the Laplace transform of the transient equations and the perturbations of the velocity and density are obtained at each region in terms of the inlet velocity perturbation \(\delta V_i\) to the channel.

STEADY-STATE PROPERTY PROFILES

The general forms of the steady state velocity, density, and enthalpy profiles for each region are obtained from the solution of Eqs. 31 through 34. Specific closed form expressions are then obtained for both a uniform heat flux profile and an exponential heat flux profile where the exponential profile comes from assuming a constant and uniform heat transfer coefficient for each region of the secondary fluid. The general forms are as follows.

Subcooled liquid region

The continuity and state equations yield

\[G_0 = G_1 = \text{constant} \tag{38}\]

\[\rho_0 = \rho_1(T_0(z)) \tag{39}\]

\[\rho_0(z_0) = \rho_i; \rho_0(z_1) = \rho_f. \tag{40}\]

Integrating the energy equation, Eq. 33, for a given heat flux profile \(q_{\delta 0}\), the steady-state enthalpy profile is obtained as

\[i_0 = i_i + \frac{\zeta Q_\delta(z)}{m}, \tag{41}\]

where

\[Q_\delta(z) = \int_{z_0}^{z} q_\delta(y) \, dy. \tag{42}\]
Two-phase mixture region I

The steady-state continuity and energy equations are integrated to obtain

\[ G_0 = G_i = \rho_0 v_0 \]  
\[ i_0 = i_f + \frac{nQ(g)(z)}{i} \]

where

\[ Q(g)(z) = \int_{z_1}^{z} q(y) \, dy. \]

The steady-state density and velocity profiles are obtained by using the state equation which can be obtained by combining Eqs. 15 and 16 to give

\[ \rho_0 = \frac{1}{\frac{1}{\rho_f} + \frac{\pi_1 Q(g)(z)}{G_i}} \]

where

\[ \pi_1 = \frac{V_{fr} \cdot c}{A_{fr}}. \]

Now Eq. 43 can be used to obtain the velocity

\[ v_0 = \frac{G_i}{\rho_0} = \frac{G_i}{\rho_f} + \pi_1 Q(g)(z) \]

or equivalently

\[ v_0 = N_i v_i + \pi_1 Q(g)(z), \]

where

\[ N_i = \frac{\rho_0(z_0)}{\rho_0(z_1)} = \frac{\rho_i}{\rho_f}. \]

Two-phase mixture region II

In this region the steady-state velocity, density, and enthalpy equations are obtained in the same way as for the previous region. They are

\[ v_0 = N_i v_i + \pi_1 Q(g)(z_2) + \pi_1 Q_i(z) \]
\[ \rho_0 = \frac{G_i}{v_0} \]
\[ i_0 = i_f + x_i i_{fr} + \frac{nQ_i(z)}{i} \]
where $x_e$ is the quality at which dry-out occurs and

$$Q_e(z) = \int_{z_2}^{z} q_{e0}(y) \, dy.$$  \hspace{1cm} (54)

**Superheated vapor region**

Since only the vapor phase is present in this region, the solutions of the continuity, energy and state equations are similar to the subcooled liquid region equations and are

$$G_0 = G_i,$$  \hspace{1cm} (55)

$$\rho_0 = \rho_k(T_0(z)),$$  \hspace{1cm} (56)

$$i_0 = i_x + \frac{Q_d(z)}{\dot{m}},$$  \hspace{1cm} (57)

where

$$Q_d(z) = \int_{z_2}^{z} q_{d0}(y) \, dy.$$  \hspace{1cm} (58)

**THE TRANSFER FUNCTION**

The transient behavior of the system is investigated by introducing an unknown perturbation into the inlet velocity and determining the pressure drop response in each region. In the single phase regions the time variations of the density is neglected by assuming that the density is a function of the steady state enthalpy. The corresponding solutions are presented below.

