Modal Analysis of a Cable-stayed Bridge

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Abstract

The project involved in this paper is a single-tower double-cable-plane bridge, and the gulf nearby the bridge is in a typical typhoon-affected zone. Therefore, modal analysis of the cable-stayed bridge should be carried out. In this paper, based on the structural vibration theory and the theory of finite element model (FEM), the space FEM model of the cable-stayed bridge are established with APDL — ANSYS Parametric Design Language. Vibration mode and its frequency can be concluded by the calculating of the cable-stayed bridge’s FEM model.

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Keywords: cable-stayed bridge; finite element method; modal analysis; vibration characteristics

1. Outline

As one of the most competitive bridge in modern times, the cable-stayed bridge is usually a hub for transportation projects. Once destroyed in all kinds of adverse natural conditions, it will result in enormous economic losses. Consequently, the reliability of the overall structure and the complexity of the dynamic characteristics should be fully taken into consideration when the cable-stayed bridge structure is designed. And thus the wind resistance studies, aseismatic design and maintenance of the bridge can be carries out rightly [1]. Modal parameters are the main parameters of structural dynamic characteristics. Therefore, modal analysis is important for the study of dynamic characteristics.

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2. Project overview

The bridge is designed as a single-tower double-cable-planes cable-stayed bridge. It is shown in Figure 1. With parts of the approach spans, the bridge is 600 meters long. The longitudinal slope of the bridge floor is also designed as 3%.

![Elevation of the whole cable-stayed bridge](image)

The girder is designed as fish-bellied prestressed concrete box girder, considering that fish-bellied prestressed concrete box beam have a good visual effects and it is beneficial to wind resistance.

The stay cables are designed to double sector cable faces. The spacing of the cables is 8 meters. There are 16 pairs of stay cables on each cable face. The stay cables adopt galvanized steel wire (φ7) protected with PE.

The tower of the bridge adopts pylon tower with herringbone pattern. The pylon tower is 91.7 meters higher than the bearing platform. The cross-section of the pylon tower is hollow rectangular.

The material property of each section is shown in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Structural Members</th>
<th>Elastic Modulus/MPa</th>
<th>Poisson's Ratio</th>
<th>Bulk Density /kN/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>Girder, Crossbeam</td>
<td>3.5×10⁴</td>
<td>0.2</td>
<td>26</td>
</tr>
<tr>
<td>Concrete</td>
<td>Bridge Tower, Pier</td>
<td>3.5×10⁴</td>
<td>0.2</td>
<td>26</td>
</tr>
<tr>
<td>Concrete</td>
<td>Bridge Deck</td>
<td>3.5×10⁴</td>
<td>0.2</td>
<td>26</td>
</tr>
<tr>
<td>Galvanized Steel Wire</td>
<td>Stay Cable</td>
<td>1.9×10⁵</td>
<td>0.3</td>
<td>78.5</td>
</tr>
</tbody>
</table>


The vibration characteristics decide the dynamic response characteristics. Consequently the modal analysis of the cable-stayed bridge is essential to study the dynamic behavior of cable-stayed bridge. The structure of the cable-stayed bridge is complex, and the cables are flexible, lightweight, low damping, etc. All these features above make the mode of the cable-stayed bridge a practical engineering issue of worth attention.

The structure of the cable-stayed bridge is a system with continuous distribution of mass and stiffness. It should be divided into finite elements with limited DOF. Because of the complexity of cable-stayed bridge structure, the result of three-dimensional FEM analysis is more comprehensive and more reliable than the result of the traditional empirical formula.

