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Palatini–Born–Infeld gravity, bouncing universe, and black hole formation

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ABSTRACT

We consider the Palatini formalism of the Born–Infeld gravity. In the Palatini formalism, the propagating mode is only graviton, whose situation is different from that in the metric formalism. We discuss the FRW cosmology by using an effective potential. Especially we consider the condition that the bouncing could occur. We also give some speculations about the black hole formation

1. Introduction

In recent years, many kinds of modified gravity theories have been proposed and investigated with several motivations. First motivation could be the quantum gravity. We have not obtained any consistent quantum theory of the gravity. Although Einstein's general relativity is a successful theory of the classical theory of gravity, we cannot obtain the quantum theory based on the general relativity due to the non-renormalizability. Motivated by the quantum gravity, many kinds of modification of the Einstein gravity have been proposed and investigated. On the other hand, motivated by the accelerating expansion of the present universe, we are considering many kinds of gravity theories beyond the Einstein gravity (for review, see [1]). As a model of such modified gravities, in this paper, we consider the Born–Infeld gravity [2] in the Palatini formalism [3]. The Born–Infeld type theory was first proposed as a non-linear model of electro-magnetics [4]. In the Maxwell electro-magnetics, the action of the electromagnetic field has no dimensional parameter but in the Born–Infeld model, a scale can be introduced. Because the action includes the square root, there appears the upper limit in the strength given by the scale, which may have suggested that there might not appear the divergence in the quantum field theory different from the standard quantum electro-dynamics. The Born–Infeld type deformation was also considered in gravity theory [2] and it has been expected that the quantum theory of the Born–Infeld gravity might be finite and there might not appear any divergence because there could be an upper limit in the magnitude of the curvature. By including the idea by Eddington [5], the Born–Infeld gravity has been formulated in the Palatini formalism, where the connections are variables independent of the metric tensor. The corresponding action is given by

$$S = \frac{1}{\kappa^2 b}\int d^4x \left\{ \sqrt{\left|\det\left(g_{\mu\nu} + bR_{\mu\nu}\right)\right|} - \sqrt{\left|\det\left(g_{\mu\nu}\right)\right|} \right\} + S_{\text{matter}} \,. \tag{1}$$

Here κ is the gravitational constant corresponding to the Einstein gravity and we introduce a new parameter b, which has the dimension of the square of the length. In Eq. (1), S_{matter} is an action for the matters and $R_{\mu\nu}$ is the Ricci tensor assumed to be symmetric and defined by $R_{\mu\nu} = -\Gamma^\rho_{\mu\rho,\nu} + \Gamma^\rho_{\mu\nu,\rho} - \Gamma^\eta_{\mu\rho}\Gamma^\rho_{\nu\eta} + \Gamma^\eta_{\mu\nu}\Gamma^\rho_{\rho\eta}$ in terms of the connection $\Gamma^\rho_{\mu\nu}$. We regard the connection $\Gamma^\rho_{\mu\nu}$ as a variable independent of the metric $g_{\mu\nu}$. The theory described by the above action (1) is often called the Eddington inspired Born–Infeld gravity. By the variations of the action with respect to the metric $g_{\mu\nu}$ and the connection $\Gamma^\lambda_{\mu\nu}$, we obtain the following equations, respectively,

$$0 = \sqrt{-P}\left(P^{-1}\right)^{\mu\nu} - \sqrt{-g}g^{\mu\nu} - b\kappa^2\sqrt{-g}T^{\mu\nu} \,, \tag{2}$$

$$0 = \nabla_\lambda\left(\sqrt{-P}\left(P^{-1}\right)^{\mu\nu}\right) = 0 \,. \tag{3}$$

Here $T^{\mu\nu}$ is the energy–momentum tensor of the matters and $P_{\mu\nu}$ is defined by

$$P_{\mu\nu} \equiv g_{\mu\nu} + bR_{\mu\nu} \,, \tag{4}$$

and P^{-1} is the inverse of the matrix $P_{\mu\nu}$, that is, $\left(P^{-1}\right)^{\mu\rho} P_{\rho\nu} = \delta^\mu_{\ \nu}$. Then P^{-1} can be expressed by the infinite power series of

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$R_{\mu\nu}$, $\left(P^{-1}\right)^{\mu\nu} = g^{\mu\nu} - b g^{\mu\rho} R_{\rho\sigma} g^{\sigma\nu} + b^2 g^{\mu\rho} R_{\rho\sigma} g^{\sigma\tau} R_{\tau\eta} g^{\eta\nu} - \cdots$. On the other hand $P^{\mu\nu}$ is defined by $P^{\mu\nu} \equiv g^{\mu\rho} P_{\rho\sigma} g^{\sigma\nu} = g^{\mu\nu} + b R^{\mu\nu}$.

