ELSEVIER
CrossMark
Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 884 (2014) 344–356
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

The cosmological constant problem and re-interpretation of time

M.J. Luo

Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China

Received 15 April 2014; received in revised form 24 April 2014; accepted 25 April 2014

Available online 30 April 2014

Editor: Stephan Stieberger

Abstract

We abandon the interpretation that time is a global parameter in quantum mechanics, replace it by a quantum dynamical variable playing the role of time. This operational re-interpretation of time provides a solution to the cosmological constant problem. The expectation value of the zero-point energy under the new time variable vanishes. The fluctuation of the vacuum energy as the leading contribution to the gravitational effect gives a correct order to the observed "dark energy". The "dark energy" as a mirage is always seen comparable with the matter energy density by an observer using the internal clock time. Conceptual consequences of the re-interpretation of time are also discussed.

1. Introduction

The cosmological constant problem is a crisis of physics [1,2]. It arises as a severe problem because anything that contributes to the energy density of the vacuum behaves like a cosmological constant. For example, contributions come from the potential of scalar Higgs boson, which is about (200 GeV)^4; the chiral symmetry breaking of QCD is about (300 MeV)^4; the behavior of electrons is well understood up to energies of order 100 GeV, so the contribution of electron loops up to this scale contribute of order (100 GeV)^4 to the vacuum energy; and other conjectured scale, like the supersymmetry breaking scale gives at least (1 TeV)^4 to the vacuum energy. Among various known contributions, the most severe trouble comes from the so-called zero-point vacuum energy predicted from our well-tested quantum field theory. We know from the quantum field theory that our vacuum is rather non-trivial, the sum of the zero-point energies of

http://dx.doi.org/10.1016/j.nuclphysb.2014.04.024

all normal modes of quantum fields gives $\sum_k^{k_{max}} \frac{1}{2}\hbar\omega_k \approx \frac{k_{max}^4}{16\pi^2}$. If we believe the general relativity up to the Planck scale $k_{max} \sim 10^{19}$ GeV, it would give about $(10^{19}\ \text{GeV})^4$. However, the present observations measure the effective vacuum energy and give a very small value, about $(10^{-12}\ \text{GeV})^4$ [3,4].

It is very disappointing for this large difference (about 10^{120}) from the prediction in quantum field theory. This would need to be canceled almost, but no exactly, by an equally large counter term with opposite sign, using the standard renormalization procedure of the quantum field theory. The cancellation of the quartic divergence and large magnitude of quantum correction compared with its small bare value leads to a severe fine-tuning. We have no reason why these large amount of quantum corrections of vacuum do not gravitate. If we trust the well-tested equivalence principle of the general relativity, all kinds of energies gravitate. Indeed, we know that the electron vacuum energy coming from the vacuum polarization (measured by the famous Lamb shift experiment) does gravitate [5]. Explaining why these quantum corrections do not gravitate is only one side, why they leave a small remnant gravitational effect is another side, since some supersymmetric theories really require an exact zero vacuum energy (although it does not help because the supersymmetry must be broken). Current cosmological observation raises the third part of the cosmological constant problem called the cosmic coincidence problem or "why now" problem [6], i.e. why it is comparable to the matter energy density now. Because the vacuum energy or anything behaving like it does not redshift like matter. In the past, the matter density is large, and the vacuum energy can be ignored, but in the latter time, matter will gradually be diluted by the expansion of the universe, but the vacuum energy density remains, so the percentage of the vacuum energy become large. These two can be comparable only in a particular epoch, but we have no idea why it is now?

What the cosmological constant problem actually implies for our most fundamental concepts and understanding of the world is not yet clear. But one thing is clear, if one wish to talk about the notion of energy, one must bear in mind very carefully that a mathematical description of it has no physical meaning unless one really clears what the "time" means [7–10]. In its usual sense, if the notion of time is defined as "the position of the pointer of the clock in my hand", we place this clock as close as the system, and the energy exists as an abstract and mysterious mathematical quantity that you find it always the same under the position change of the clock pointer. This is true, but not true enough. In certain situations, the position of the clock pointer becomes fuzzy. Beside that, if the system is very far from us, and the clock by definition is placed far apart as well, certain technique is needed to compare the readings of it and that of the clock in my hand, since there is no further assumption to compare how fast these two clocks are, even if they have already been locally synchronized. To measure the energy at a distance, we have to take into account that all our judgments in which energy or time concerns are always the judgments of the synchronization of clocks in a distance. However, can the synchronization between spatially separated clocks be precisely realized? The answer from our current understanding of the spacetime based on the classical relativity is "yes". But what is the case when the spacetime is quantum mechanical? In principle, this kind of question cannot be fully answered unless a consistent quantum theory of spacetime is discovered. But it is because the lacking of the complete theory of quantum spacetime, the cosmological constant problem may be an important clue to find the theory, the notion of time may be a key to the problem.

Before we discuss the problem, let's first re-examine what the concept of time actually is in Section 2, in which we re-interpret the notion of time and present our framework. Then we discuss the consequences of the re-interpretation of time and its implication to the cosmological constant problem in Section 3. In Section 4, we generalize the idea to a more general version of quantum reference frame. Finally, we draw the conclusions of the paper in Section 5.