It can be assumed that the structural of the cable-stayed bridge has N DOF. The dynamic equilibrium equation of the model can be list as:
Where \( [M] \) = mass matrix of the architecture, \( [C] \) = damping matrix of the architecture, \( [K] \) = stiffness matrix of the architecture, \( [u] \) = displacement vector of each node, \( [\dot{u}] \) = velocity vector of each node, \( [\ddot{u}] \) = acceleration vector of each node. Ignoring the resistance, we can get a dynamic equilibrium equation:

\[
[M][\ddot{u}] + [C][\dot{u}] + [K][u] = \{p(t)\}
\]  

(1)

Take \( [\ddot{u}] = 0 \), and then solve the differential equations. We can obtain:

\[
[M][\dot{u}] + [K][u] = 0
\]  

(2)

We can get natural frequencies of the system \( \omega_i (i = 1, 2, \ldots, N) \) and the vibration modals of the structure \( \{\Phi_i\} (i = 1, 2, \ldots, N) \) from this equation.

4. Establish FEM model [3]

Modelling is critical to structural analysis. Whether the model can reflects the structural stiffness and mass system accurately may seriously affect the accuracy of the calculation. Structural stiffness, mass and boundary conditions should be fully considered in modal analysis, because these three main factors are directly related to characteristics of the structural. Establishing a very fine FEM model wastes manpower and material resources, and it may be not unnecessary. However, the speed of the computer and the efficiency of the software have greatly improved now, we can adopt a better model. FEM analysis is the final goal of the establishment of FEM model, so a reasonable FEM model should accurately reflect the nature and characteristics of the structure.

The selection of element type is listed in the Table 2.

<table>
<thead>
<tr>
<th>structure</th>
<th>element type</th>
</tr>
</thead>
<tbody>
<tr>
<td>longitudinal girder, crossbeam, bridge tower</td>
<td>BEAM44</td>
</tr>
<tr>
<td>bridge deck</td>
<td>SHELL181</td>
</tr>
<tr>
<td>stay cable</td>
<td>LINK10</td>
</tr>
</tbody>
</table>

The initial force in the cables and the gravity of the bridge has little effect on the dynamic characteristics of the structure [4]. Therefore, the initial force in the cable and gravity of the bridge can be ignored when modal analysis carried out. By constraining the corresponding nodes’ degrees, the boundary conditions of the model can be defined.

Taking into account the purpose of this analysis is to study the modals of the main span, only the main object of the bridge is established. Based on the structural vibration theory and the theory of FEM model, the space FEM model of cable-stayed bridge is established with APDL. The FEM model of the cable-stayed bridge with constraints is shown in Figure 2.
5. **FEM calculate and modal analysis**

The methods of modal calculation include Subspace method, Block Lanczos method, Power Dynamics method, etc. Subspace method adopts the subspace iteration technique, and calculation samples showed the exactitude and efficiency of Subspace method. What’s more, Subspace method is suitable to extract a few of modals from large model [5].

In this paper, modal calculation adopts Subspace method, the convergence error takes default value, and the number of iterations is 30. Parts of modal shapes are shown in Figure 3 to 5.

![Fig. 3. (a) 1st-step vibration mode; (b) 2nd-step vibration mode](image1)

![Fig. 4. (a) 3th-step vibration mode; (b) 5th-step vibration mode](image2)
The natural frequencies and the features of mode shapes are listed in the Table 3.

