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Spinless bosons embedded in a vector Duffin–Kemmer–Petiau oscillator

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ABSTRACT

Some properties of minimal and nonminimal vector interactions in the Duffin–Kemmer–Petiau (DKP) formalism are discussed. The conservation of the total angular momentum for spherically symmetric nonminimal potentials is derived from its commutation properties with each term of the DKP equation and the proper boundary conditions on the spinors are imposed. It is shown that the space component of the nonminimal vector potential plays a crucial role for the confinement of bosons. The exact solutions for the vector DKP oscillator (nonminimal vector coupling with a linear potential which exhibits an equally spaced energy spectrum in the weak-coupling limit) for spin-0 bosons are presented in a closed form and it is shown that the spectrum exhibits an accidental degeneracy.

1. Introduction

The first-order Duffin–Kemmer–Petiau (DKP) formalism [1,2] describes spin-0 and spin-1 particles. The DKP equation for a free boson is given by [2] $(\beta^\mu p_\mu - m)\psi = 0$ (with units in which $\hbar = c = 1$), where the four beta matrices satisfy the algebra $\beta^\mu\beta^\nu\beta^\lambda + \beta^\lambda\beta^\nu\beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\lambda\nu}\beta^\mu$ and the metric tensor is $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. The algebra expressed by those matrices generates a set of 126 independent matrices whose irreducible representations are a trivial representation, a five-dimensional representation describing the spin-0 particles and a ten-dimensional representation associated to spin-1 particles. A well-known conserved four-current is given by $j^\mu = \overline{\psi}\beta^\mu\psi/2$, where the adjoint spinor $\overline{\psi}$ is given by $\overline{\psi} = \psi^\dagger\eta^0$ with $\eta^\mu = 2\beta^\mu\beta^\mu - g^{\mu\mu}$ in such a way that $(\eta^0\beta^\mu)^\dagger = \eta^0\beta^\mu$ (the matrices β^μ are Hermitian with respect to η^0). Despite the similarity to the Dirac equation, the DKP equation involves singular matrices, the time component of j^μ is not positive definite and the case of massless bosons cannot be obtained by a limiting process. Nevertheless, the matrices β^μ plus the unit operator generate a ring consistent with integer-spin algebra [3] and j^0 may be interpreted as a charge density. The factor $1/2$ multiplying $\overline{\psi}\beta^\mu\psi$, of no importance regarding the conservation law, is in order to hand over a charge density conformable to that one used in the Klein–Gordon theory and its nonrelativistic limit (see e.g. [4]). Then the normalization condition $\int d\tau\, j^0 = \pm 1$ can be expressed as $\int d\tau\, \overline{\psi}\beta^0\psi = \pm 2$, where the plus (minus) sign must be used for a positive (negative) charge.

The DKP formalism has been used to analyze relativistic interactions of spin-0 and spin-1 hadrons with nuclei. A number of different couplings in the DKP formalism, with scalar and vector couplings in analogy with the Dirac phenomenology for proton-nucleus scattering, has been employed in the phenomenological treatment of the elastic meson-nucleus scattering at medium energies with a better agreement to the experimental data when compared to the Klein–Gordon and Proca based formalisms [5–10]. Recently, there has been an increasing interest on the so-called DKP oscillator [11–15]. That system is a kind of tensor coupling with a linear potential which leads to the harmonic oscillator problem in the weak-coupling limit. A nonminimal vector potential, added by other kinds of Lorentz structures, has already been used successfully in a phenomenological context for describing the scattering of mesons by nuclei [5,6,8,10], and a sort of vector DKP oscillator (nonminimal vector coupling with a linear potential [14,16]) has also been an item of recent investigation. Vector DKP oscillator is the name given to the system with a Lorentz vector coupling which exhibits an equally spaced energy spectrum in the weak-coupling limit. The name distinguishes from that system called DKP oscillator with Lorentz tensor couplings of Refs. [11–15]. The nonminimal vector coupling with square step [17] and smooth step potentials [18] have also appeared in the literature.