2. Operational definition of time at quantum level

The modern physics from his father Galileo and Newton was built beginning with the physical realization of time. Galileo gave a decisive contribution to the discovery of the modern clock. The small oscillations of a pendulum can be used as a standard clock that "takes equal time", although he had no standard "time" to tell him whether its oscillation periods take equal time, he just checked the pendulum against his pulse (which was thought as the only "standard" he could use). It was Newton who made the conceptual idea clear up. He assumed the existence of a global parameter measured by the device invented by Galileo, which flows to infinity absolutely as the pendulum oscillates forever. By using this global parameter ("time"), the law of motion could be simplified. The observable quantities can be parametrized as functions of Newton's time $X(t), Y(t), Z(t)\dots$, called evolution. The complex motions we observed in nature are strongly simplified by a few fundamental laws that governs the form of these functions. The physics was then built up to predict the behavior of $X(t), Y(t), Z(t)$ at temporal distance, in analog with the Euclidean geometry (before Newton's mechanics thousand of years) which predicts the behavior of points, lines and angles at spatial distance. This framework of the universe was challenged when the law of electrodynamics discovered experimentally was found conflict with the relativity of Galileo. It was Einstein who cleared up the issue conceptually that we must re-examine the simultaneity in different reference frames. He found that the clock readings (by his invention of "light clock") in different reference frames are not Newton's global parameter t but each reference frame has each parameter time, as a consequence of the constant speed of light. The physical quantities can be rewritten as $X(\tau), Y(\tau), Z(\tau)\dots$ together with his light clock $T(\tau)$, if a global parameter τ is also assumed to be exist. He abandoned the unobserved global time, and replaced it by physical clocks readings from his light clock in each reference frame. Then the evolution in each reference frame is seen as functionals $X[T(\tau)], Y[T(\tau)], Z[T(\tau)]\dots$ instead of the functions $X(t), Y(t), Z(t)\dots$.

Newton's global parameter in certain sense is still alive in the quantum mechanics even the quantum fields theories after the discovery of Einstein's re-interpretation of time. Heisenberg abandoned the unobserved spatial trajectories of electron in atom, and only used the observable such as light spectrum that induced from the transition between two states. As a consequence the spatial coordinate of electron as a number was replaced by a square matrix relating two states. Although the spatial coordinates was re-interpreted, the time in quantum mechanics is still the classical Newton's parameter. The quantum fields theories replaced the only one global parameter in quantum mechanics by four interpreted as the spacetime coordinates in order to keep the Lorentz invariance. The quantum mechanics presupposes an external classical observer measuring the global parameter time, which makes several intrinsic difficulties, e.g. the quantum mechanics cannot be applied to study the whole universe, since the universe has no outside by its definition. And such division of the universe into a quantum world that to-be-measured and a classical world describing the measuring instruments, makes the quantum mechanics needs extra assumptions or axioms to justify the process of measurement, such as the argument of the collapse of wavefunction in the Copenhagen interpretation of quantum mechanics.

As a general believe, the difficulty of quantizing general relativity is deeply rooted in the very different treatment of the concept of time in general relativity and in quantum mechanics. The lessons we have learned about our world from these two theories is that we need to carefully reexamine our fundamental notion. The relativity teaches us that the time is nothing but an artificial notion invented to simplify our thinking about motion, in classical physics the clock is always imagined as an idea or perfect motion that as a reference to other more complex motions. While the quantum mechanics teaches us that there is no idea or perfect motion in our world, the physical quantities are always fluctuating quantum mechanically. However, the non-existence of a perfect motion as a standard clock in quantum mechanics may not be important, in practice, all clocks including the quantum or atom clocks we have invented in laboratories are not perfect, they are just used as reference like the relativity had taught us, the important thing is the relation between different motions, they could be bridged by an imagined perfect motion or not by it, whether the bridge exists or not the relations are still there, just like that we still could exchange our goods without money.

A physical theory works in such a way, a formal mathematical apparatus joints with a physical interpretation. The great progresses in the history of physics are made not so much through a deeper understanding of the nature as a deeper understanding of the science itself. Combining the spirits of the relativity and quantum mechanics, the physical clocks time $T(\tau)$ used in the relativity must be treated quantum mechanically, and then the global parameter τ in quantum mechanics can no longer be interpreted as time. This re-interpreted time variable is relativistic, since it is physical operational defined; it is quantum, since the physical clock $T(\tau)$ is treated quantum mechanically and has quantum fluctuation. As a key new ingredient, the classical relativistic simultaneity cannot be realized precisely due to the intrinsic quantum fluctuation, just like Newton's simultaneity cannot be precisely realized when the speed of light is a constant.

What does it mean when we consider the physical clock $T(\tau)$ is quantum mechanical, and what is the meaning when we talk about the quantum version of the evolution $X[T(\tau)], Y[T(\tau)], Z[T(\tau)]\ldots$ Let us consider a Hamiltonian H_X governing the behavior of the physical quantities $X(\tau)$, and a Hamiltonian H_T governing the physical clock $T(\tau)$. They share the global parameter τ in quantum mechanical treatment, whether or not τ has any physical meanings is not important in our setting. There is no interaction with the field $X(\tau)$ and $T(\tau)$, it is a separable system, each field independently evolves under the parameter τ, so the Hilbert space of the system is a direct product of these two Hilbert spaces $\mathcal{H}=\mathcal{H}_X\otimes\mathcal{H}_T$, the state vector can be written as $|\Psi\rangle=\sum_\tau c_\tau|X(\tau)\rangle\otimes|T(\tau)\rangle$ which is the eigenstate of $H=H_X+H_T$. The statement that the system is separable does not necessarily mean that they are always independent, since when we initialize an experiment we need to adjust the instruments, which makes an instant interacting between $X(\tau)$ and $T(\tau)$ at early stage, and hence the state $|\Psi\rangle$ is not simply a direct product state $\sum_\tau a_\tau|X(\tau)\rangle\otimes\sum_{\tau'}b_{\tau'}|T(\tau')\rangle$, in most cases, it is an entangled state. This argument suggest that the evolution $X[T(\tau)]$, at quantum level, is replaced by the entangled state $\sum_\tau c_\tau|X(\tau)\rangle\otimes|T(\tau)\rangle$. The squared norm of the coefficient of the entangled state $|c_\tau|^2$ measures the joint probability when the clock is at state $|T(\tau)\rangle$ and the physical quantity is at $|X(\tau)\rangle$, which is a quantum version of the process that one reads the clock and sees the evolution of X. Only when the clock is classical and deterministic, $|c_\tau|^2$ reduces to the textbook probability of $|X(\tau)\rangle$. The relational probabilistic interpretation of $|X(\tau)\rangle\otimes|T(\tau)\rangle$ replaces the deterministic interpretation of $X[T(\tau)]$, and the Schrodinger equation governing the quantum evolution of $|X(\tau)\rangle$ and $|T(\tau)\rangle$ with τ is replaced by the Wheeler–DeWitt equation