**Inlet restriction**

Considering an orifice as the inlet restriction the pressure drop response to an inlet velocity perturbation $v_i(t)$ is obtained. The density and enthalpy of the fluid across the restriction are assumed to be constant. Assuming the steady-state pressure drop equations are approximately valid for the transient flow through the inlet restriction, its pressure drop response is

$$\Delta P_{io} + \delta P_i = \frac{K_i(\rho_i + \delta \rho_i)}{g_c} [v_i^2 + 2v_i \delta v_i + \delta v_i^2].$$  \hspace{1cm} (59)

Linearizing and using the steady-state relations, the perturbed pressure drop response in the Laplace domain is

$$\overline{\delta P_i} = \frac{2K_iG_i \overline{\delta v_i}}{g_c}.$$  \hspace{1cm} (60)

**Subcooled liquid region**

In this region, the density is assumed to be a function of only the steady-state enthalpy;
hence
\[ \delta \rho = 0. \]  
\hspace{2cm} (61)

Using this in Eq. 18 yields
\[ \rho_0 v_0 + \rho_0 \delta v = \rho_i v_i + \rho_i \delta v_i. \]  
\hspace{2cm} (62)

Hence
\[ \delta v = \frac{\rho_i}{\rho_0} \delta v_i. \]  
\hspace{2cm} (63)

Using Eqs. 61 and 63 together with a Laplace transformation and integration of the perturbed momentum equation in space yields
\[ \Delta \delta \rho_a = \frac{g_i \delta v_i}{g_c} \left[ \frac{s_{L_a}}{v_i} + 2(N_i - 1) + \frac{f_a(1 + N_i)l_a}{2D_i} \right]. \]  
\hspace{2cm} (64)

Here the bar over the variables denote the Laplace transformation
\[ \delta v_i = (\delta v_i(t)). \]  
\hspace{2cm} (65)

**Two-phase mixture region I**

In this region, the nonlinear continuity and energy equations are used to obtain the velocity and the density responses. First, the void fraction \( \alpha \) is obtained from Eq. 15 as
\[ \alpha = \frac{\rho_f - \rho}{\rho_f - \rho_g} \]  
\hspace{2cm} (66)

and substitution of \( \alpha \) into Eq. 16 and taking the differential yields
\[ di = \frac{i_{fg}}{V_w} \delta \rho \]  
\hspace{2cm} (67)

Substituting this into the energy equation yields
\[ \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} = - \pi_i q_{bo} \rho. \]  
\hspace{2cm} (68)

The continuity equation is rewritten as
\[ \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} = - \frac{\partial v}{\partial z} \rho. \]  
\hspace{2cm} (69)

Hence, Eqs. 68 and 69 yield
\[ \frac{\partial v}{\partial z} = \pi_i q_{bo}. \]  
\hspace{2cm} (70)
The integration of this equation together with the boundary condition gives

\[ v = v_0 + \delta v = N_i v_i + \pi_i Q \beta(z) + N_i \delta v_i. \]  

(71)

Using the steady-state velocity relation gives

\[ \delta v = N_i \delta v_i(t). \]  

(72)

Since the velocity response is obtained the continuity equation can be used to obtain the density response. Let

\[ \phi(z, t) = \ln \frac{\rho(z, t)}{\rho_f}, \]  

(73)

then

\[ \delta \phi = \frac{\delta \rho}{\rho}. \]  

(74)

Using these relations, the Laplace transform of the linearized perturbed continuity equation becomes

\[ \frac{d}{dz} \frac{\delta \phi}{v_0} + \frac{s \delta \phi}{v_0} = \frac{\delta v}{v_0} \frac{dv_0}{dz}. \]  

(75)

The boundary condition can be evaluated by using the density response in the previous region, which yields

\[ \overline{\delta \rho}(z, s) = 0, \]  

(76)

hence,

\[ \overline{\delta \phi}(z, s) = 0. \]  

(77)

By defining

\[ E_\delta(z) = \int_{z_1}^{z} \frac{dy}{v_0(y)}, \]  