Table 3. the natural frequencies and the features of mode shapes

<table>
<thead>
<tr>
<th>step</th>
<th>frequency /Hz</th>
<th>cycle time/s</th>
<th>features of mode shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.33793</td>
<td>2.95919</td>
<td>girder antisymmetric vertical bending, bridge tower longitudinal bending</td>
</tr>
<tr>
<td>2</td>
<td>0.43574</td>
<td>2.29495</td>
<td>girder antisymmetric torsion, bridge tower no bending</td>
</tr>
<tr>
<td>3</td>
<td>0.47876</td>
<td>2.08873</td>
<td>girder symmetrical vertical bending, bridge tower no l bending</td>
</tr>
<tr>
<td>4</td>
<td>0.68722</td>
<td>1.45514</td>
<td>girder symmetrical torsion, associated with lateral bending</td>
</tr>
<tr>
<td>5</td>
<td>0.80838</td>
<td>1.23704</td>
<td>girder antisymmetric torsion, associated with lateral bending</td>
</tr>
<tr>
<td>6</td>
<td>0.84881</td>
<td>1.17812</td>
<td>girder symmetrical torsion, associated with lateral bending</td>
</tr>
<tr>
<td>7</td>
<td>0.98465</td>
<td>1.01559</td>
<td>girder antisymmetric vertical bending, bridge tower longitudinal bending</td>
</tr>
<tr>
<td>8</td>
<td>1.0655</td>
<td>0.93853</td>
<td>girder symmetrical vertical bending, bridge tower no bending</td>
</tr>
<tr>
<td>9</td>
<td>1.2865</td>
<td>0.77730</td>
<td>girder antisymmetric vertical bending, bridge tower longitudinal bending</td>
</tr>
<tr>
<td>10</td>
<td>1.4715</td>
<td>0.67958</td>
<td>girder symmetrical torsion, bridge tower lateral bending</td>
</tr>
<tr>
<td>15</td>
<td>2.6894</td>
<td>0.37183</td>
<td>girder antisymmetric torsion, bridge tower no bending</td>
</tr>
<tr>
<td>20</td>
<td>3.1971</td>
<td>0.31278</td>
<td>girder antisymmetric vertical bending, bridge tower longitudinal bending</td>
</tr>
<tr>
<td>25</td>
<td>4.1657</td>
<td>0.24006</td>
<td>girder symmetrical torsion, bridge tower lateral bending</td>
</tr>
<tr>
<td>30</td>
<td>5.7320</td>
<td>0.17446</td>
<td>girder symmetrical torsion, bridge tower lateral bending</td>
</tr>
</tbody>
</table>

According to Table 6-1 above, we can draw the conclusions about the single-tower double-cable-planes cable-stayed bridge as follows:

- The results are completely symmetrical or antisymmetrical, which accord with the structure of the cable-stayed bridge.
- The circle time of the first step is 2.95919s, which is longer than the circle time of general civil engineering structures. So this cable-stayed bridge is a flexible structure. The long circle time indicates that the bridge is a flexible engineering structure. However, this cycle time is still less than 5 seconds, it is still short compared with other long-span cable-stayed bridges [6]. Consequently, this program of the bridge is not economic enough.
- Two kinds of vertical bending mode shapes appear in the first three steps, which is due to the weak vertical stiffness.
The first-step torsion mode of vibration has a great relationship with the critical flutter velocity of cable-stayed bridge. The first torsional frequency of this bridge is 0.43574Hz. The torsion vibration mode of girder appeared earlier and frequently, which indicates that the fish-bellied prestressed concrete box girder’s torsional stiffness is weak. What’s more, torsion and curvature of girder couple significantly, which indicates that the tension in the cables makes the cable-stayed bridge structure a space system.

Most of steps are complex-coupled vibration mode. What’s more, the mode shapes show that the higher the step number is the more complex mode shape is.

The girder’s first-step anti-symmetric lateral bending occurs late, which indicates that the bridge deck is wide, the girder is rigid.

The spectrum of the cable-stayed bridge is dense. Therefore, more steps modals should be analysis than general civil engineering structures.

6. Conclusion

In this paper, the space FEM model of cable-stayed bridge is established, and modal analysis is carried out for the cable-stayed bridge. The results can be used as the basic data of the bridge on various complex dynamic response analyses, long-term health monitoring and state assessment. The method of the cable-stayed bridge FEM model’s establishment can provide a reference for similar structure modelling, and the analysis results can provide a useful reference for other cable-stayed bridge’s safety design.

The bridge tower and the girder are compression members. When taking into account nonlinear effects, the stiffness of the members will reduce, the deflection will be larger and frequency will be reduced. What’s more, buckling may appear. All these conditions need take into account.

The results in this paper are based on the initial design FEM model. Therefore, the finite element model needs to be validated and corrected in the future.

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References