The one-dimensional vector DKP oscillator was treated in Ref. [16] but we show in this Letter that the three-dimensional case has some very special features such as the question of conservation of the total angular momentum $\vec{J}$, boundary conditions on the spinor and degeneracy of the spectrum. The conservation of $\vec{J}$ is derived from its commutation properties with each term of the DKP equation. The proper boundary condition at the origin follows from the absence of Dirac delta potentials, avoiding in this manner to recourse to plausibility arguments regarding

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the self-adjointness of the momentum and the finiteness of the kinetic energy, as done by Greiner [19] in the case of the nonrelativistic harmonic oscillator. The exact solutions are presented in a closed form and the spectrum presents, beyond the essential degeneracy omnipresent for any central force field, an accidental degeneracy.

2. Vector interactions in the DKP equation

With the introduction of interactions, the DKP equation can be written as

$$(\beta^\mu p_\mu - m - V)\psi = 0 \tag{1}$$

where the more general potential matrix V is written in terms of 25 (100) linearly independent matrices pertinent to the five(ten)-dimensional irreducible representation associated to the scalar (vector) sector. In the presence of interactions j^μ satisfies the equation

$$\partial_\mu j^\mu + \frac{i}{2}\bar{\psi}\left(V - \eta^0 V^\dagger \eta^0\right)\psi = 0. \tag{2}$$

Thus, if V is Hermitian with respect to η^0 then the four-current will be conserved. The potential matrix V can be written in terms of well-defined Lorentz structures. For the spin-0 sector there are two scalar, two vector and two tensor terms [20], whereas for the spin-1 sector there are two scalar, two vector, a pseudoscalar, two pseudovector and eight tensor terms [21]. The tensor terms have been avoided in applications because they furnish noncausal effects [20,21]. Considering only the vector terms, V is in the form

$$V = \beta^\mu A_\mu^{(1)} + i[P, \beta^\mu] A_\mu^{(2)} \tag{3}$$

where P is a projection operator ($P^2 = P$ and $P^\dagger = P$) in such a way that $\bar{\psi} P \psi$ behaves as a scalar and $\bar{\psi}[P, \beta^\mu]\psi$ behaves like a vector. $A_\mu^{(1)}$ and $A_\mu^{(2)}$ are the four-vector potential functions. Notice that the vector potential $A_\mu^{(1)}$ is minimally coupled but not $A_\mu^{(2)}$. One very important point to note is that this matrix potential leads to a conserved four-current but the same does not happen if instead of $i[P, \beta^\mu]$ one uses either $P\beta^\mu$ or $\beta^\mu P$, as in [5,6,8,10,12]. As a matter of fact, in Ref. [5] is mentioned that $P\beta^\mu$ and $\beta^\mu P$ produce identical results.

The DKP equation is invariant under the parity operation, i.e. when $\vec{r} \to -\vec{r}$, if $\vec{A}^{(1)}$ and $\vec{A}^{(2)}$ change sign, whereas $A_0^{(1)}$ and $A_0^{(2)}$ remain the same. This is because the parity operator is $\mathcal{P} = \exp(i\delta_P) P_0 \eta^0$, where δ_P is a constant phase and P_0 changes $\vec{r}$ into $-\vec{r}$. Because this unitary operator anticommutes with $\vec{\beta}$ and $[P, \vec{\beta}]$, they change sign under a parity transformation, whereas β^0 and $[P, \beta^0]$, which commute with η^0, remain the same. Since $\delta_P = 0$ or $\delta_P = \pi$, the spinor components have definite parities. The charge-conjugation operation changes the sign of the minimal interaction potential, i.e. changes the sign of $A_\mu^{(1)}$. This can be accomplished by the transformation $\psi \to \psi_c = \mathcal{C}\psi = CK\psi$, where K denotes the complex conjugation and C is a unitary matrix such that $C\beta^\mu = -\beta^\mu C$. The matrix that satisfies this relation is $C = \exp(i\delta_C)\eta^0\eta^1$. The phase factor $\exp(i\delta_C)$ is equal to ± 1, thus $E \to -E$. Note also that $j^\mu \to -j^\mu$, as should be expected for a charge current. Meanwhile C anticommutes with $[P, \beta^\mu]$ and the charge-conjugation operation entails no change on $A_\mu^{(2)}$. The invariance of the nonminimal vector potential under charge conjugation means that it does not couple to the charge of the boson. In other words, $A_\mu^{(2)}$ does not distinguish particles from antiparticles. Hence, whether one considers spin-0 or spin-1 bosons, this sort of interaction cannot exhibit Klein's paradox.