In the metric formalism of the Born–Infeld gravity, theory includes a ghost in general and we need to tune the action by adding the higher derivative terms so that the ghost does not appear [2]. In the Palatini formalism, however, there does not appear ghost [3] and the only propagating mode is massless graviton. Then in the leading order, the standard Newton law can be reproduced although there also appear some corrections [6]. We should note that the Born–Infeld gravity in the Palatini formalism is equivalent to the Einstein gravity in the vacuum although they are different from each other when the matter exists.

The cosmology by the Born–Infeld gravity in the Palatini formalism has been considered in [6,7] by including matters. The development of the universe in the Born–Infeld gravity in the Palatini formalism shows the behavior which is different from that in the Einstein gravity and there often appears the bouncing universe, where the shrinking universe turns to expand. We claim, however, that the treatments in the previous papers were not complete and we re-examine the cosmology and we show that there anyway appears the bouncing universe as a solution.

Even in the Palatini–Born–Infeld gravity, the Schwarzschild space–time and the Kerr black hole space–time are exact solutions. Because these solutions are vacuum solution, they are equivalent to the solutions in the Einstein gravity and therefore, for example, the expressions of the black hole entropies are identical with those in the Einstein gravity (in case of the metric formalism, see [8]).

In this paper, we investigate the FRW cosmology. We should note that in the previous works, there were too strong constraints on the variables but we consider more general treatment in this paper. We consider the cosmology by including dust as a matter and show that the bouncing universe can be realized, whose behavior is, in some sense, similar to that in the loop quantum gravity [9–11]. We also give some speculations about the formation of the black hole by considering the collapse of the sphere of the dust. Because the pressure of the dust vanishes, we can regard the inside of the sphere as the FRW universe. Then by using the results in the FRW universe, we show that the small black hole could not be formed by the bouncing although the large black holes might be created. We should note, however, we do not use the junction conditions in the Born–Infeld gravity in the Palatini formalism but we use those in the Einstein gravity. This is because the junction conditions in the Born–Infeld gravity in the Palatini formalism are very complicated and, to be honest, we have not found the full expressions of the junction conditions. We will discuss this point in more detail later. Therefore the analyses in this paper could not be justified in some cases but by using the obtained results, we can give some realistic speculations.

2. FRW universe with dust

We consider the FRW space–time with flat spacial part,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} \left(dx^i\right)^2 , \tag{5}$$

and assume that the non-vanishing components of the connection are given by

$$\Gamma^t_{tt} = A(t) , \quad \Gamma^t_{ij} = a(t)^2 B(t) \delta ij , \quad \Gamma^i_{jt} = \Gamma^i_{tj} = C(t) \delta^i{}_j . \tag{6}$$

In the Einstein gravity, the metric (6) implies $A = 0$, $B = C = H \equiv \dot a/a$.

In the previous works [6,7], the FRW metric was assumed for $P_{\mu\nu}$ in Eq. (4),

$$ds_P^2 = \sum_{\mu,\nu=0}^{4} P_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \tilde a(t)^2 \sum_{i=1,2,3} \left(dx^i\right)^2 , \tag{7}$$

in [6] and

$$ds_P^2 = \sum_{\mu,\nu=0}^{4} P_{\mu\nu} dx^\mu dx^\nu = -\tilde b(t)^2 dt^2 + \tilde a(t)^2 \sum_{i=1,2,3} \left(dx^i\right)^2 , \tag{8}$$

in [7] and the connection $\Gamma^\lambda_{\mu\nu}$ was given by $P_{\mu\nu}$:

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} \left(P^{-1}\right)^{\lambda\rho} \left(\partial_\mu P_{\rho\nu} + \partial_\nu P_{\mu\rho} - \partial_\rho P_{\mu\nu}\right) , \tag{9}$$

which, however, reduces the degrees of freedom in $\Gamma^\lambda_{\mu\nu}$. We should note that $P_{\mu\nu}$ is not the fundamental variable in the Palatini formalism but $\Gamma^\lambda_{\mu\nu}$ is the fundamental one. As we find in Eq. (6), even if we assume homogeneous and isotropic universe, there appear three undetermined variables A, B, and C but in Eq. (7), only one undetermined variable $\tilde a$ appears and in Eq. (9), two variables $\tilde b$ and $\tilde a$. Therefore the assumption (7) or (9) may conflict with the equations of the Born–Infeld gravity in the Palatini formalism. In fact, if we assume Eq. (7), we find $A = 0$ and $B = C$, which conflict with the analysis given later in this paper. Even if we assume Eq. (8), there occur conflictions.[1] Although there might be cases that the assumption (7) or (8) could be justified, we do not know any a priori reason. Therefore we like to re-investigate the cosmology under the assumptions (5) and (6).

