Inflation with multiple sound speeds: A model of multiple DBI type actions and non-Gaussianities

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A B S T R A C T

In this Letter we study adiabatic and isocurvature perturbations in the frame of inflation with multiple sound speeds involved. We suggest this scenario can be realized by a number of generalized scalar fields with arbitrary kinetic forms. These scalars have their own sound speeds respectively, so the propagations of field fluctuations are individual. Specifically, we study a model constructed by two DBI type actions. We find that the critical length scale for the freezing of perturbations corresponds to the maximum sound horizon. Moreover, if the mass term of one field is much lighter than that of the other, the entropy perturbation could be quite large and so may give rise to a growth outside sound horizon. At cubic order, we find that the non-Gaussianity of local type is possibly large when entropy perturbations are able to convert into curvature perturbations. We also calculate the non-Gaussianity of equilateral type approximately.

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1. Introduction

Inflationary cosmology has become the prevalent paradigm to understand the early stage of our universe, with its advantages of resolving the flatness, homogeneity and monopole problems [1,2], and predicts a scale-invariant primordial power spectrum consistent with current cosmological observations [3] very well. However, a single field inflation model often suffers from fine tuning problems on the parameters of its potential, such as the mass and the coupling constant.

In recent years, people has noticed that, when a number of scalar fields are involved, they can relax many limits on the single scalar inflation model [4]. Usually, these fields are able to work cooperatively to give an enough long inflationary stage, even none of them can sustain inflation separately. Models of this type have been considered later in Refs. [5–8]. The main results show that both the e-folding number $\mathcal{N}$ and the curvature perturbation ζ are approximately proportional to the number of the scalars N. Later, the model of N-flation was proposed by Dimopoulos et al. [9], which showed that a number of axions predicted by string theory can give rise to a radiatively stable inflation. This model has explored the possibility for an attractive embedding of multi-field inflation in string theory.

Over the past several years, based on the recent developments in string theory, there have been many studies on its applications to the early universe in inflationary cosmology. However, people still often encounter fine tuning and inconsistency problems when they try to combine string theory with cosmology as reviewed in Ref. [10]. Facing to these embarrassments, it is usually suggested that N-flation is able to relax these troubles and so can let stringy cosmology survive. A good example is that Piao et al. have successfully applied assisted inflation mechanism to amend the problems of tachyon inflation [11]. There are also many works on investigating multi-field inflation models in stringy cosmology, for example see Refs. [12–17].

Recently, an interesting inflation model, which has a non-canonical kinetic term inspired by string theory, was studied intensively in the literature. Due to a non-canonical kinetic term, the propagation of field fluctuations in this model is characterized by a sound speed parameter and the perturbations get frozen not on Hubble radius, but the sound horizon instead. One specific realization of this type of models can be described by a Dirac–Born–Infeld-like (DBI) action [18,19]. Based on brane inflation [20], the model with a single DBI field was investigated in detail [21–23], which has explored a window of inflation models without flat potentials. In this model, a warping factor was applied to provide a speed limit which keeps the inflaton near the top of a potential even if the potential is steep.

In this Letter, we study an inflation model involving multiple sound speeds with each sound speed characterizing one field fluctuation. We suggest this scenario can be realized by a number of

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general scalar fields with arbitrary kinetic forms, and these scalars have their own sound speeds, respectively [24]. Therefore, we call this model as *Multi-Speed Inflation*. In this model, the propagations of field fluctuations are individual, and the usual conceptions in multi-field inflation models might be not suitable in this scenario. For example, in a usual generalized N-flation model, the length scale for perturbations being freezed takes the unique sound horizon; however, in our model it corresponds to the maximum sound horizon.