For massive spinless bosons the projection operator is given by [20]

$$P = \frac{1}{3}\left(\beta^\mu \beta_\mu - 1\right). \tag{4}$$

Defining $P^\mu = P\beta^\mu$ and ${}^\mu P = \beta^\mu P$, one can obtain the follow relations [22]

$$\begin{aligned} &\beta^\mu = P^\mu + {}^\mu P, \qquad P^\mu \beta^\nu = P g^{\mu\nu}, \\ &\left(P^\mu\right)P = P\left({}^\mu P\right) = 0, \qquad \left(P^\mu\right)\left(P^\nu\right) = \left({}^\mu P\right)\left({}^\nu P\right) = 0. \end{aligned} \tag{5}$$

Applying P and P^ν to the DKP equation and using the relations (5), we have

$$i\left(D_\mu - A_\mu^{(2)}\right)\left(P^\mu \psi\right) = m(P\psi) \tag{6}$$

and

$$i\left(D_\mu + A_\mu^{(2)}\right)(P\psi) = m(P_\mu \psi), \tag{7}$$

respectively. Here, $D_\mu = \partial_\mu + iA_\mu^{(1)}$. Combining these results we obtain

$$\left[D^\mu D_\mu + m^2 + \left(\partial^\mu A_\mu^{(2)}\right) - \left(A^{(2)}\right)_\mu \left(A^{(2)}\right)^\mu\right](P\psi) = 0. \tag{8}$$

On the other hand, by using (5) j^μ can be written as

$$j^\mu = -\frac{1}{m}\,\mathrm{Im}\left[(P\psi)^\dagger D^\mu (P\psi)\right]. \tag{9}$$

One sees that $A_\mu^{(2)}$ does not intervene explicitly in the current and, in the absence of the nonminimal potential, (8) reduces to the Klein–Gordon equation in the presence of a minimally coupled potential and that all elements of the column matrix $P\psi$ are scalar fields of mass m. It is instructive to note that the form of the two distinct vector couplings in the generalized Klein–Gordon equation has become obvious because the interaction operates under the umbrella of the DKP theory. Otherwise, only the minimal vector coupling could be obtained by applying the minimal substitution $\partial_\mu \to \partial_\mu + iA_\mu^{(1)}$ to the free Klein–Gordon equation.

3. The nonminimal vector interaction

In this stage, we concentrate our efforts in the nonminimal vector potential $A_\mu^{(2)} = A_\mu$ and use the representation for the β^μ matrices given by [11,23]

$$\beta^0 = \begin{pmatrix} \theta & \bar{0} \\ \bar{0}^T & \mathbf{0} \end{pmatrix}, \qquad \vec{\beta} = \begin{pmatrix} \tilde{0} & \vec{\rho} \\ -\vec{\rho}^{\,T} & \mathbf{0} \end{pmatrix} \tag{10}$$

where

$$\begin{aligned} &\theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ &\rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{11}$$

$\bar{0}$, $\tilde{0}$ and $\mathbf{0}$ are 2×3, 2×2 and 3×3 zero matrices, respectively, while the superscript T designates matrix transposition. The five-component spinor can be written as $\psi^T = (\psi_1, \ldots, \psi_5)$. With this representation the projection operator is $P = \mathrm{diag}(1,0,0,0,0)$. In this case P picks out the first component of the DKP spinor.

If the terms in the potential A^μ are time-independent one can write $\psi(\vec{r}, t) = \phi(\vec{r})\exp(-iEt)$, where E is the energy of the boson, in such a way that the time-independent DKP equation becomes

$$\left[\beta^0 E + i\beta^i \partial_i - \left(m + i[P, \beta^\mu] A_\mu\right)\right]\phi = 0. \tag{12}$$

In this case $j^\mu = \bar{\phi}\beta^\mu\phi/2$ does not depend on time, so that the spinor ϕ describes a stationary state. In the time-independent case (7) becomes

$$\phi_2 = \frac{1}{m}(E + iA_0)\phi_1, \tag{13}$$

$$\vec{\zeta} = (\vec{\nabla} - \vec{A})\phi_1 \tag{14}$$

where

$$\vec{\zeta} = \frac{m}{i}(\phi_3, \phi_4, \phi_5)^T \tag{15}$$

and (8) furnishes

$$\left(-\nabla^2 + \vec{\nabla}\cdot\vec{A} + \vec{A}^2\right)\phi_1 = k^2\phi_1 \tag{16}$$

where

$$k^2 = E^2 - m^2 + A_0^2. \tag{17}$$

Meanwhile,

$$j^0 = \frac{E}{m}|\phi_1|^2, \qquad \vec{j} = \frac{1}{m}\,\mathrm{Im}\left(\phi_1^*\vec{\nabla}\phi_1\right). \tag{18}$$