(78)

the solution is obtained as

\[ \overline{\delta \phi} = N_i w_\delta(z, s) \delta v_i, \]  

(79)

where

\[ w_\delta(z, s) = \int_{z_1}^{z} \left( \frac{dv_0}{v_0^2} \right)^{\frac{1}{2}} \exp \left[ s E_\delta(y) - s E_\delta(z) \right] dy. \]  

(80)

Using the velocity and the density response in the momentum equation the Laplace
transform of the pressure drop in this region can be expressed as

$$\Delta \delta P_\delta = \frac{N_i \delta v_i}{g_e} \left\{ \int_{z_1}^{z_2} \rho_\delta(\tau) \left[ s + \frac{dv_0(\tau)}{dz} \right] d\tau + \int_{z_1}^{z_2} g_e \frac{dP_0}{dz} w_\delta(z, s) dz \right\} \quad (81)$$

Two-phase mixture region II

Using the same approach as for the previous region, the velocity, density, and pressure drop responses are obtained and given below:

$$\overline{\delta v} = N_i \overline{\delta v_i} \quad (82)$$

$$\overline{\delta \phi} = \overline{\delta \phi(z_2, s)} \exp[sE_c(z_2) - sE_c(z)] + w_e N_i \overline{\delta v_i}, \quad (83)$$

where

$$E_c(z) = \int_{z_2}^{z} \frac{dy}{v_0(y)} \quad (84)$$

$$w_e = \int_{z_2}^{z} \frac{(dv_0/dy)}{v_0^2} \exp[sE_c(y) - sE_c(z)] dy \quad (85)$$

$$\overline{\delta \phi(z_2, s)} = N_i w_h(z_2, s) \overline{\delta v_i}. \quad (86)$$

The corresponding pressure drop response can be obtained from the following equation:

$$\overline{\Delta \delta P_c} = \frac{N_i \overline{\delta v_i}}{g_e} \left\{ \int_{z_1}^{z_2} \rho_\delta(z) \left[ s + \frac{dv_0(z)}{dz} \right] dz + \frac{f_b G L_0}{D_h} \right.$$ 

$$- \left. \int_{z_1}^{z_2} g_e \frac{dP_0}{dz} \left[ w_h(z, s) \exp[sE_h(z) - sE_h(z)] + w_e(s, z) \right] dz \right\}. \quad (87)$$

Superheated vapor region

The governing equations for this region are similar to the equations of the subcooled liquid region. Since the density is assumed to be a function of only the steady state enthalpy, the continuity equation yields

$$\rho_\delta v_0 + \rho_0 \delta v = G(t). \quad (88)$$

For the two-phase mixture region II we have

$$G = \rho_0(z_3)v_0(z_3) + \rho_0(z_3)\delta v(z_3) + v_0(z_3)\delta \rho(z_3). \quad (89)$$

Equating Eqs. 88 and 89 gives

$$\rho_0 \delta v = v_0(z_3)\delta \rho(z_3) + \rho_0(z_3)\delta v(z_3) \quad (90)$$
or
\[
\frac{\delta v}{\delta u} = N_i \delta v_i \left\{ \frac{G_i \delta \phi (z_2, s) \exp[sE_i(x)] - sE_i(z_3)] + w_i(z_3, s) + \frac{\rho}{\rho_0} \right\} (91)
\]

\[
\frac{\delta \rho}{\delta v} = 0. \quad (92)
\]

Note that these values of the velocity and density gives a discontinuity in their values at \( z = z_j \). However, this approximation simplifies the problem and takes into account the movement of this boundary. This approximation is used throughout this paper.