$$(H_X+H_T)|\Psi\rangle=(H_X+H_T)\sum_\tau c_\tau\big|X(\tau)\big\rangle\otimes\big|T(\tau)\big\rangle=0. \tag{1}$$

The quantum physical clock time $T(\tau)$ reduces to the classical relativistic time when we use the mean field approximation. The Schrodinger equation emerges as an approximation from the above Wheeler–DeWitt equation.

Consider the action of the separable system $S = S_X + S_T$, where the physical clock by convention is chosen as the quantum pendulum, i.e. the continuous free quantum fields or infinite many quantum harmonic oscillators located on the parameter background τ, $S_T = \frac{1}{2}\int d^d\tau(\partial_\tau T)^2$, the action of X can be in general written as conventional kinetic part and potential energy part $S_X = \int d^d\tau \frac{1}{2}(\partial_\tau X)^2 - V[X(\tau)]$. The action can be written as

$$S = \int d^d\tau \left[\frac{1}{2}(\partial_\tau X)^2 - V[X] + \frac{1}{2}(\partial_\tau T)^2\right] \tag{2}$$

$$= \int dT \left\| \frac{\partial\tau}{\partial T} \right\| \left[\frac{1}{2}(\partial_\tau T)^2\left[1 + \left(\frac{\delta X}{\delta T}\right)^2\right] - V[X]\right], \tag{3}$$

where d is the dimension of the parameter space τ, the $\|\frac{\partial\tau}{\partial T}\|$ is the Jacobian determinant. Then the partition function is

$$Z = \int \mathcal{D}X\mathcal{D}T \exp\bigl(-S\bigl[X(\tau), T(\tau)\bigr]\bigr) \overset{MF}{\approx} \int \mathcal{D}X \exp\bigl(-S_{\mathit{eff}}\bigl[X[T]\bigr]\bigr), \tag{4}$$

where the effective action under the mean field approximation is

$$S_{\mathit{eff}}\left[X, \frac{\delta X}{\delta T}\right] = \int dT \frac{1}{2}\mathcal{M}\left(\frac{\delta X}{\delta T}\right)^2 - V[X] + \text{const}, \tag{5}$$

where $\mathcal{M} = \langle\|\frac{\partial\tau}{\partial T}\|(\partial_\tau T)^2\rangle_{MF}$ is a constant depending on the integration constant of the mean field value of $T(\tau)$. Up to a constant, the mean field effective action reproduces the classical action of X, $S = \int dt \frac{1}{2}m(\frac{dx}{dt})^2 - V(x)$, by using the T as time of the system. An obvious observation from this effective action is that, the functional derivative formally replaces the conventional derivative, since the clock time $T(\tau)$ now is certain dynamical variable playing the role of time. The evolution of X is now with respect to the physical quantity T.

The notion of time and energy are closely correlated to each other, this is true not only in quantum mechanics, but also in classical physics. The textbook Schrodinger equation strongly relies on the notion of energy, but in our setting, strictly speaking there is no notion of time and energy at fundamental level. Only when the quantum fluctuated clock time $T(\tau)$ is treated semi-classically as a classical parameter time, the Schrodinger equation emerges as an approximation from the Wheeler–DeWitt equation, and the conventional notion of "time" in the induced Schrodinger equation emerges from the timeless Wheeler–DeWitt equation. The effective action Eq. (5) and the emergent Schrodinger equation are only approximations, as a consequence, the notion of unitarity of the Schrodinger equation is also an approximation. That is not to say that the probability does not conserve any more, it suggests that certain relational interpretation connected to the entangled state solution of the Wheeler–DeWitt equation must be introduced, replacing the absolute probability in the textbook quantum mechanics. The relational interpretation just cares about the mutual relation between the to-be-measured system $|X(\tau)\rangle$ and measuring instrument (the clock) $|T(\tau)\rangle$, the absolute individual state $|X(\tau)\rangle$ or $|T(\tau)\rangle$ defined as a function of τ has no individual physical meaning. In the standard Schrodinger picture, a state is defined as a function of the parameter time, the entangled state of the Wheeler–DeWitt equation can also be defined as a function of its global parameter, but it is not interpreted as time any more. The state of the to-be-measured system is defined (to be) entangled with the state of the clock, it is their relation that are observable. It is reasonable that only the relation between the to-be-measured system and the measuring instruments is important, but their individual absolute states, when the measuring instrument such as the clock is inevitably be treated quantum mechanically.