If we consider spherically symmetric potentials

$$A^\mu(\vec{r}) = \left(A_0(r), A_r(r)\hat{r}\right), \tag{19}$$

then the DKP equation permits the factorization

$$\phi_1(\vec{r}) = \frac{u_\kappa(r)}{r} Y_{lm_l}(\theta, \varphi) \tag{20}$$

where Y_{lm_l} is the usual spherical harmonic, with $l = 0, 1, 2, \ldots$, $m_l = -l, -l+1, \ldots, l$, $\int d\Omega\, Y_{lm_l}^* Y_{l'm_{l'}} = \delta_{ll'}\delta_{m_l m_{l'}}$ and κ stands for all quantum numbers which may be necessary to characterize ϕ_1. For $r \neq 0$ the radial function u obeys the radial equation

$$\frac{d^2u}{dr^2} + \left[k^2 - 2\frac{A_r}{r} - \frac{dA_r}{dr} - \frac{l(l+1)}{r^2} - A_r^2\right]u = 0 \tag{21}$$

and because $\nabla^2(1/r) = -4\pi\delta(\vec{r})$, unless the potentials contain a delta function at the origin, one must impose the homogeneous Dirichlet condition $u(0) = 0$ [24]. Furthermore, from the normalization condition $\int d\tau\, j^0 = \pm 1$ one sees that u must be normalized according to

$$\frac{E}{m}\int_0^\infty dr\,|u|^2 = \pm 1. \tag{22}$$

Therefore, for motion in a central field, the solution of the three-dimensional DKP equation can be found by solving a Schrödinger-like equation. The other components are obtained through of (13) and (14). Note that the DKP spinor is an eigenstate of the parity operator. This happens because $\eta^0 = \mathrm{diag}(1, 1, -1, -1, -1)$ and the parity of $\vec{\zeta}$ is opposite to that one of ϕ_1 and ϕ_2. Furthermore, the spin operator $S_k = i\varepsilon_{klm}\beta^l\beta^m$ [2] satisfies the commutation relations

$$\begin{aligned}
&[S_k, \beta^0] = [S_k, [P, \beta^0]] = 0, \\
&[S_k, \beta^l] = i\varepsilon_{klm}\beta^m, \qquad [S_k, [P, \beta^l]] = i\varepsilon_{klm}[P, \beta^m]
\end{aligned} \tag{23}$$

so that the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ satisfies

$$[\vec{J}, \beta^\mu p_\mu] = [\vec{J}, \beta^\mu A_\mu^{(1)}] = [\vec{J}, [P, \beta^\mu]A_\mu^{(2)}] = \vec{0} \tag{24}$$

in such a way that the DKP spinor is also an eigenstate of $\vec{J}^2$ and J_3. Accordingly, the DKP spinor can be classified by the parity, by the total angular momentum, and its third component, quantum numbers. As a matter of fact,

$$\vec{S} = \begin{pmatrix} \tilde{0} & \bar{0} \\ \bar{0}^T & \vec{s} \end{pmatrix} \tag{25}$$

where s_k are the 3×3 spin-1 matrices $(s_k)_{lm} = -i\varepsilon_{klm}$. As a result, $\vec{S}$ does not act on the two upper components of the DKP spinor. This means that the orbital angular momentum quantum numbers of ϕ_1 and ϕ_2 are good quantum numbers. With the orbital angular momentum quantum number l referring to the two upper components of the DKP spinor, as before, $\vec{\zeta}$ in (14) can be written as [25]

$$\begin{aligned}
\vec{\zeta} = {}& \vec{Y}_{l,l-1,m_l}\sqrt{\frac{l}{2l+1}}\left(\frac{d}{dr} + \frac{l+1}{r} - A_r^{(2)}\right)\frac{u(r)}{r} \\
& - \vec{Y}_{l,l+1,m_l}\sqrt{\frac{l+1}{2l+1}}\left(\frac{d}{dr} - \frac{l}{r} - A_r^{(2)}\right)\frac{u(r)}{r}.
\end{aligned} \tag{26}$$

