CRITICAL COOLANT FLOW VELOCITIES
IN REACTORS HAVING PARALLEL
FUEL PLATES

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Abstract—The response of flat, thin, parallel, metal fuel elements to loads excited by the coolant flow
through reactor core passages is investigated for the existence of plate divergence at velocities higher than a
 critical value.

1. INTRODUCTION

It is always useful to increase power densities in reactors. This can be achieved by using either
high coolant velocities or thinner fuel plates, which both cause instability in the reactor. For a
given reactor structure, it is necessary to define a critical coolant velocity, beyond which
instability occurs in the system.

The critical velocity is obtained by Miller[1], by solving the neutral equilibrium problem in
which the fluid pressure forces and plate restoring forces are in balance so that deflections are
not time dependent. This implies that higher velocities will cause plate divergence, that is
deflections increasing with time.

In the analysis of Rosenberg and Youngdahl[2], the dynamic terms are included in the
plate-deflection. Time variable and fluid inertia are also considered in the problem. However, in
the present study, the hydrodynamic forces acting on the plates are described by the classical
linearized, potential theory. The critical velocity is obtained as a minimum flow velocity which
causes divergence at least in one fuel plate.

The assumptions for the analysis are:
(a) The plates are homogeneous, isotropic, elastic, initially flat, uniformly spaced and free
of unidentified sources of deformation.
(b) The plate behaviour is accounted for by small deflection plate theory, shear deforma-
tions are neglected.
(c) The panels are infinitely long in the flow direction, in the same direction the plates have
built-in edges and the other two ends are free.
(d) The leakage between channels is not considered.
(e) The coolant is incompressible and inviscid and there exists a single phase.

2. ANALYSIS

The equation of motion of the ith plate (see Fig. 1) is

\[ D \left( \frac{\partial^4 w_i}{\partial x_i^4} + 2 \frac{\partial^4 w_i}{\partial x_i^2 \partial y_i^2} + \frac{\partial^4 w_i}{\partial y_i^4} \right) + \rho_m h \frac{\partial^2 w_i}{\partial t^2} + 2 \gamma \rho_m h \frac{\partial w_i}{\partial t} + p_{ii} - p_{2i} = 0 \]  

(2.1)

where \( w_i \) is the deflection of the ith plate in the \( z_i \) direction and \( D = Eh^3/12(1 - \nu^2) \) = plate stiffness,
\( E \) = modulus of elasticity, \( \nu \) = Poisson’s ratio, \( h \) = plate thickness, \( \rho_m \) = plate density,
\( \gamma \) = damping coefficient, \( p_{ii} \) = pressure on the positive side of the plate, and \( p_{2i} \) = pressure on
the negative side of the plate, \( x_i \) and \( y_i \) are space variables and \( t \) is time. The velocity potential

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\( \phi_i \) for the coolant in the \( i \)th channel satisfies the equation

\[
\frac{\partial^2 \phi_i}{\partial x_i^2} + \frac{\partial^2 \phi_i}{\partial y_i^2} + \frac{\partial^2 \phi_i}{\partial z_i^2} = 0
\]  
(2.2)

where the fluid is considered incompressible.

The boundary conditions at built-in boundaries are:

\[
w_i(x_i - b/2, t) = w_i(x_i + b/2, t) = 0, \quad (\text{2.3})
\]

\[
\frac{\partial w_i}{\partial y_i} (x_i - b/2, t) = \frac{\partial w_i}{\partial y_i} (x_i + b/2, t) = 0 \quad (\text{2.4})
\]

where \( b \) is the width of the fuel plate and at each end of the plate

\[
w_i(-\infty, y_i, t) = w_i(\infty, y_i, t) = \text{finite}. \quad (\text{2.5})
\]

The deflection can be chosen with respect to conditions (2.3), (2.4) and (2.5),

\[
w_i = w_i^0 [1 + \cos(2\pi y_i/b) \exp[i(\omega t - kx_i)]] \quad (\text{2.6})
\]

where \( \omega, k, w_i^0 \) are the frequency, the wave number and a constant, and \( i = \sqrt{-1} \), a simple form for \( w_i \). Since inclusion of more terms causes quite a small change in \( w_i \) the retention of these terms is hardly justified.