The action Eq. (2) and the related Wheeler–DeWitt equation (1) are the precise theories we need to study quantum mechanically. We hence face a boundary of the textbook Schrodinger equation and beyond, in this sense, the functional approach is more useful than the operator approach. Beyond the mean field approximation, the quantum fluctuations of the clock time become important, which will lead to a departure from the prediction of the parameter time. A salient departure gives rise to the first order prediction of the groundstate energy, which exhibits the importance of the quantum fluctuation of the clock, the classical relativistic simultaneity cannot by precisely realized due to the intrinsic quantum fluctuation. These consequences will be discussed in detail in the next section.

3. Consequences – the cosmological constant problem

In this section, we will discuss the consequences of the operational re-interpreted time, which leads to a solution of the cosmological constant problem. True enough, as a fundamental variable of physics, any modification of the notion of time will cause salient changes and consequences in almost all aspects of physics. The most directly related notion is no doubt the energy.

Considering the whole universe is divided into two sub-regions, the finite region-1 with a sphere of radius R, and the outside region, denoted as region-2. And considering a dynamical system with a total action be written as $S[X_1, X_2] = S_1[X_1] + S_2[X_2]$, where $S_1[X_1]$ is the action of the system obtained by integrating the Lagrangian density within region-1 and S_2 is the action of the outside region, region-2. The $X_1(x)$ and $X_2(x)$ are quantum scalar field variables in region-1 and region-2 respectively, x are global parameters shared by these fields. These two fields independently live in each region and do not couple with each other. Since there is no external classical observer outside the whole region (outside the region-1 and region-2), the parameter time $t = x_0$ cannot be interpreted any more as the notion of time with any physical meaning, the total energy of the whole system cannot be observed.

In such a closed quantum system without outside, measuring a subsystem means that an observer stands outside the subsystem and uses measuring instruments to "watch" it from the external of the subsystem. Now let us consider an observer in the region-1 performs measurements to "watch" the system of the region-2, by using a device described by a field X_1 being a clock in his/her hands. The energy of the region-2 can only be measured by the observer's clock readings X_1, which is a quantity defined as his/her clock time shift invariant. The energy density of region-2 is considered to be continued to the point x when the region-1 tends to shrink to the point x,

$$\langle E_2(x)\rangle = \frac{\delta S_{eff}}{\delta X_1(x)}, \tag{6}$$

where the effective action is $S_{eff} = -\ln Z = -\ln \int \mathcal{D}X_1 \mathcal{D}X_2 e^{-(S_1+S_2)}$. Here the functional derivative w.r.t. the clock time fields replaces the conventional derivative w.r.t. the global parameter time in defining the energy $E = \frac{\partial S}{\partial t}$. The latter global energy can only be measured by an external observer outside the whole system, so it is completely unobservable in our setting.

In principle, a clock is just a reference, any dynamical variable could be chosen and defined as a clock, but by convention, the simplest and practical ones are the periodic systems. Now let us consider the physical clock X_1 is the coordinate of a periodic quantum harmonic oscillator or a continuous free quantum field (infinite many quantum harmonic oscillators located at the continuous parameter x), the action of the physical clock is written as

$$S_1[X_1] = \frac{1}{2} \int\limits_{|x|<R} d^4x (\partial_x X_1)^2. \tag{7}$$

The first consequence of the framework is that the vacuum energy of E_2 measured by the observer in region-1 is vanished. Note that the subsystem S_1 and S_2 are independent, i.e. only S_1 contains explicitly the field variable X_1, so in fact the energy E_2 density is just the conjugate momentum density p_1 of the field X_1,

$$\langle E_2 \rangle = \frac{\delta S_{eff}}{\delta X_1} = \langle p_1 \rangle. \tag{8}$$

Let $|0\rangle = |0\rangle_1 \otimes |0\rangle_2$ be the groundstate which is the eigenstate of the Hamiltonian of the whole system. It is known that the expectation value of momentum p_1 at groundstate is trivially vanished $\langle 0|p_1|0\rangle = 0$, so we find

$$\langle 0|E_2|0\rangle = 0. \tag{9}$$

This result exhibits that the observer does not feel any zero-point energy density of the system of the region-2. The divergent zero-point energy density predicted by conventional quantum field theories $\frac{1}{2}\hbar \sum_\omega \omega$ can only be seen by an external classical observer outside the universe using the global parameter time. Only the energy of a subsystem can be measured, one needs to stand outside the subsystem, and use the physically fluctuating field variable outside the subsystem as the physical clock time. When the clock X_1 in region-1 is treated quantum mechanically, it also undergoes zero-point fluctuation. As a consequence, we cannot feel the zero-point fluctuations by using a zero-point fluctuating clock, or equivalently standing on a zero-point fluctuating reference frame. In summary, we abandon the unobserved parameter time and use the operational defined quantum clock variable as time, the zero-point energy automatically vanishes, then there are no such divergent contributions to the cosmological constant.

The second consequence from the re-interpretation of time is that when we consider the quantum fluctuations of the physical clock time, the intrinsic quantum uncertainty in the notion of simultaneity between two clocks will result in intrinsic quantum fluctuation of energy density, leading to an observed order of the mysterious energy density (so-called the "dark energy") that drives the accelerating expansion of the universe. In our setting, the effective energy density is completely due to the quantum effects of the re-interpreted time variable, and there is no need for the extra assumption of dark energy. To deduce such consequence, note that although $\langle E_2 \rangle = \frac{\delta S}{\delta X_1} = 0$, we have a non-vanished zero-point energy fluctuation $\langle \delta E_2^2 \rangle = \langle E_2^2 \rangle - \langle E_2 \rangle^2 = \langle E_2^2 \rangle = \frac{\delta^2 S}{\delta X_1^2} \neq 0$.