In this last expression, $\vec{Y}_{Jlm_J}(\theta, \varphi)$ are the so-called vector spherical harmonics. They result from the coupling of the three-dimensional unit vectors in spherical notation to the eigenstates of orbital angular momentum, form a complete orthonormal set and satisfy

$$\begin{aligned}
&\vec{J}^2\vec{Y}_{Jlm_J} = J(J+1)\vec{Y}_{Jlm_J}, \qquad \vec{L}^2\vec{Y}_{Jlm_J} = l(l+1)\vec{Y}_{Jlm_J}, \\
&J_3\vec{Y}_{Jlm_J} = m_J\vec{Y}_{Jlm_J}
\end{aligned} \tag{27}$$

and $\vec{Y}_{l,l\pm1,m_l}$ transforms under parity as

$$\vec{Y}_{l,l\pm1,m_l}(\theta - \pi, \varphi + \pi) = (-1)^{l+1}\vec{Y}_{l,l\pm1,m_l}(\theta, \varphi). \tag{28}$$

One sees that if the two upper components of the DKP spinor are eigenfunctions of $\vec{L}^2$ with an orbital angular momentum quantum number l, the three lower components will be a linear superposition of two types of eigenfunctions of $\vec{L}^2$. One of those with orbital angular momentum quantum number $l+1$ and the other with $l-1$. The fact that the upper and lower components of the DKP spinor have different orbital angular momentum quantum numbers is related to the fact that $\vec{L}$ is not a conserved quantity in the DKP theory. Nevertheless, the orbital angular momentum quantum number of the first component of the DKP spinor equals the total angular momentum quantum number of the DKP spinor, as it should be since ϕ_1 describes a spinless particle. It follows that the parity of the DKP spinor is given by $(-1)^l$.

4. The vector DKP oscillator

Let us consider a nonminimal vector linear potential in the form

$$A_0^{(2)} = m^2\lambda_0 r, \qquad A_r^{(2)} = m^2\lambda_r r \tag{29}$$

where λ_0 and λ_r are dimensionless quantities. Our problem is to solve (21) for u and to determine the allowed energies.

One finds that u obeys the second-order differential equation

$$\frac{d^2u}{dr^2} + \left[K^2 - \lambda^2r^2 - \frac{l(l+1)}{r^2}\right]u = 0 \tag{30}$$

where

$$K = \sqrt{E^2 - m^2 - 3m^2\lambda_r}, \qquad \lambda = m^2\sqrt{\lambda_r^2 - \lambda_0^2}. \tag{31}$$

With $u(0) = 0$ and $\int_0^\infty dr\,|u|^2 < \infty$, the solution for (30) with K and λ real is precisely the well-known solution of the Schrödinger equation for the three-dimensional harmonic oscillator (see, e.g.

[19]). For $\lambda = i|\lambda|$ (and $K = |K|$ or $K = i|K|$), the case of an inverted harmonic oscillator, the energy spectrum will consist of a continuum corresponding to unbound states. We shall limit ourselves to study the case of bound-state solutions.

The asymptotic behavior of (30) and the conditions $u(0) = 0$ and $\int_0^\infty dr\,|u|^2 < \infty$ dictate that the solution close to the origin valid for all values of l can be written as being proportional to r^{l+1}, and proportional to $e^{-\lambda r^2/2}$ as $r \to \infty$. It is convenient to introduce the following new variable and parameters:

$$z = \lambda r^2, \qquad a = \frac{1}{2}\left(l + \frac{3}{2} - \frac{K^2}{2\lambda}\right), \qquad b = l + \frac{3}{2} \tag{32}$$

so that the solution for all r can be expressed as $u(r) = r^{l+1} \times e^{-\lambda r^2/2} w(r)$, where w is a regular solution of the confluent hypergeometric equation (Kummer's equation) [26]

$$z\frac{d^2 w}{dz^2} + (b - z)\frac{dw}{dz} - aw = 0. \tag{33}$$

The general solution of (33) is given by [26]