We now assume that the matter is given by the dust whose pressure p vanishes and the energy density is denoted by ρ. One of the reasons why we consider the dust as the matter is for the simplicity although it is not so difficult if we consider general perfect fluid as the matter. Another reason is because we like to compare the black hole formation in this model with the Oppenheimer–Snyder collapse in the general relativity [14], where the inside of the collapsing star is described by the FRW universe filled with dust. Because the dust evolves in the space–time whose metric is given by $g_{\mu\nu}$, we obtain the conservation law identical with that in the Einstein gravity, $\nabla^{(g)\mu} T_{\mu\nu} = 0$. Here $\nabla^{(g)\mu}$ is the covariant derivative where the connection is given in terms of $g_{\mu\nu}$ as in the standard Einstein gravity but not by $P_{\mu\nu}$. Then in the FRW universe (5), we obtain the standard conservation law

$$\dot\rho + 3Hp = 0 , \tag{10}$$

and we find

$$\rho = \rho_0 a^{-3} . \tag{11}$$

Then we obtain the following equations (the derivations of the following equations are given in Appendix A):

$$A = C - H = -\frac{3}{4} b\kappa^2 H\rho \left(1 + b\kappa^2\rho\right)^{-1} , \tag{12}$$

$$B = H + \frac{b\kappa^2}{4} H\rho , \tag{13}$$

$$C = H - \frac{3}{4} b\kappa^2 H\rho \left(1 + b\kappa^2\rho\right)^{-1} , \tag{14}$$

$$b\kappa^2\rho = \left\{1 + b\left(\dot H + 3H^2 + \frac{b\kappa^2}{2}\dot H\rho\right)\right\}^2 - 1 . \tag{15}$$

[1] Even in case of Eq. (8), we find $A = \dot{\tilde b}/\tilde b$, $B = \dot{\tilde a}/ab^2$, $C = \dot{\tilde a}/a$, which satisfy the relation $2A = \dot C/C - \dot B/B$. If we use this relation for Eqs. (12), (13), and (14) with Eq. (10), we obtain

$$b\kappa^2\rho = -8 ,$$

which is inconsistent. Furthermore we should note that the difference between Eqs. (7) and (9) is nothing but the parametrization of the time coordinate.

When $b<0$, Eq. (15) tells that there is an upper limit ρ_u for the energy density ρ:

$$\rho_u = -\frac{1}{b\kappa^2}\,. \tag{16}$$

Because $H=\dot a/a$ and $\dot H=\ddot a/a-\dot a^2/a^2$, by using Eq. (11), Eq. (15) can be rewritten as

$$b\kappa^2\rho_0 a^{-3} = \left\{1+b\left(\frac{\ddot a}{a}+2\left(\frac{\dot a}{a}\right)^2 + \frac{b\kappa^2}{4}\left(\frac{\ddot a}{a}-\left(\frac{\dot a}{a}\right)^2\right)\rho_0 a^{-3}\right)\right\}^2 - 1\,, \tag{17}$$

which is a single equation with respect to the scale factor a. If we use the e-foldings N defined by $a=e^N$, we obtain

$$b\kappa^2\rho_0 e^{-3N} = \left\{1+b\left(\ddot N+3\dot N^2+\frac{b\kappa^2}{4}\ddot N\rho_0 e^{-3N}\right)\right\}^2 - 1\,, \tag{18}$$

or

$$\ddot N = -\frac{3\dot N^2}{1+\frac{b\kappa^2\rho_0}{4}e^{-3N}} - \frac{1-\sqrt{1+b\kappa^2\rho_0 e^{-3N}}}{b\left(1+\frac{b\kappa^2\rho_0}{4}e^{-3N}\right)}\,. \tag{19}$$

By an analogy with the Newton equation in the classical mechanics, the first term in the r.h.s. could be an analogue of the drag force in the fluid when the Reynolds number is large and the second term could be a force by a potential, which we denote by $F(N)$ as

$$F(N) \equiv -\frac{1-\sqrt{1+b\kappa^2\rho_0 e^{-3N}}}{b\left(1+\frac{b\kappa^2\rho_0}{4}e^{-3N}\right)}\,. \tag{20}$$