Consistent with the assumed panel deflection shape, the velocity potential is written as,

\[
\phi_i = \phi_i(0, y_i, z_i) \exp[i(\omega t - kx_i)]. \quad (\text{2.7})
\]

The symmetry and boundary conditions for \( \phi_i \) are

\[
\frac{\partial \phi_i}{\partial y_i} |_{y_i=0} = \frac{\partial \phi_i}{\partial y_i} |_{y_i=\infty} = 0, \quad (\text{2.8})
\]

and

\[
\frac{\partial \phi_i}{\partial z_i} |_{z_i=0} = \frac{\partial w_i}{\partial t} + U \frac{\partial w_i}{\partial x_i}, \quad (\text{2.9})
\]

\[
\frac{\partial \phi_i}{\partial z_i} |_{z_i=-d} = \frac{\partial w_i}{\partial t} + U \frac{\partial w_i}{\partial x_i}, \quad (\text{2.10})
\]

where \( U \) is the flow velocity which is assumed to be equal to the entrance velocity of the coolant at every channel and \( d \) is the channel thickness. For simplicity, \( \phi_i \) is considered as

\[
\phi_i = \phi_{ia} + \phi_{ib}. \quad (\text{2.11})
\]

Then, conditions (2.8), (2.9) and (2.10) become for \( \phi_{ia} \) and \( \phi_{ib} \), respectively,

\[
\frac{\partial \phi_{ia}}{\partial y_i} |_{y_i=0} = \frac{\partial \phi_{ia}}{\partial y_i} |_{y_i=\infty} = 0 \quad (\text{2.12})
\]

\[
\frac{\partial \phi_{ia}}{\partial z_i} |_{z_i=0} = \frac{\partial w_i}{\partial t} + U \frac{\partial w_i}{\partial x_i}, \quad (\text{2.13})
\]

\[
\frac{\partial \phi_{ia}}{\partial z_i} |_{z_i=-d} = 0; \quad (\text{2.14})
\]

Fig. 1. Cross section of the fuel plate assemblies
Critical coolant flow velocities

\[ \frac{\partial \phi_{ib}}{\partial y} |_{y=0} = \frac{\partial \phi_{ib}}{\partial y} |_{y=-\frac{b}{2}} = 0. \]  
(2.15)

\[ \frac{\partial \phi_{ib}}{\partial z} |_{z=0} = 0, \]  
(2.16)

\[ \frac{\partial \phi_{ib}}{\partial z} |_{z=-d} = \frac{\partial w_{i+1}}{\partial t} + U \frac{\partial w_{i+1}}{\partial x_{i+1}}. \]  
(2.17)

These conditions yield that

\[ \phi_{ia} = -\exp \left[ i(\omega t - kx_i) \right] \left[ \left( \frac{\cosh [k(d - z_i)]}{k \sinh k d} + \frac{\cosh [\nu_i(d - z_i)]}{\nu_i \sinh \nu_i d} \cos \left( \frac{2\pi y_i}{b} \right) \right) \right] \]  
(2.18)

\[ \phi_{ib} = \exp \left[ i(\omega t - kx_i) \right] \left[ \left( \frac{\cosh k z_i}{k \sinh k d} + \frac{\cosh \nu_i z_i}{\nu_i \sinh \nu_i c} \cos \left( \frac{2\pi y_i}{b} \right) \right) \right] \]  
(2.19)

where \( \nu_i^2 = k^2 + 4\pi^2/b^2 \), and noting \( x_0 = x_1 = \ldots = x_n \) and \( y_0 = y_1 = \ldots = y_n \).

The fluid pressures on and under the fuel plate \( p_{ii} \) and \( p_{2i} \) are determined from the well-known Bernoulli equation,

\[ p_{ii} = -\rho \left[ (\partial \phi_i/\partial t) + U (\partial \phi_i/\partial x_i) \right] |_{y=0} \]  
(2.20)

and

\[ p_{2i} = -\rho \left[ (\partial \phi_{i-1}/\partial t) + U (\partial \phi_{i-1}/\partial x_i) \right] |_{y=-d}. \]  
(2.21)

Substituting \( w_i \), \( p_{ii} \) and \( p_{2i} \) into eqn (2.1) and using eqns (2.6) and (2.18–2.21) and then applying the method of Galerkin to the resulting equation, Fung[3], i.e. multiplying the equation by \( 1 + \cos(2\pi y/b) \) and integrating over the panel width, the equation becomes

\[ A w^0_{i-1} + B w^0_i + A w^0_{i+1} = 0 \]  
(2.22)

where

\[ A = \mu (U^* - C^*)^2 \{1/(b/l) \sinh (2\pi \epsilon /b/l) \} \]  
\[ + 1/2 (1 + b^2/l^2)^{1/2} \sinh [2\pi \epsilon (1 + b^2/l^2)^2]/2\pi \]  
(2.23)