$$w = A\,M(a, b, z) + B\,z^{1-b} M(a - b + 1, 2 - b, z) \tag{34}$$

with arbitrary constants A and B. The confluent hypergeometric function (Kummer's function) $M(a, b, z)$, or ${}_1F_1(a, b, z)$, is

$$M(a, b, z) = \frac{\Gamma(b)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(b+n)} \frac{z^n}{n!}, \tag{35}$$

and $\Gamma(z)$ is the gamma function. The second term in (34) has a singular point at $z = 0$, so we set $B = 0$. In order to furnish normalizable ϕ_1, the confluent hypergeometric function must be a polynomial. This is because $M(a, b, \lambda r^2)$ goes as $e^{\lambda r^2}$ as r goes to infinity unless the series breaks off. This demands that $a = -n$, where n is a nonnegative integer. We put $N = 2n + l$, whence the requirement $a = -n$ implies into

$$|E| = m\sqrt{1 + 3\lambda_r + (2N + 3)\sqrt{\lambda_r^2 - \lambda_0^2}}, \quad N = 0, 1, 2, \ldots. \tag{36}$$

Note that $M(-n, b, \lambda r^2)$ is proportional to the generalized Laguerre polynomial $L_n^{(b-1)}(\lambda r^2)$, a polynomial of degree n. Therefore, by using the normalization condition $|\int d\tau\, j^0| = |E|/m \int d\tau\, |\phi_1|^2 = 1$, with $|E| \neq 0$, and that the generalized Laguerre polynomial is standardized as [26]

$$\int_0^\infty d\xi\, \xi^\alpha e^{-\xi} \left[L_n^{(\alpha)}(\xi)\right]^2 = \frac{\Gamma(\alpha + n + 1)}{n!} \tag{37}$$

one determines A in (34) and obtains

$$\phi_1 = \sqrt{\frac{2m\lambda^{l+3/2}(\frac{N-l}{2})!}{|E|\Gamma(\frac{N+l+3}{2})}}\, r^l e^{-\lambda r^2/2} L_{\frac{N-l}{2}}^{(l+1/2)}(\lambda r^2) Y_{lm_l}(\theta, \varphi). \tag{38}$$

Note that l can take the values $0, 2, \ldots, N$ when N is an even number, and $1, 3, \ldots, N$ when N is an odd number. All the energy levels with the exception of that one for $N = 0$ are degenerate. The degeneracy of the level of energy for a given principal quantum number N is given by $(N + 1)(N + 2)/2$. Notice that the condition $\lambda \in \mathbb{R}$ requires that $|\lambda_r| > |\lambda_0|$, meaning that the space component of the potential must be stronger than its time component. There is an infinite set of discrete energies (symmetrical about $E = 0$ as it should be since A_μ does not distinguish particles from antiparticles) irrespective to the sign of λ_0. In general, $|E|$ is higher for $\lambda_r > 0$ than for $\lambda_r < 0$. It increases with the principal quantum number and it is a monotonically decreasing function of λ_0. For $\lambda_r < 0$ and $\lambda_0 = 0$ the spectrum acquiesces $|E| = m$ for $N = 0$. In order to insure the reality of the spectrum, the coupling constants λ_0 and λ_r satisfy the additional constraint $\sqrt{\lambda_r^2 - \lambda_0^2} > -(1 + 3\lambda_r)/(2N + 3)$ in such a way that there can be no bound states for $\lambda_r < 0$ with small principal quantum numbers and $|\lambda_r|$ enough small. This means that for $\lambda_r < 0$ and $|\lambda_r|$ enough small a number of solutions with the smallest principal quantum numbers does not exist. For $|\lambda_r| \simeq |\lambda_0|$ we have a very high density of very delocalized states (because $\lambda \simeq 0$). For $|\lambda_r| \gg |\lambda_0|$ one has that

$$|E| \simeq m\sqrt{1 + 3\lambda_r + (2N + 3)|\lambda_r|} \tag{39}$$

so that $|E| > m$ for $\lambda_r > 0$. Concerning $\lambda_r < 0$, as far as λ_r decreases, the spectrum moves towards $E = 0$, except for $\lambda_0 = 0$ which maintains $|E| \geqslant m$. On the other hand, in the weak-coupling limit, $|\lambda_r| \ll 1$ and $|\lambda_0| \ll 1$, $|E| \simeq m$ for small quantum numbers, and (36) becomes

$$|E| \simeq m\left[1 + \frac{3}{2}\lambda_r + \left(N + \frac{3}{2}\right)\sqrt{\lambda_r^2 - \lambda_0^2}\right]. \tag{40}$$