We should note that the potential force $F(N)$ is positive, which does not depend on the sign of the parameter b and therefore the force acts so that the e-foldings N increase. Then even if the universe is shrinking, it may turn to expand. Because

$$F'(N) \equiv \frac{3\kappa^2\rho_0 e^{-3N}\left(1-\frac{b\kappa^2\rho_0}{2}e^{-3N}+\sqrt{1+b\kappa^2\rho_0 e^{-3N}}\right)}{4\left(1+\frac{b\kappa^2\rho_0}{4}e^{-3N}\right)^2\sqrt{1+b\kappa^2\rho_0 e^{-3N}}}\,, \tag{21}$$

we find:

- When $b>0$, there is a maximum $F(N)=2/3b$ at $b\kappa^2\rho_0 e^{-3N}=8$. We also find that $F(N)\to 0$ when $N\to+\infty$ and $F(N)\to 0$ when $N\to-\infty$.
- When $b<0$, we find $F'(N)<0$ and therefore there is a maximum $F(N)=-4/3b$ at $b\kappa^2\rho_0 e^{-3N}=-1$ and $F(N)\to 0$ when $N\to+\infty$, again.

Eq. (19) can be further rewritten in the following form:

$$0=\frac{d^2}{dt^2}\left(\frac{e^{3N}}{3}+\frac{b\kappa^2\rho_0}{4}N\right) + \frac{e^{3N}\left(1-\sqrt{1+b\kappa^2\rho_0 e^{-3N}}\right)}{b}\,, \tag{22}$$

which tells that there is a conserved quantity E,

$$\begin{aligned} E &= \frac{1}{2}\left\{\frac{d}{dt}\left(\frac{e^{3N}}{3}+\frac{b\kappa^2\rho_0}{4}N\right)\right\}^2 + \int^N dN\,\frac{e^{3N}\left(1-\sqrt{1+b\kappa^2\rho_0 e^{-3N}}\right)\left(e^{3N}+\frac{b\kappa^2}{4}\rho_0\right)}{b} \\ &= \frac{1}{2}\left\{\frac{d}{dt}\left(\frac{e^{3N}}{3}+\frac{b\kappa^2\rho_0}{4}N\right)\right\}^2 + V(N)\,, \end{aligned} \tag{23}$$

which corresponds to the total energy in the classical mechanics. Here $V(N)$ is given by

$$V(N) = \frac{e^{6N}}{6b}\left(1+\frac{1}{2}b\kappa^2\rho_0 e^{-3N}\right)\left(1-\sqrt{1+b\kappa^2\rho_0 e^{-3N}}\right) - \frac{b\kappa^2\rho_0}{12b}e^{3N}\sqrt{1+b\kappa^2\rho_0 e^{-3N}}\,. \tag{24}$$

When N is positive and large, $V(N)$ behaves as

$$V(N) \sim -\frac{\kappa^2\rho_0 e^{3N}}{6}\,. \tag{25}$$

In case $b>0$, when N is negative and large, we find

$$V(N) \sim -\frac{\left(b\kappa^2\rho_0\right)^{\frac{3}{2}} e^{\frac{3}{2}N}}{6b}\,. \tag{26}$$

On the other hand, in case $b<0$, there is a maximum in $V(N)$ when $1+b\kappa^2\rho_0 e^{-3N}=0$:

$$V(N) = V_{\rm max} \equiv \frac{\left(b\kappa^2\rho_0\right)^2}{12b} < 0\,. \tag{27}$$

We now assume that the universe may have started from $N\to+\infty$ and after that the universe has started to shrink. Then from the above results, by the analogy with the classical mechanics, we find the following:

- In case $b>0$, if $E<0$, the shrinking of the universe will stop and turn to expand. On the other hand if $E>0$, the universe will continue to shrink and the scale factor a vanishes in the infinite future.
- In case $b<0$, if $E<V_{\rm max}$, the shrinking of the universe will stop and turn to expand. On the other hand if $E>V_{\rm max}$, the universe will reach the singular point at $1+b\kappa^2\rho_0 e^{-3N}=0$.

In order to estimate E, we now solve Eq. (19) by assuming $N\gg 1$. Then Eq. (19) can be rewritten as

$$\ddot N + 3\dot N^2 - \frac{\kappa^2\rho_0 e^{-3N}}{2} = \frac{3}{4}b\kappa^2\rho_0 e^{-3N}\dot N^2 - \frac{\left(b\kappa^2\rho_0\right)^2}{4b}e^{-6N} + \mathcal{O}\left(b^2\right)\,. \tag{28}$$

Then in the limit $b\to 0$, we find

$$N=\frac{2}{3}\ln\left|\frac{t}{t_0}\right|\,,\quad t_0^2\equiv\frac{4}{3\kappa^2\rho_0}\,. \tag{29}$$