The zero-point energy fluctuation can be understood as follows. Considering the physical clocks X_1 at spatially separated points x and y, with clock readings are $X_1(x)$ and $X_1(y)$. To compare these two clocks quantum mechanically, a quantitative description is by a probabilistic correlation function, from Eq. (7) we have

$$\left\langle X_1(x) X_1(y) \right\rangle = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} e^{ik\cdot(x-y)} \sim \frac{1}{4\pi^2 |x-y|^2}, \tag{10}$$

which measures the correlation between the clock at x reads $X_1(x)$ and at y reads $X_1(y)$. Note that the correlation between the two clocks decays with their spatial distance. The decorrelation between the clocks indicates the fact that the rates of these two clocks are unable to be synchronized precisely at quantum level. There is an intrinsic uncertainty in synchronizing two spatially separated clocks, which sets a universal limit in a measurement of remote time. If the clock at y is considered standard (zero-uncertainty), then the remote clock at $|x-y|$ is inevitably seen uncertain. In a homogeneous, isotropic, flat and empty space, considering a standard clock with reading $X_1(y)$ is transported from place y to x, then the wavefunction that one finds the clock at the remote place x with reading $X_1(x)$ is given by

$$\int_{X_1(y)}^{X_1(x)} \mathcal{D}X_1 e^{-S_1[X_1]} = \frac{V^2}{4\pi^2|x-y|^2} e^{-2V\frac{[X_1(x)-X_1(y)]^2}{|x-y|}} = \frac{1}{\sigma^4(2\pi)^2} e^{-\frac{4[X_1(x)-X_1(y)]^2}{2\sigma^2}}, \tag{11}$$

where $\int \mathcal{D}X_1$ is Feynman's path integral of the physical clock. The spatial evolution of the clock broadens the wavefunction from a standard clock (delta distribution) to a wavefunction with finite width. The width σ^2 describes the uncertainty of the reading $X_1(x)$ of the remote clock at x with respect to the standard clock at y, which is given by

$$\sigma^2 = \langle \delta X_1^2(x-y) \rangle = \frac{1}{V}|x-y|, \tag{12}$$

where V is a 3-volume IR cut-off. Therefore, a remote simultaneity defined by the physical clock $\langle X_1 \rangle = \text{const}$ has an intrinsic uncertainty proportional to the distance between the remote clock and the observer. The distance dependence of the clock uncertainty is important when the 3-volume is not infinity. Because the IR cut-off 3-volume V is as large as the cosmic scale, the uncertainty of simultaneity can be ignored in our ordinary observation, while it is significant when the spatial interval $|x-y|$ is also at cosmic scale. By dimensional consideration, the remote time/simultaneity uncertainty can be written as

$$\langle \delta t^2 \rangle \sim L_H^{-3} L_P^4 |x-y|, \tag{13}$$

where $L_H \sim V^{1/3}$ and L_P are the IR and UV cut-offs chosen as the Hubble and Planck scale. In general, if we consider the time is measured by a quantum physical clock, but a global parameter, an intrinsic quantum uncertainty of remote simultaneity is inevitable. There are two important points to emphasize: (1) the effect is different from the time dilation, it does not change the central value $\langle t \rangle$ of the remote time, it only makes the time fuzzy with a non-vanishing $\langle \delta t^2 \rangle$. (2) Different from those time effects predicted from relativity, in which time are different in different reference frames or in a curved space, here, the effect even happens in one reference frame and/or in a flat space. This quantum effect that a remote clock must be uncertain Eq. (13) provides a new explanation to the observed dark energy, the density of which can be roughly estimated: of order $L_H^{-3}\sqrt{\langle \delta t^2 \rangle^{-1}} \sim \mathcal{O}(L_P^{-2} L_H^{-2})$, if one considers $|x-y| \sim \mathcal{O}(L_H)$.

Clear, the farther the distance, the weaker the clocks' correlation, the more uncertain the time or simultaneity, so the larger the energy fluctuations seen by the observer distance separated. It looks like there is an apparent standard deviation of energies emerging out of the void. The deviation or uncertainty the observer feels from the remote clock introduces a remote energy uncertainty, according to the uncertainty principle.

To describe this phenomenon quantitatively, we can find from Eq. (7), the observer feels energy fluctuations in a 4-volume element,

$$\big\langle \delta E_2(x)\delta E_2(0)\big\rangle d^4x = \frac{\delta^2 S_{eff}}{\delta X_1(x)\delta X_1(0)} d^4x \approx \frac{\delta^2 S}{\delta X_1(x)\delta X_1(0)} d^4x = \partial_x^2 \delta^4(x) d^4x. \tag{14}$$

We have written the S_{eff} by using the classical action S at tree level approximation, so the leading contribution to the energy fluctuations is expressed in terms of a widthless Dirac delta function, while it actually has a non-vanishing width. The calculation can be regulated when we first rewrite the Dirac delta function as a limit of a Gaussian distribution, performing the derivatives and finally taking the zero width limit of the Gaussian distribution back to the delta distribution,

$$\big\langle \delta E_2(x)\delta E_2(0)\big\rangle d^4x \approx \lim_{a\to 0} \partial_x^2 \left(\frac{1}{a^4\pi^2} e^{-\frac{4x^2}{a^2}}\right) d^4x = 64a^{-4}|x-0|^2 \delta^4(x) d^4x, \tag{15}$$

where a is the UV cut-off. To regulate the result, both UV and IR cut-offs are needed, a natural UV cut-off is the Planck length $a = L_P$. Since the formula is proportional to $|x-0|^2$, the fluctuations become important when the IR cut-off is at cosmic scale, a natural choice is the Hubble scale $|x-0| = L_H$ as the cosmic horizon. Let us keep the squared norm $|x-0|^2 = L_H^2$ fix and integrate over x, then the fluctuation of the total energy of a Hubble scale volume is given by

$$\big\langle \delta E_2^2\big\rangle \approx 64 \int d^4x L_P^{-4} L_H^2 \delta^4(x) = 64 L_P^{-4} L_H^2. \tag{16}$$