Specifically, we consider a double-field inflation model, with each field being described by a DBI action and the total action is constructed by the sum of they two. It is worth emphasizing that our model is different from the usual DBI N-flation in which only multiple moduli fields are involved in one DBI action [25–31], but ours is constructed by multiple DBI type actions (DBIs), as proposed in Ref. [24]. The model we considered in the paper can be achieved as follows. We consider two D3-branes in a background metric field with negligible covariant derivatives of field strengths and we assume that these branes are decoupled from others. Besides, we also need to neglect the backreaction of those branes on the background geometry as is usually done in brane inflation models. Specifically we are interested in two phenomenological scenarios. The first one is these two scalars work cooperatively like those in usual N-flation models, which gives the predictions on primordial perturbations the similar as those obtained in single field DBI inflation. The second one is the scenario of cascade inflation with one scalar providing first several efolding numbers and then the other finishing the rest. For the second scenario, if the mass term of one field is much lighter than that of the other, the entropy perturbation could be quite large and so may give rise to a growth outside sound horizon. At cubic order, we find that the non-Gaussianity of local type is possibly large if entropy perturbations can be converted into curvature perturbations. We also calculate the non-Gaussianity of equilateral type approximately.

The Letter is organized as follows. In Section 2, we propose a model of *Multi-Speed Inflation*, then study its generic background dynamics. In Section 3, we study its linear perturbations using Arnowitt–Deser–Misner (ADM) formalism, and show that different field fluctuations are governed by different sound speeds respectively. In this case we provide a new decomposition on adiabatic and isocurvature perturbations. In Section 4, we analyze a specific inflation model constructed by two DBI type actions, first investigate its background evolution, and then give its curvature perturbation and entropy perturbation, and finally its non-Gaussianity is addressed.

In the Letter we take the normalization $M_p^2 = 1/8\pi G = 1$ and the sign of metric is adopted as $(-, +, +, +)$ in the following.

2. The model

Our starting point is the action as follows,

$$S = \int d^4 x \sqrt{-g} \left[\frac{1}{2} R + \sum_I P_I(X_I, \phi_I) \right], \tag{1}$$

with

$$X_I \equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi_I \nabla_\nu \phi_I, \tag{2}$$

defined as the kinetic term of the Ith scalar field ϕ_I. This model involves multiple k-essence-type fields. For simplicity, we assume that there are no couplings between scalar fields, so each field evolves independently except for gravity coupling.[1]

An inflation model constructed with a single k-essence was originally proposed by [34] and later its perturbation theory has been studied [35]. In the literature this type of model has been widely studied, and one of the most significant features is that there is an effective sound speed describing the propagation of the perturbations [36–39]. However, one may already notice that in our model, for each a k-essence field there is one sound speed correspondingly. Therefore, the field fluctuations in our model do not propagate synchronously. In the current Letter, our main interests focus on the effects of multiple sound speeds in perturbation theory. However, before studying the perturbations, we first take an investigation on the background equations.

By varying the action with respect to the metric, we can obtain the energy–momentum tensor of the form

$$T^{\mu\nu} = \sum_I \left(P_I g^{\mu\nu} + P_{I,X_I} \nabla^\mu \phi_I \nabla^\nu \phi_I \right), \tag{3}$$

where P_{I,X_I} denotes the partial derivative of P_I with respect to X_I. Moreover, the scalar fields satisfy generalized Klein–Gordon equations, which are given by

$$\nabla_\mu \left(P_{I,X_I} \nabla^\mu \phi_I \right) + P_{I,I} = 0, \tag{4}$$

where $P_{I,I}$ is the partial derivative of P_I with respect to the scalar ϕ_I.

Considering a spatially flat Friedmann–Robertson–Walker (FRW) spacetime with its metric

$$ds^2 = -dt^2 + a^2(t) \, d\vec{x}^2, \tag{5}$$

we can read the energy density and pressure of a field ϕ_I from the energy–momentum stress

$$\rho_I = 2 X_I P_{I,X_I} - P_I, \qquad p_I = P_I. \tag{6}$$

The equations of motion for the scalar fields reduce to

$$\ddot{\phi}_I + \left(3H + \frac{\dot{P}_{I,X_I}}{P_{I,X_I}} \right) \dot{\phi}_I - \frac{P_{I,I}}{P_{I,X_I}} = 0, \tag{7}$$