Then for the finite b, by writing $N=\frac{2}{3}\ln\frac{t}{t_0}+\delta N$ and by using Eq. (28), we find

$$\delta\ddot N + \frac{4}{t}\delta\dot N + \frac{1}{t}\delta N = \mathcal{O}\left(b^2\right) + \mathcal{O}\left((\delta N)^2\right)\,, \tag{30}$$

whose solution is given by

$$\delta N = C_+\left|t\right|^{\frac{-3+\sqrt{7}}{2}} + C_-\left|t\right|^{\frac{-3-\sqrt{7}}{2}} + \mathcal{O}\left(b^2\right)\,. \tag{31}$$

Here $C_\pm$ are arbitrary constants. Because the first and the second terms do not depend on b, we may put $C_\pm = 0$. In fact if we keep C_+, we find E diverges and therefore physically not acceptable. On the other hand, even if we keep C_-, this term does not contribute to E. Then for the large N, by using the expression of E in Eq. (23) with Eq. (24), we find

$$E = -\frac{\left(b\kappa^2\rho_0\right)^2}{16b} . \tag{32}$$

Therefore when $b > 0$, the shrinking of the universe will always stop and turn to expand, that is, we obtain the bouncing universe. On the other hand, when $b < 0$, the shrinking universe always reaches the singular point at $1 + b\kappa^2\rho_0 e^{-3N} = 0$.

When $b > 0$, we may estimate N when the shrinking universe turns to expand and therefore $V(N) = E$. When $b\kappa^2\rho_0 \gg 1$, by using the expression of $V(N)$ in Eq. (25) and E in Eq. (32), we find the minimum $N_{\min}$ of N,

$$e^{3N_{\min}} \sim \frac{3}{8} b\kappa^2\rho_0 . \tag{33}$$

On the other hand, when $b\kappa^2\rho_0 \ll 1$, by using Eq. (26), we find

$$e^{3N_{\min}} \sim \frac{9}{64} b\kappa^2\rho_0 . \tag{34}$$

We should note that Eq. (23) can be identified with the first FRW equation because $H = \dot{N}$ and rewritten as

$$\frac{3}{\kappa^2} H^2 = \frac{6}{\kappa^2} e^{-6N} \left(1 + \frac{b\kappa^2\rho_0}{4} e^{-3N}\right)^2 (E - V(N)) . \tag{35}$$

For large N, the r.h.s. of Eq. (35) can be expanded as a power series with respect to e^{-3N} and we find

$$\frac{3}{\kappa^2} H^2 = \rho\left(1 - \frac{\rho}{\rho_l}\right) + \mathcal{O}\left(e^{-9N}\right) ,$$
$$\rho = \rho_0 e^{-3N} , \quad \rho_l \equiv \frac{2}{b\kappa^2} . \tag{36}$$

The above structure is similar to that in the loop quantum cosmology [9–11]. In case of the loop quantum cosmology, instead of Eq. (36), we have

$$\frac{3}{\kappa^2} H^2 = \rho\left(1 - \frac{\rho}{\rho_c}\right) . \tag{37}$$

Here ρ_c is the critical density and the energy density ρ is always equal to or smaller than ρ_c, $\rho \leq \rho_c$. In the loop quantum cosmology, the shrinking universe turns to expand when $\rho = \rho_c$. Even in our model, if we define the critical density $\rho_c^{\rm BI}$ by the density satisfying $E = V(N)$ in Eq. (35), the shrinking universe turns to expand when $\rho = \rho_c^{\rm BI}$ but we find $\rho_c^{\rm BI} \neq \rho_l$ in Eq. (35) due to the correction of $\mathcal{O}\left(e^{-9N}\right)$ and by using Eq. (35) or (36), we obtain $\rho_c^{\rm BI}$ as follows,

$$\rho_c^{\rm BI} = \begin{cases} \frac{8}{3b\kappa^2} & \text{when } b\kappa^2\rho_0 \gg 1 \\ \frac{64}{9b\kappa^2} & \text{when } b\kappa^2\rho_0 \ll 1 \end{cases} . \tag{38}$$

Therefore the obtained behavior of the bouncing is similar to that in the loop quantum gravity, although there are quantitative differences.

3. Black hole formation by the collapse of dust

In the last section, we have concluded that the FRW Universe filled with dust would bounce in the Born–Infeld gravity with positive b. On the other hand, the result could be applied to a gravitational collapse of uniform and spherical ball of dust as in [14]. In this section, we consider if black hole can be formed by the collapse of dust. We now assume there is a spherically symmetric and uniform ball made of dust and consider the collapse of ball. In the Einstein gravity, this assumption is valid because the pressure of the dust vanishes. If the falling matter fluid has a pressure, the density of ball cannot be uniform because the pressure should vanish at the boundary between the ball and bulk, which is assumed to be vacuum. This assumption can be justified in the Einstein gravity by using the junction conditions. In the Born–Infeld gravity in the Palatini formalism, however, we do not know the exact expressions of the junction condition. Then the arguments below might not be justified quantitatively but we may expect that the results obtained in this section could be correct qualitatively.