The proportional to the horizon area L_H^2 of the result known as the area scaling is a generic feature argued by many literature [11–13]. The physical reason is transparent, since the unobservability of the total energy of the systems inside and outside the Hubble volume, the Wheeler–DeWitt equation provides that it is zero, then the fluctuations of their total energies have the relation

$$\big\langle \delta E_{inside}^2\big\rangle = \big\langle \delta E_{outside}^2\big\rangle. \tag{17}$$

This relation can also be proved mathematically. On the other hand, because the inside and outside systems only share an identical bounding surface, the relation suggests that either dispersion are proportional to the area of the surface which scales as L_H^2.

We thus find a non-zero total vacuum energy fluctuation in a Hubble scale volume (the 3-ball with fixed radius $|x-0| = L_H$) Eq. (16), so the averaged vacuum energy density is

$$\rho_\Lambda = \frac{\sqrt{\langle \delta E_2^2\rangle}}{\frac{4\pi}{3} L_H^3} \approx \frac{6}{\pi} L_P^{-2} L_H^{-2} \propto \frac{H^2}{G}, \tag{18}$$

where H is the Hubble constant, and G is the Newton gravitational constant. This result gives a correct order to the observed energy density driving the accelerating expansion of the universe. The numerical proportional coefficient depends on the precise nature of the cut-off, so we would better not predict the precise value of ρ_Λ unless the factors are cooked up. If I must do so, I would rather guess $L_P^2 = 8\pi G$, $L_H^{-1} = H$, then it predicts the fraction within the critical energy density $\rho_c = \frac{3H^2}{8\pi G}$ is $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{2}{\pi} \approx 0.637$, which is consistent with current cosmic observation.

The key new features arising from the re-interpretation of time shown in this section are as follows. Now the vacuum expectation value of the energy density related to one state is vanished, however, the energy fluctuation related to two states in the vacuum is the leading order contribution to the gravitational effect [14,15], which is a pure quantum effect originated from the intrinsic uncertainty of the physical clock simultaneity Eq. (12). These consequences provide a solution to the cosmological constant problem. First, it explains why the conventionally predicted zero-point energy has no gravitational effect, since the parameter time is unobservable in the universe, so is the zero-point energy corresponding to it. Second, it gives a prediction with correct order.

Further more, it leads to a third consequence, it answers the "cosmic coincidence" problem, because the effective vacuum energy density is an apparent effect due to the intrinsic fluctuations of remote clocks but the real vacuum energy of the region-2 (like the Lorentz contraction, which is only a visual effect but a real contraction by a force). Note that the time X_1 here is a local internal clock time, the functional derivative of ρ_Λ, Ω_Λ and the Hubble constant $H = L_P^{-1}$ w.r.t. the clock time X_1 vanishes, so in this sense, they are really constants and do not vary with time. The ρ_Λ is always comparable with the critical density ρ_c and matter density ρ_M. In a flat universe $\Omega_K = 0$, the fraction of the matter density is then always seen $\Omega_M \approx 1 - \Omega_\Lambda$ by an internal observer at any epoch.

At first glance, it seems that the statement "always comparable" contradicts the standard picture in which the "dark energy" is constant while matter are gradually diluting. How could these two statements are both true, please do not immediately make an arbitrary judgment that it must be wrong. In fact, the expansion of the universe is a relative concept but absolute. The key is again that we are using a "local internal clock time" in the framework, but an "absolute external time". These two kinds of times predict different situations. We consider the universe is divided into two parts, one is a finite regime A in which an observer lives, and the regime B is the rest of the universe. The notion "now" in principle is a limit of regime A shrinking to infinitely small, but in practice the regime can be considered finite, it is the notion "near now" or "a near epoch". The change in the regime B is defined relative to the clock in regime A who is an external observer (w.r.t. regime B). While the change in the regime A is relative to the clock also in regime A who is an internal observer (w.r.t. regime A). As a result, the internal observers do not see any density change of regime A with respect to their internal clocks, although it is expanding seen by an external observer. Because we as observers always live in the regime A (the "near now" regime), although we as external observers can see changes in regime B, we as internal observers cannot see any matter diluting in the regime A, since our rulers and clocks expand accordingly, in this sense, the regime A seems like an expansion "static" regime. That is the reason we always see that the matter density does not vary with internal time and is always comparable with the apparent "dark energy".