where we define the Hubble parameter $H \equiv \dot{a}/a$. Moreover, we introduce the sound speed parameters,

$$c_{sI}^2 \equiv \frac{p_{I,X_I}}{\rho_{I,X_I}} = \frac{P_{I,X_I}}{P_{I,X_I} + 2 X_I P_{I,X_I X_I}}, \tag{8}$$

and also use the dimensionless parameters

$$s_I \equiv \frac{\dot{c}_{sI}}{H c_{sI}}, \tag{9}$$

for each of which measures the variation of the sound speed c_{sI} in one Hubble time.

3. Linear perturbations

Now we start to study the linear perturbations of the model introduced in the previous section. Since we are working in the frame of a cosmological system, the metric perturbations ought to be included as well as the field fluctuations. However, one can eliminate one degree of freedom by taking a suitable gauge. We would like to expand the action to the second order with the ADM formalism [40]. In this formalism, we can eliminate one extra degree of freedom of perturbations at the beginning of the calculation, by choosing the spatially flat gauge with the spatial curvature vanishing as $^{(3)}R = 0$.

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[1] We refer Refs. [32,33] for a discussion on initial condition of inflation.

3.1. ADM formalism and the action at quadratic order

To start, the spacetime metric in the ADM formalism is written as

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{10}$$

with N being the lapse function and N_i the shift vector. Substituting this metric into the original expression of the action, we get

$$S = \int dt\, dx^3 \sqrt{h} \left[N \sum_I P_I + \frac{1}{2N}\left(E_{ij}E^{ij} - E^2\right) \right], \tag{11}$$

where $h = \det(h_{ij})$, and the tensor E_{ij} is defined as

$$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i), \tag{12}$$

which is related to the extrinsic curvature of the spatial slice with $K_{ij} = N^{-1}E_{ij}$. We use the lowercase index i to denote the spatial coordinates.

To vary the action with N, we obtain the Hamiltonian constraint,

$$2\sum_I P_I - \frac{1}{N^2}\left(E_{ij}E^{ij} - E^2 + 2\sum_I P_{I,X_I} v_I v_I\right) = 0, \tag{13}$$

where there is

$$v_I = \dot{\phi}_I - N^i \partial_i \phi_I; \tag{14}$$

while the variation of the action with respect to N_i yields the momentum constraint as follows,

$$\nabla_j\left[N^{-1}\left(E_i^j - E\delta_i^j\right)\right] = N^{-1}\sum_I P_{I,X_I} v_I \partial_i \phi_I. \tag{15}$$

We have already restricted ourselves to the spatially flat gauge of the FRW background with the spatial metric as $h_{ij} = a^2(t)\delta_{ij}$. Therefore, the degrees of freedom merely comes from the field fluctuations as the following decomposition,

$$\phi_I(t,\vec{x}) \rightarrow \phi_I(t) + \delta\phi_I(t,\vec{x}). \tag{16}$$

Meanwhile, we need to expand the lapse function and shift vector in form of,

$$N = 1 + \alpha, \qquad N_i = V_i + \partial_i \beta, \tag{17}$$

where the scalar functions α and β can be expressed in terms of the field fluctuations $\delta\phi_I$, and V_i belongs to the vector modes and so can be eliminated to second order.

By solving the linearized constraint equations (13) and (15), we have

$$\alpha = \frac{1}{2H}\sum_I P_{I,X_I} v_I \delta\phi_I, \tag{18}$$

and

$$\partial^2 \beta = \frac{a^2}{2H}\sum_I \left[-\frac{P_{I,X_I}}{c_{sI}^2} v_I \delta v_I + (P_{I,I} - 2X_I P_{I,IX_I})\delta\phi_I \right.$$
$$\left. + \frac{P_{I,X_I}}{H}\left(\frac{X_I P_{I,X_I}}{c_{sI}^2} - 3H^2\right)v_I \delta\phi_I \right], \tag{19}$$

and here $\delta v_I = \delta\dot{\phi}_I$ to linear order.