If we can regard that the space–time inside the ball of dust could be the shrinking FRW universe as in the last section, the results in the last section could tell that there would be a bouncing. If the radius of the ball at the bouncing is larger than the Schwarzschild radius, the black hole cannot be formed.

We assume the ball of dust with radius R at $N = N_0$. We choose N_0 to be large enough. Then the total mass M is given by

$$M = \frac{4\pi}{3} R^3 \rho_0 e^{-3N_0} . \tag{39}$$

Here $\rho_0 e^{-3N_0}$ is the energy density of the ball at $N = N_0$. We now consider the case that $b > 0$. First we assume

$$b\kappa^2\rho_0 = \frac{3b\kappa^2 M e^{3N_0}}{4\pi R^3} \gg 1 . \tag{40}$$

Then by using Eq. (33), we find $N = N_b$ at the bouncing is given by

$$e^{3N_b} \sim \frac{9b\kappa^2 M e^{3N_0}}{32\pi R^3} , \tag{41}$$

which gives the radius R_b at the bouncing by

$$R_b^3 = R^3 e^{3(N_b - N_0)} = \frac{9b\kappa^2 M}{32\pi} . \tag{42}$$

On the other hand, the Schwarzschild radius R_s is given by

$$R_s = \frac{\kappa^2 M}{4\pi} . \tag{43}$$

Then we find

$$\frac{R_b^3}{R_s^3} = \frac{18\pi^2 b}{\kappa^4 M^2} . \tag{44}$$

Therefore large black hole, where $M^2 \gg \frac{b}{\kappa^4}$, can be formed because $R_b \ll R_s$ and therefore the bouncing can occur after the formation of the horizon.

Instead of Eq. (40), we may also consider the case

$$b\kappa^2\rho_0 = \frac{3b\kappa^2 M e^{3N_0}}{4\pi R^3} \ll 1 . \tag{45}$$

Then by using Eq. (34), we find that the bouncing occurs when

$$e^{3N} \sim e^{3\tilde{N}_b} \sim \frac{27b\kappa^2 M e^{3N_0}}{256\pi R^3} , \tag{46}$$

and the radius $\tilde{R}_b$ at the bouncing is given by

$$R_b^3 = R^3 e^{3(N_b - N_0)} = \frac{27b\kappa^2 M}{256\pi}, \tag{47}$$

and we obtain

$$\frac{R_b^3}{R_s^3} = \frac{27\pi^2 b}{4\kappa^4 M^2}. \tag{48}$$

Therefore small black hole, where $M^2 \ll \frac{b}{\kappa^4}$, cannot be formed because $R_b \gg R_s$.

We now consider the case that $b < 0$. In this case, there is a maximum $\rho_{\max}$ in the energy density ρ given by Eq. (16). We now consider the meaning of the density $\rho_{\max}$ in the black hole formation. We now assume that the black hole is formed by the collapse of the star made of the dust with radius r. Then the energy density ρ is given by

$$\rho = \tilde{\rho}_0 r^{-3}. \tag{49}$$

Here $\tilde{\rho}_0$ is a constant. Then the mass M and the Schwarzschild radius R_s of the star is given by

$$M = \frac{4\pi}{3}\rho r^3 = \frac{4\pi}{3}\tilde{\rho}_0, \quad R_s = \frac{\kappa^2 M}{4\pi} = \frac{\kappa^2 \tilde{\rho}_0}{3}. \tag{50}$$

Then Eq. (16) tells that the minimum of r is given by

$$r_{\min} = (-3bR_s)^{\frac{1}{3}}. \tag{51}$$

The black hole cannot be formed if $r_{\min} > R_s$, that is

$$R_s^2 < -3b. \tag{52}$$

Therefore small black holes may be prohibited if $b < 0$ but large ones are not prohibited. This result may tell that the creation of the primordial black holes might be prohibited.

An important question is how the horizon formation could be consistent with the bouncing of the universe. In case of the loop quantum gravity, an expectation is the formation of the inner horizon as in the Reissner–Nordstrom space–time. The ball of the dust goes through the inner horizon and the ball might appear in another world as in the space–time of the Reissner–Nordstrom black hole or the Kerr black hole. Another possibility could be that the star which has bounced come back to the asymptotic region through the regular black hole structure such as discussed in [16]. At present, we do not have any definite answer and this question could be a future problem.