The Laplace transform of the momentum equation and its integration in this region gives the pressure drop response for this region:

\[
\Delta \delta P_d = \frac{1}{g_c} \left\{ \int_{z_j}^{z_i} \left[ s\rho_0 + \rho_0 \frac{\delta v_0}{\delta z} + \frac{f_2 g}{D_h} \right] \delta v \, dz + G_k \delta v(z_4) - \delta v(z_3) \right\}. (93)
\]

**Exit restriction**

The pressure drop at the exit restriction is

\[
\Delta \delta P_e = \frac{2K_c \delta \nu(z_4)}{g_c}. \quad (94)
\]

The total pressure drop response is obtained by summing the pressure drop responses of each region and the restrictions as

\[
\Delta \delta P = \Delta \delta P_t + \Delta \delta P_d + \Delta \delta P_b + \Delta \delta P_c + \Delta \delta P_e + \Delta \delta P_r = H(s) \delta v_i. \quad (95)
\]

Since the pressure drop responses of each region are expressed in terms of the inlet velocity variation, the following relation can be obtained. Here,

\[
\frac{1}{H(s)} = \frac{\delta v_i}{\Delta \delta P_t}
\]

is called the transfer function of the system and is used to analyze the stability of the system.

**STABILITY ANALYSIS**

The integration is carried out to obtain specific forms of the transfer functions for both the uniform and the exponential heat flux profile cases. Even though these transfer functions have different parameters their general form can be written as follows:

\[
\frac{\delta v_i}{\Delta \delta P_t} = \frac{G_i(s)}{G_c(s, e^{\nu})}. \quad (97)
\]

This function contains exponential terms which indicate that the response of the inlet velocity at any instant is coupled with the past history of the process.

The function \( G_c(s, e^{\nu}) \) is called the characteristic function of the system. The characteristic equation is obtained by setting this characteristic function equal to zero. A system is stable if and only if all the roots of the characteristic equation lie in some closed subset of the open
left-half s-plane. Hence, most of the techniques for the stability analysis of a system search for the location of the roots of the characteristic equation. Since the poles of the transfer function are the roots of the characteristic equation, determining the number of poles of the transfer function in the right half of the s-plane gives the stability information about the system. These techniques use the results of the so-called encirclement theorem. This theorem gives a relation between the number of zeros, number of poles and the number of encirclements of the origin for a rational complex function $G(s)$ under the following mapping:

$$w = G(s).$$  \hspace{1cm} (98)

Let $c$ be a simple closed curve in the s-plane and $c_1$ is the image of $c$ under this mapping. If $c$ is traversed once in the clockwise direction, then the number of encirclements $N$ of the origin by $c_1$ in the counterclockwise direction is given by

$$N = P - Z,$$  \hspace{1cm} (99)

where $P$ is the number of poles of $G(s)$ and $Z$ the number of zeros of $G(s)$. This theorem is valid only if no poles or zeros lie on the contour $c$.

**COMPARISON WITH EXPERIMENTAL RESULTS**

The theoretical model developed in the previous sections is used to analyze the stability of three different experimental works. These experiments consider a uniformly heated, horizontal two-phase flow system, a uniformly heated vertical two-phase flow system and a nonuniformly heated vertical steam generator.

Stenning, Veziroglu and Callahan [10] experimentally investigated the stability of a uniformly heated, horizontal two-phase flow system and used freon-11 as the working fluid. Several operating conditions are tested for stability at heat inputs of both 343 W (1170 Btu/h) and 375 W (1280 Btu/h). In Figs. 3 and 4 the experimentally observed density wave oscillations are superimposed on the observed lower frequency pressure drop oscil-

![Fig. 3. Pressure drop and mass flow rate oscillations for a heat input of 1170 Btu/h [10].](image-url)
A stability model for steam generators

Theoretical estimations of the density wave oscillations are also indicated in these figures and are in almost excellent agreement with the experimental oscillations.

The work of Dijkman [2] involves the experimental investigation of density wave oscillations in a uniformly heated, vertical two-phase flow system.

The theoretical estimations of the density wave oscillations are comparable with the experimental results. However, as the inlet subcooling increases, the theoretical estimations of the threshold of oscillations deviate more from the experimental results.