Because of this equally spaced energy spectrum, it can be said that the linear potential given by (29) describes a genuine vector DKP oscillator. It is obvious that, despite the effective harmonic oscillator potential appearing in (30) and the spectrum given by (40), in a nonrelativistic scheme would appear the sum of the two intervening potentials in the Schrödinger equation and no bound-state solutions would be possible for $\lambda_r < 0$ and $|\lambda_r| > |\lambda_0|$. Therefore, the weak-coupling limit does not correspond to the nonrelativistic limit and so we can say that the nonminimal vector linear potential given by (29) is an intrinsically relativistic potential in the DKP theory.

5. Conclusions

We showed that minimal and nonminimal vector interactions behave differently under the charge-conjugation transformation. In particular, nonminimal vector interactions have no counterparts in the Klein–Gordon theory. The conserved charge current plus the charge conjugation operation are enough to infer about the absence of Klein's paradox under nonminimal vector interactions, or its possible presence under minimal vector interactions. Although Klein's paradox cannot be treated as unworthy of regard in the DKP theory with minimally coupled vector interactions, it never makes its appearance in the case of nonminimal vector interactions because they do not couple to the charge. Nonminimal vector interactions have the very same effects on both particles and antiparticles and so in the case of a pure nonminimal vector coupling, both particle and particle energy levels are members of the spectrum, and the particle and antiparticle spectra are symmetrical about $E = 0$. If the interaction potential is attractive (repulsive) for bosons it will also be attractive (repulsive) for antibosons. However, there is no crossing of levels because possible states in the strong field regime with $E = 0$ are in fact unnormalizable. These facts imply that there is no channel for spontaneous boson–antiboson creation and for that reason the single-particle interpretation of the DKP equation is ensured. The charge conjugation operation allows us to migrate from the spectrum of particles to the spectrum of antiparticles and vice versa just by changing the sign of E. This change induces no change in the nodal structure of the components of the DKP spinor and so the nodal structure of the four-current is preserved.

We showed that nonminimal vector couplings have been used improperly in the phenomenological description of elastic meson-nucleus scatterings potential by observing that the four-current is not conserved when one uses either the matrix $P\beta^{\mu}$ or $\beta^{\mu}P$, even though the bilinear forms constructed from those matrices behave as true Lorentz vectors. The space component of the nonminimal vector potential cannot be absorbed into the spinor and we showed that the space component of the nonminimal vector potential could be irrelevant for the formation of bound states for potentials vanishing at infinity but its presence is an essential ingredient for confinement.

The complete solution of the DKP equation with spherically symmetric nonminimal vector potentials was found by recurring to vector spherical harmonics due to the expression appearing in (14) with $\vec{\nabla}$ in spherical coordinates acting on a function of r multiplied by $Y_{lm_l}(\theta,\varphi)$. A similar procedure resulting in a set of coupled differential equations for the components of the spinor has already appeared in the literature [23]. Here, instead of a set of coupled first-order equations, the DKP equation was mapped into a Sturm–Liouville problem for the first component of the spinor and the remaining components were expressed in terms of the first one in a simple way. In this process, the conserved four-current was also expressed in terms of the first component of the DKP spinor in such a way that the searching for the solutions of the DKP equation becomes more clear and transparent. The conservation of the total angular momentum was derived from its commutation properties with each term of the DKP equation.

The solution for a nonminimal linear potential was found by solving a Schrödinger-like problem for the nonrelativistic harmonic oscillator for the first component of the spinor. The behavior of the solutions for this sort of DKP oscillator was discussed in detail. Instead of imposing boundary conditions at the origin by recurring to plausibility arguments regarding the self-adjointness of the momentum and the finiteness of the kinetic energy, as done by Greiner [19] in the case of the nonrelativistic harmonic oscillator, the proper boundary conditions were imposed in a simple way by observing the absence of Dirac delta potentials. The exact solutions were presented in a closed form and the spectrum presents, beyond the essential degeneracy omnipresent for any central force field, an accidental degeneracy. That model reinforced the absence of Klein's paradox for nonminimal vector interactions.

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