In more realistic way, the above problem about the consistency between the horizon formation and the bouncing of the universe might be solved by the junction condition. In fact, there is an application of the Born–Infeld gravity to study the spherical symmetric compact star [12] by using the junction conditions, where the Darmois–Israel formulation [13] has been used. We cannot, however, consider the junction conditions in a similar way because the second junction condition could not be formulated in our case. The effective repulsive force which causes the bouncing universe makes the dust to move to the outside of the star and then violates the homogeneity we assumed. This inhomogeneity could be realized by introducing the shell on the boundary of the star. Therefore there should appear the surface term proportional to a δ-function in the energy–momentum tensor. Furthermore, because the surface term depends on time during the collapse, we cannot use simple shell model which has a constant tension. Therefore it could be a difficult but essential task to construct an appropriate model and solve the dynamics of the star.

We do not know the exact expression of the junction condition but we could be able to use the junction condition in the Einstein gravity as an approximation. Therefore although the obtained results could not be completely justified, we may give the following speculation: The dust on the boundary between the sphere of the dust and the vacuum may move by the geodesic of the Schwarzschild space–time. Inside the sphere, there appears the effective repulsive force between the dust due to the correction by the Born–Infeld gravity, which may lead to the bouncing in the FRW universe. As mentioned above, the repulsive force makes the dust inhomogeneous and pushes the dust which was inside of the sphere, out of the boundary. The dust pushed out from the boundary will move by the geodesic of the Schwarzschild space–time but the geodesic is not always infalling one but going outward. Therefore if the bouncing occurs before the dust goes inside the horizon, there will occur the bouncing and the black hole might not be formed. On the other hand, even if the bouncing occurs inside the horizon, the dust may spread in the time direction, which is space like inside the horizon and the dust may not appear outside the horizon.

4. Summary

In the Born–Infeld gravity by using the Palatini formalism, we have investigated the FRW cosmology where the matter is dust and we have shown that when $b > 0$, there occurs the bouncing. The cosmology in the Palatini–Born–Infeld gravity has been investigated in several papers, but in the most of the previous works, the connections are assumed to be given by $P_{\mu\nu}$, which resembles the metric of the FRW universe but this requirement is too strong and we considered more general case. By applying the results in the FRW universe, we also gave some speculations about the collapse of the sphere of dust and the black hole formation. Then we have shown that although the large black hole might be formed but the small black holes could be prohibited to be formed. This naive speculation about the singularity avoidance requires more quantitative arguments and we need to solve the junction conditions for the metric and connection so that the internal FRW space–time connects smoothly to the external Schwarzschild space–time. There is an application to the spherically symmetric and static compact star in the Palatini formalism by using the Darmois–Israel junction condition. In our case, however, there is another kind of difficulty. As mentioned in the last section, we have assumed that the dust could be homogeneous but because there appears the repulsive force between the dust in the Born–Infeld gravity, the dust becomes inhomogeneous during the collapse when there is a boundary between the dust and vacuum. In fact, if we solve the equations by assuming that the dust is homogeneous inside the boundary, there should appear a kind of shell, where the energy–momentum tensor diverges as a δ-function. If we assume the junction condition in the Einstein gravity as an approximation, the approximation could be valid when the density of the dust is small and the total mass is large enough because the equations in the Born–Infeld gravity coincide with the Einstein equations in the limit that the energy–momentum tensor vanishes, that is, the energy density goes to zero. Furthermore, the repulsive force could be weak when the curvature at the horizon is small enough because the corrections from the Einstein gravity become small when $bR \ll 1$. Therefore the junction condition in the Einstein gravity can be a leading order approximation of the full junction condition in the Born–Infeld gravity in the Palatini formalism although the full and exact analysis could be complicated. We will consider this problem in the future work.

It could be interesting to investigate the possibility that a spherically symmetric space–time without singularity, as is expected to appear in the final stage of dust collapse, may be an exact solution in the Born–Infeld gravity. We should note that some regular black hole solutions are already known [15–17]. In the Einstein gravity, those solutions are not vacuum solutions, but a non-linear electromagnetic source is responsible to obtain the regular black holes [18–20]. On the other hand, we found that the Born–Infeld gravity in the Palatini formalism may have the black hole solution without singularity from the speculation about the dust collapse. If so, the non-linearity in gravitational action itself would work as the origin to remove the singularity as in case of the non-linear electromagnetic source. Additionally, if we could find the static configuration of the non-singular space–time with matter fields, it could be an great interest to study the stability, the equation of state of matter inside the black hole, and the energy condition [21].