Dijkman also predicted the instability of the density waves by developing his own theoretical model. His theoretical predictions, the theoretical predictions of this work and his experimental observations of the threshold of the density wave instability are compared below:

<table>
<thead>
<tr>
<th>Subcooling, °C (°F)</th>
<th>Experimental threshold power, kW (Btu/h) (Dijkman)</th>
<th>Theoretical threshold power, kW (Btu/h) (Dijkman)</th>
<th>Theoretical threshold power, kW (Btu/h) (present work)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>190</td>
<td>190</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>(648,000)</td>
<td>(648,000)</td>
<td>(614,000)</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>155</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>(580,000)</td>
<td>(529,000)</td>
<td>(631,000)</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>—</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>(580,000)</td>
<td>—</td>
<td>(745,000)</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>(683,000)</td>
<td>(745,000)</td>
<td>(745,000)</td>
</tr>
</tbody>
</table>

**Fig. 4. Pressure drop and mass flow rate oscillations for a heat input of 1280 Btu/h [10].**

The data for nonuniform heat flux profile systems, especially for once-through steam generators, is not available in the open literature. However, unpublished proprietary data was obtained for a once-through steam generator from a company in the power generation field.

The stability analysis is developed by approximating each steady state heat flux by mathematical functions. The predictions of this analysis are compared with the experimental
CONCLUSION

In this study a method is developed to analyze the stability of any nonuniform heat flux profile system where, if necessary, the heat flux profile can be approximated by suitable mathematical expressions. This method is illustrated by the technique used to predict the stability of the once-through steam generator. The theoretical analysis is generalized to apply to a wider variety of operating conditions and systems such as a heat exchanger. An equivalent system is developed by decoupling the transient effects of the primary fluid and the tube wall on the stability of a secondary vaporizing fluid. This is demonstrated by imposing the steady-state heat flux profile on the secondary fluid to investigate its stability. It is observed that the homogeneous model is capable of predicting the instability of the
A stability model for steam generators

various systems even though the drift number of the two-phase flowing fluid is in the range which may suggest the drift flux model is appropriate.

The results of this theoretical analysis are compared with the results of three different experimental works. These experiments have different geometrical and operational arrangements and two of them have a uniform heat flux profile while the third has nonuniform, but approximately exponential, heat flux profiles. The theoretical predictions of the steady-state conditions and of the system stability agree quite well both qualitatively and quantitatively with the experimental observations.

The present analysis can also be used to analyze the stability of condensing two-phase flow systems, to give information about the transient behavior of two-phase flow systems for small changes of the variables and to give an insight into the transient behavior of a system for large changes of the variables.

The mechanism leading to the density wave oscillations of two-phase flow systems is still not clearly explained in the literature even though there are some "rules of thumb" criteria regarding their stability. The present analysis gives the solution of the perturbed property profiles in the Laplace domain and can provide some necessary ingredients for a future theoretical study of the mechanism of density wave oscillations.