\[ B = [(3/2) (2b/l)^2 + 4 + 8(l/2b)^2]/4 - 3C^*/2 + (3 \gamma l/2b) C^* - D \]  
(2.24)

\[ D = \mu (U^* - C^*)^2 \{1/(b/l) \tanh (2\pi \epsilon b/l) + 1/2 (1 + b^2/l^2)^{1/2} \tanh [2\pi \epsilon (1 + b^2/l^2)]/\pi \} \]  
(2.25)

and \( l \) is the wavelength of the disturbance, \( k = 2\pi/l, \omega = 2\pi C/l, \mu = \rho b \rho m h, U^* = U/C, \) \( C^* = C/C_0, C \) is the wavespeed, \( C_0 = 2\pi (D/\rho_m h^2)^{1/2}, \epsilon = d/b, \gamma = 2\gamma \omega_0 \) and \( \omega_0 = \pi C_0/l \).

For an \( m \)-plate fuel assembly, assuming the bordering walls to be rigid, i.e. \( w^0 = w^0_{m+1} = 0 \), eqn (2.22) becomes a system of equations, giving the wave velocities \( C_i^* \)'s for the system,

\[ \Delta w^0 = 0 \]  
(2.26)

where \( \Delta \) and \( w \) are the \( m \) by \( m \) and \( m \) by 1 matrices,

\[ \Delta = \begin{bmatrix} B & A & 0 & \ldots & 0 & 0 \\ A & B & A & \ldots & 0 & 0 \\ 0 & A & B & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & \ldots & A & B \\ \end{bmatrix}, \quad w^0 = \begin{bmatrix} w^0_1 \\ \vdots \\ w^0_{m-1} \\ \vdots \\ w^0_m \end{bmatrix} \]  
(2.27)

and \( 0 \) is the \( m \) by 1 zero matrix.

In order to have a non-trivial solution

\[ \det (\Delta) = D_m = 0 \]  
(2.28)
where
\[ D_m = B \, D_{m-1} - A^2 \, D_{m-2}. \] (2.29)
also
\[ D_1 = B \quad \text{and} \quad B_2 = B^2 - A^2. \] (2.30)

3. THE CRITICAL COOLANT VELOCITY THROUGH THE DIVERGENCE CRITERION

Equation (2.28) gives a function
\[ F(n, \varepsilon, \mu, \eta, l/b, C^*, U^*) = 0 \] (3.1)
by which the coolant entrance velocity to the channels can be evaluated. It is a fact that \( C^* = 0 \) is the condition for the onset of divergence, Dowell[4]. Then, using this conditions for equation (3.1), the critical coolant velocity
\[ U^* = U^*(n, \varepsilon, \mu, l/b), \] (3.2)
which is the highest value for \( U^* \), that would not cause divergence for the reactor. The minimum value for \( U^*_c \) is obtained when \( n = \infty \) and \( l/b = \infty \). This result is obtained by evaluating the \( U^* \) values versus \( n, \varepsilon, \mu, \) and \( l/b \), where \( \varepsilon \) and \( \theta \) are the structural coefficients. By considering the superscripts as the number of plates in the assemblies, it is also seen that the value of
\[ U^*_c \approx \pi (e/2\mu)^{1/2} \] (3.3)
which is almost equal to \( U^*_c \) when \( e^* = 2\varepsilon^* \), \( n = 1 \) and \( l/b = \infty \), i.e.
\[ U^*_c = \pi (e/\mu)^{1/2}. \] (3.4)

This could also be concluded by considering the individual divergence of a simple plate surrounded by two coolant channels whose thicknesses are equal to half the thickness of channels of the infinite plate assembly, noting \( d^* = 2d^i \) and \( e = d/l \).

The values for coolant velocity given by Miller[1] for the plates with built-in edges are
\[ V^m = (45\pi/7\pi^2\mu)^{1/2} \quad \text{and} \quad V^i = (90\pi/7\pi^2\mu)^{1/2} \] (3.5)
where \( V^m \) and \( V^i \) denote the coolant velocity for an infinite plate assembly and a single plate, respectively. \( V^m \) and \( V^i \) are almost equal to the coolant velocities given by eqns (3.3) and (3.4), respectively. It should be noted that the values obtained in this analysis are four percent more than the values given in [1] through a different analysis. As a result of the analysis performed in this paper, it could be concluded that the Miller velocity for reactor design is quite a safe value. However, it should be used with a safety factor for design
\[ V_{\text{design}} = \alpha V_c \] (3.6)
where \( \alpha \) is chosen with respect to design policy so that \( 0 < \alpha \leq 1 \). It could also be noted that the experimental studies lend support Miller’s results.

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REFERENCES