It is worth emphasizing that "always comparable" does not mean these two as real components of the universe would be scaled in the same way under expansion, since it is impossible to be consistent with many observations such as the growth of large scale structure. However, the cosmic acceleration in fact is an apparent (quantum) cosmic variance, no matter in any epoch an (internal) observation is performed, the mirage "dark energy" is always seen being of the order of the matter density. The evolution of the observable universe gives place to the evolution with redshift. If one considers the fraction of matter density evolves as $\Omega_M(1+z)^3$ from now (could be any epoch) to a relative redshift z, then an internal observer at any epoch, always "sees" the vacuum energy density become comparable with the matter density at a relatively small redshift $z_c \approx (\frac{\Omega_\Lambda}{1-\Omega_\Lambda})^{1/3} - 1 \approx 0.3$. "Why now" seems to be a problem because of the breaking of external time translation invariance of the scale factor, however, here the scale factor $a \sim \partial X$ (see next section) is invariant under internal time X translation, the standard meaning that the scale factor evolves with time is lost. In the standard external observer's interpretation, it is a problem "why now", but in a local internal observer's view, the densities do not vary with their clocks, and the coincident redshift z_c is always relatively small. All in all, the resolution of the coincidence problem is not dynamical, it is again a consequence of using the local internal clock time.

4. Generalization – quantum reference frame

To our knowledge, although the quantum field theories on a flat and/or a curved spacetime background are achieved, it is difficult to treat a quantum field system on a dynamical fluctuated spacetime background. This is in analog with that the hydrogen atom had been explained by the quantum mechanics, but people had no idea to the Lamb shift and other effects due to the intrinsic fluctuations of the quantum electrodynamics background. The status is deeply rooted in the fact that we do not have a consistent quantum theory of spacetime. The operational re-interpretation of time can be generalized to an operational definition of a quantum reference frame, in which the notion of time and space coordinates could be put on an equal footing. Since in the spirit of relativity, the spacetime itself is nothing but the property of metric operationally measured by physical instruments (described by quantum mechanics). The idea provides a promising approach to treat a system on a quantum fluctuated spacetime background. Considering reference frame (scalar) coordinate fields $X_\mu(x)$ ($\mu = 0, 1, 2, 3$) defined on a flat fixed parameter background x_i with metric η_{ij} ($i, j = 0, \dots, d-1$), and a quantum field shared the parameter background is written as $\varphi(x)$. They are considered independent and the system is separable. These two fields φ and X_μ are defined on the parameter background, but the parameter background does not necessarily has any physical meaning, the important thing is the relation between φ and the quantum reference frame system X_μ. At classical level, it means the action is a functional of φ and $\frac{\delta\varphi}{\delta X_\mu}$, but at quantum level, it means that they can be written as a separable system,

$$S[\varphi, X_\mu] = \int d^d x \left[\frac{1}{2}\eta^{ij}\partial_i\varphi\partial_j\varphi - V[\varphi] + \frac{\lambda}{2} g_{\mu\nu}\eta^{ij}\partial_i X_\mu \partial_j X_\nu\right], \tag{19}$$

in which the first two terms are the actions of the field φ, the third term describes the reference frame fields, the $g_{\mu\nu}$ is the metric of the frame manifold, i.e. $g^{\mu\nu} = \langle \eta^{ij}\partial_i X_\mu \partial_j X_\nu\rangle$, and we have already effectively written a cosmological constant λ in front of the reference frame term (can be viewed as a $(d-1)$-volume averaged renormalized mass of the reference frame fields, i.e. $\lambda = m/V_{d-1}$). Obviously, the action is formally invariant under the frame coordinates transformation $X'_\mu(x) = \omega^\nu_\mu X_\nu(x) + b_\mu$. Note that if we do not presuppose the mean field metric $g^{\mu\nu} = \langle \eta^{ij}\partial_i X_\mu \partial_j X_\nu\rangle$, but write it explicitly in terms of vierbein $e^\mu_i = \partial_i X_\mu$, the precise full action is highly nonlinear.

By using the mean field approximation, it is easy to verify that this action Eq. (19) can be reduced to our familiar form that a quantum field lives on a curved background,

$$S_{eff} = \int d^4 X \left\| \frac{\partial x}{\partial X} \right\| \left[\frac{1}{4}\left(g_{\mu\nu}\eta^{ij}\partial_i X_\mu \partial_j X_\nu\right)\left(\frac{1}{2} g^{\mu\nu}\frac{\delta\varphi}{\delta X_\mu}\frac{\delta\varphi}{\delta X_\nu} + \lambda\right) - V[\varphi]\right] \tag{20}$$

$$= \int d^4 X \sqrt{\det g}\left[\frac{1}{4}\mathcal{N}\left(\frac{1}{2} g^{\mu\nu}\frac{\delta\varphi}{\delta X_\mu}\frac{\delta\varphi}{\delta X_\nu} + \lambda\right) - V[\varphi]\right], \tag{21}$$

where $\mathcal{N} = \langle g_{\mu\nu}\eta^{ij}\partial_i X_\mu \partial_j X_\nu\rangle_{MF}$ is a constant calculated from the mean field value of $X_\mu(x)$. If you note $\mathcal{N} = \langle g_{\mu\nu} g^{\mu\nu}\rangle_{MF}$, it is in fact a topological invariant related to the dimension of the reference frame. The formula Eq. (21) is a generalization of Eq. (5). The Jacobian determinant

$\|\frac{\partial x}{\partial X}\|$ requires the metric being a square matrix, thus leading to the dimensions of the parameter space is equal to that of the reference frame fields, i.e. $d = 4$. If we do not demand such semi-classical limit, d will not necessarily be 4. A possible implication of this fact is that a topological non-classifiable manifold at certain dimensions may be able to reformulate as a topological classifiable manifold in other dimensions at quantum level. It provides an alternative route to the non-classifiable problem [16–18] of quantum gravity in 4-dimensions, in which the ergodicity does not require to be abandoned.