NOMENCLATURE

\[ A = \text{cross sectional area, m}^2 \ (\text{ft}^2) \]
\[ C_D = \text{discharge coefficient} \]
\[ C_p = \text{specific heat, J/(kg} \cdot \text{K) (Btu/(lb m} \cdot \text{OF})} \]
\[ D_h = \text{hydraulic diameter, m (ft)} \]
\[ E(y) = \int [dy/u_0(y)], \text{a time parameter, s} \]
\[ F_i(t) = \text{total pressure drop forcing function, Pa (lb f/ft}^2) \]
\[ f = \text{Darcy friction factor} \]
\[ G = \text{mass flux, kg/(m}^2-s) \ [\text{lb m/(ft}^2-s)] \]
\[ G_i = \text{inlet mass flux, kg/(m}^2-s) \ [\text{lb m/(ft}^2-s)] \]
\[ g = \text{gravitational body force field, m/s}^2 \ (\text{ft/s}^2) \]
\[ H(s) = \Delta \delta P_i/\delta v, \text{characteristic function, N-s/m}^3 \ (\text{lb f-s/ft}^3) \]
\[ h = \text{heat transfer coefficient, J/(m}^2-s} \cdot \text{K) [Btu/(ft}^2-s} \cdot \text{OF}] \]
\[ i = \text{enthalpy, J/kg (Btu/lb m)} \]
\[ i_{\text{evap}} = \text{enthalpy of evaporation, J/kg (Btu/lb m)} \]
\[ K = \text{restriction coefficient} \]
\[ k = \text{thermal conductivity, J/(m-s} \cdot \text{K) [Btu/(ft-s} \cdot \text{OF}]} \]
\[ L = \text{length, m (ft)} \]
\[ m, M = \text{mass flow rate, kg/s (lb m/s)} \]
\[ N_i = \rho_l/\rho_f, \text{inlet density ratio} \]
\[ N_0 = \rho_s/\rho_f, \text{density ratio} \]
\[ P = \text{pressure, Pa (lb f/ft}^2) \]
\[ q_{l0} = \text{steady-state heat flux profile for subcooled liquid region, W/m}^2 \ [\text{Btu/(ft}^2-s)] \]
\[ q_{\text{bo}} = \text{steady-state heat flux profile for two-phase mixture region I, W/m}^2 \ [\text{Btu/(ft}^2-s)] \]
\[ q_{l0} = \text{steady-state heat flux profile for two-phase mixture region II, W/m}^2 \ [\text{Btu/(ft}^2-s)] \]
\[ q_{\text{d0}} = \text{steady-state heat flux profile for superheated vapor region, W/m}^2 \ [\text{Btu/(ft}^2-s)] \]
\[ Re = \rho v D_u/\mu, \text{Reynolds number} \]
\[ r_1 = \text{outside diameter, m (ft)} \]
\[ r_2 = \text{inside diameter, m (ft)} \]
\[ s = \text{Laplace's variable, s}^{-1} \]
\[ T = \text{temperature, °K (°F)} \]
\[ t = \text{time, s} \]
\( V_s = \) specific volume, m\(^3\)/kg (ft\(^3\)/lb m)

\( v = \) velocity, m/s (ft/s)

\( x = \) quality

\( x_c = \) dry-out quality

\( z = \) axial coordinate, m (ft)

\( z_0 = \) location of the inlet boundary of the subcooled liquid region, m (ft)

\( z_1 = \) location of the inlet boundary of the two-phase mixture region I, m (ft)

\( z_2 = \) location of the inlet boundary of the two-phase mixture region II, m (ft)

\( z_3 = \) location of the inlet boundary of the superheated vapor region, m (ft)

\( z_4 = \) location of the exit boundary of the superheated

\( \sigma = \) void fraction

\( \delta = \) perturbation of the variables

\( \Delta P = \) pressure drop across a region, Pa (lb f/ft\(^2\))

\( \Delta P_t = \) total pressure drop, Pa (lb f/ft\(^2\))

\( \Delta \delta P_t = \) total perturbed pressure drop, Pa (lb f/ft\(^2\))

\( \zeta = \) perimeter, m (ft)

\( \pi = V_s \zeta/A_{ij} \), a parameter, m\(^2\)/J (ft\(^2\)/Btu)

\( \rho = \) density, kg/m\(^3\) (lb m/ft\(^3\))

\( \phi = \ln(\rho/\rho_0) \), dimensionless density variable

**Subscripts**

\( a = \) subcooled liquid region

\( b = \) two-phase mixture region I

\( c = \) two-phase mixture region II

\( d = \) superheated vapor region

\( e = \) exit of the channel

\( f = \) saturated liquid

\( g = \) saturated vapor

\( i = \) inlet of the channel

\( 0 = \) steady state condition

\( w = \) wall of the channel

\( w_1 = \) outside surface of the wall

\( w_2 = \) inside surface of the wall

**Superscript**

\( ' = \) primary fluid

**REFERENCES**