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Appendix A. Derivations of Eqs. (12), (13), (14), and (15)

In this appendix we derive Eqs. (12), (13), (14), and (15). By assuming Eqs. (5) and (6), we find that the Ricci tensors are given by

$$R_{tt} = -3\left(\dot{C} + C^2 - AC\right),$$
$$R_{ij} = a^2\left(\dot{B} + 2HB + BC + BA\right)\delta_{ij}, \quad R_{ti} = R_{it} = 0. \tag{A.1}$$

Then we obtain the following equations:

$$b\kappa^2\rho = \left\{1 + b\left(\dot{B} + 2HB + BC + BA\right)\right\}^{\frac{3}{2}} \times \left\{1 + 3b\left(\dot{C} + C^2 - AC\right)\right\}^{-\frac{1}{2}} - 1, \tag{A.2}$$

$$0 = \left\{1 + b\left(\dot{B} + 2HB + BC + BA\right)\right\}^{\frac{1}{2}} \times \left\{1 + 3b\left(\dot{C} + C^2 - AC\right)\right\}^{1\frac{1}{2}} - 1, \tag{A.3}$$

$$\Gamma^t_{tt} = \frac{1}{2}\frac{d}{dt}\left\{\ln\left\{1 + 3b\left(\dot{C} + C^2 - AC\right)\right\}\right\}, \tag{A.4}$$

$$\Gamma^t_{ij} = \frac{a(t)^2}{2}\left\{1 + 3b\left(\dot{C} + C^2 - AC\right)\right\}^{-1} \times \left\{2H + b\left\{4H\dot{B} + 4H^2B + 2HBC + 2HBA + \ddot{B} + 2\dot{H}B + \dot{B}(C + A) + B\left(\dot{C} + \dot{A}\right)\right\}\right\}\delta_{ij}, \tag{A.5}$$

$$\Gamma^i_{jt} = \Gamma^i_{tj} = \frac{1}{2}\frac{d}{dt}\left\{\ln\left\{a^2 + ba^2\left(\dot{B} + 2HB + BC + BA\right)\right\}\right\}\delta^i{}_j. \tag{A.6}$$

Eq. (A.2) is the (t, t) component of (2) and Eq. (A.3) is the (i, j) component. Eqs. (A.4), (A.5), and (A.6) are solutions of Eq. (3).

By using Eqs. (6), (A.3), (A.4), and (A.6), we find the first equality $A = C - H$ in Eq. (12). We may delete A by using Eq. (12) and obtain

$$b\kappa^2\rho = \left\{1 + b\left(\dot{B} + 3HB\right)\right\}^{\frac{3}{2}} \times \left\{1 + 3b\left(\dot{C} + 2C^2 - CH\right)\right\}^{-\frac{1}{2}} - 1, \tag{A.7}$$

$$0 = \left\{1 + b\left(\dot{B} + 3HB\right)\right\}^{\frac{1}{2}} \times \left\{1 + 3b\left(\dot{C} + 2C^2 - CH\right)\right\}^{\frac{1}{2}} - 1, \tag{A.8}$$

$$-C + H = \frac{1}{2}\frac{d}{dt}\left\{\ln\left\{1 + 3b\left(\dot{C} + 2C^2 - CH\right)\right\}\right\}, \tag{A.9}$$

$$B = \left\{1 + 3b\left(\dot{C} + 2C^2 - CH\right)\right\}^{-1} \times \left\{2H + b\left\{5H\dot{B} + 6H^2B + 3\dot{H}B + \ddot{B}\right\}\right\}. \tag{A.10}$$

Furthermore by using Eqs. (A.7) and (A.8), we obtain

$$b\kappa^2\rho = \left\{1 + b\left(\dot{B} + 3HB\right)\right\}^2 - 1. \tag{A.11}$$

We now delete B and C in Eqs. (A.7), (A.8), (A.9), (A.10) and obtain a single equation with respect to the scale factor a. By assuming Eq. (11) and by combining Eqs. (A.8) and (A.10), we obtain

$$4a^4B = \frac{d}{dt}\left[a^4\left\{1 + b\left(\dot{B} + 3HB\right)\right\}^2\right]. \tag{A.12}$$

Furthermore by combining (A.11) and (A.12), we find (13). On the other hand, Eqs. (A.7) and (A.8) give

$$b\kappa^2\rho = \left\{1 + 3b\left(\dot{C} + 2C^2 - CH\right)\right\}^{-2} - 1. \tag{A.13}$$

By using Eqs. (A.9) and (A.13), we obtain Eq. (14) and the second equality (12). A single equation (15) with respect to the scale factor a can be obtained by deleting B in Eq. (A.11) by using Eq. (13).

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