Therefore, at classical or action level, it demonstrates an equivalence (up to a cosmological constant λ) between the quantum reference frame theory Eq. (19) and a quantum field theory on a generic spacetime background. It is interesting to note that renormalization of λ also gives rise to a Ricci curvature $R(g)$ of the frame manifold, $\frac{d}{d\ln k}\lambda = \frac{1}{2}Rk^2$. The emerged Ricci curvature term describes the low energy dynamics of the quantum reference frame,

$$S_{eff} = \int d^4X\sqrt{\det g}\left[\frac{1}{4}\mathcal{N}\left(\frac{1}{2}g^{\mu\nu}\frac{\delta\varphi}{\delta X_\mu}\frac{\delta\varphi}{\delta X_\nu} + \lambda + \frac{1}{2}R(g)L_P^{-2}\right) - V[\varphi]\right]. \tag{22}$$

It is striking that Einstein's theory of gravity emerges as a low energy effective quantum dynamics of the reference frame, the (relational) quantum reference frame theory itself automatically contains a theory of gravity. Although we have shown the classical equivalence between the theory Eq. (19) and conventional dynamical spacetime theory, at quantum level these two theories are very different. First, the theory Eq. (19) relates to a Wheeler–DeWitt equation, and the states defined on the hypersurface of parameter background x of the equation are entangled states, entangling the state of quantum field φ with the state of the reference frame X_μ, only the relational interpretation of these states is reasonable, the entangled state suggests that the theory is a parameter background independent theory. In contrast, the theory Eq. (21) relates to an approximate Schrodinger equation, the state of it is thought defined on the hypersurface of X_μ, which can only be realized when the field X_μ are treated semi-classically, only in this case, the theory has a standard absolute probability interpretation. Second, there is no zero-point energy if you stand on the quantum reference frame, since the reference frame is also fluctuating at quantum level. Third, the most important feature is that, Eq. (19) has a well-defined quantum theory defined on the flat parameter background x, while it is difficult to treat Eq. (21) and/or Eq. (22) quantum mechanically when X_μ is a dynamical background spacetime. In this sense, Eq. (19) may be a good starting point to study a quantum theory of gravity.

5. Conclusions

In this paper, we abandon the interpretation that time is a global parameter in quantum mechanics, replace it by a quantum dynamical variable playing the role of time. The operational re-interpretation of time causes a new notion of energy and important consequences. We find (1) the expectation value of the zero-point energy under the new time variable vanishes; (2) the leading contribution to the gravitational effect is the energy fluctuation, the vacuum energy fluctuation effectively gives a correct order to the observed "dark energy", $\rho_\Lambda = \frac{6}{\pi}L_P^{-2}L_H^{-2}$; (3) the vacuum energy density is always comparable with the matter energy density seen by an observer using the local internal clock time. The three of the consequences from the time re-interpretation provides a solution to the cosmological constant problem.

The re-interpretation of time also leads to several conceptual consequences. (1) The new quantum time variable is able to reduce to conventional parameter time as a limit of semi-classical approximation. (2) The Wheeler–DeWitt equation plays a more fundamental role than the textbook

Schrodinger equation. The Schrodinger equation is a derivation under the semi-classical approximation of the Wheeler–DeWitt equation. (3) The solution of the Wheeler–DeWitt equation is in general an entangled state, which leads to the consequence that the absolute probability interpretation of the textbook quantum mechanics is required to be replaced by a relational interpretation with the help of the joint probability. (4) The entangled state solution of the Wheeler–DeWitt equation implies that not only the to-be-measured system but also the measuring instruments (such as the clock) both are required to be described by the quantum mechanics.

The idea of re-interpretation of time can be generalized to a more general version of a quantum reference frame, in which we could put the space and time on an equal footing. This framework provides us a new approach to treat the spacetime quantum dynamically, and leads to a possible route to the non-classifiable problem of quantum gravity in 4-dimensions.

Acknowledgements

This work was supported in part by the National Science Foundation of China (NSFC) under Grant No. 11205149.

References

[1] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.
[2] P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 559.
[3] A.G. Riess, et al., Supernova Search Team, Astron. J. 116 (1998) 1009, arXiv:astro-ph/9805201.
[4] Planck Collaboration, P.A.R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A.J. Banday, et al., arXiv:1303.5062, 2013.
[5] J. Polchinski, arXiv:hep-th/0603249, 2006, ArXiv High Energy Physics – Theory e-prints.
[6] R. Bousso, Gen. Relativ. Gravit. 40 (2008) 607, arXiv:0708.4231.
[7] C. Rovelli, Phys. Rev. D 42 (1990) 2638.
[8] C. Rovelli, Phys. Rev. D 43 (1991) 442.
[9] C. Rovelli, Found. Phys. 41 (2011) 1475, arXiv:0903.3832.
[10] C. Rovelli, Quantum Gravity, Cambridge University Press, 2004.
[11] L. Bombelli, R.K. Koul, J. Lee, R.D. Sorkin, Phys. Rev. D 34 (1986) 373.
[12] M. Srednicki, Phys. Rev. Lett. 71 (1993) 666.
[13] R. Brustein, D. Eichler, S. Foffa, D.H. Oaknin, Phys. Rev. D 65 (2002) 105013.
[14] T. Padmanabhan, T. Singh, Class. Quantum Gravity 4 (1987) 1397.
[15] T. Padmanabhan, Class. Quantum Gravity 22 (2005) L107, arXiv:hep-th/0406060.
[16] J. Hartle, Class. Quantum Gravity 2 (1985) 707.
[17] J. Ambjørn, Þ. Jónsson, Quantum Geometry: A Statistical Field Theory Approach, Cambridge University Press, 1997.
[18] S. Kauffman, L. Smolin, arXiv:gr-qc/9703026, 1997.