Making use of the above results, now we can expand the action to quadratic order. To do some integrations by parts and regroup the terms, the second order action takes the form

$$S_2 = \int dt\, dx^3 \frac{a^3}{2}\sum_I \left[\frac{P_{I,X_I}}{c_{sI}^2}\delta\dot{\phi}_I^2 - \frac{P_{I,X_I}}{a^2}\partial_i\delta\phi_I \partial_i\delta\phi_I \right.$$
$$\left. + 2P_{I,IX_I}\dot{\phi}_I \delta\phi_I \delta\dot{\phi}_I - \sum_J M_{IJ}\delta\phi_I \delta\phi_J \right], \tag{20}$$

where the effective mass matrix of the field fluctuations is given by

$$M_{IJ} = \frac{1}{2H}\left(\dot{\phi}_I^2 P_{I,IX_I}\dot{\phi}_J P_{J,X_J} + \dot{\phi}_I P_{I,X_I}\dot{\phi}_J^2 P_{J,JX_J}\right)$$
$$+ \frac{1}{4H^2}\sum_K \left(1 - \frac{1}{c_{sK}^2}\right)\dot{\phi}_K^2 P_{K,KX_K} P_{I,X_I} P_{J,X_J}$$
$$- \frac{1}{a^3}\frac{d}{dt}\left[\frac{a^3}{4H}\left(2 + \frac{1}{c_{sI}^2} + \frac{1}{c_{sJ}^2}\right)\dot{\phi}_I P_{I,X_I}\dot{\phi}_J P_{J,X_J}\right]$$
$$- P_{I,II}\delta_{IJ}, \tag{21}$$

which is strongly suppressed by slow-roll parameters in the frame of usual inflationary cosmology.

One may notice, if there is only one field, the above results are consistent with a model of single k-essence field as analyzed in Ref. [35]. However, in our case there are multiple sound speeds which govern the propagations of the field fluctuations. For each sound speed, there is a critical length scale which takes the form c_s/H. We would like to call this scale as *sound horizon*.

3.2. Curvature perturbations and isocurvature perturbations

Having obtained the second order action, we can proceed to study the field fluctuations by solving their perturbation equations in concrete models. However, before doing that, let us take a closer look at the kinetic terms of the field fluctuations. From Eq. (20), it is not straightforward how to define the modes of curvature and isocurvature perturbations, since the model involves more than one sound speeds which make the usual decomposition of an orthonormal basis in field spaces invalid. So we need to develop a new decomposition which should includes the information of the sound speeds. To do it, we need to go back to the basic definition of curvature perturbation in perturbed Einstein's equations.

A widely used quantity characterizing the gauge invariant curvature perturbation is given by

$$\mathcal{R} \equiv \Phi - H\frac{\delta q}{\rho + p}$$
$$= \Phi_k + H\frac{\delta\sigma_k}{\dot{\sigma}}. \tag{22}$$

Here Φ is the gravitational potential, δq represents the perturbation of momentum, and σ is the so-called adiabatic field.

One can find that this field takes the form $\dot{\sigma} = \sqrt{\sum_I \dot{\phi}_I^2}$ in usual cases. However, recall that, in our model the $(0i)$ components of the perturbed energy–momentum stress give the momentum perturbation as follows,

$$\delta q = -\sum_I P_{I,X_I}\dot{\phi}_I \delta\phi_I, \tag{23}$$

while, the background energy density and the pressure yield

$$\rho + p = \sum_I P_{I,X_I}\dot{\phi}_I^2. \tag{24}$$

Thus the adiabatic field in our model is given by

$$\dot{\sigma} = \sqrt{\sum_I P_{I,X_I}\dot{\phi}_I^2}, \tag{25}$$

and its perturbation can be expressed as

$$\delta\sigma = \sum_I P_{I,X_I} \frac{\dot{\phi}_I}{\dot{\sigma}} \delta\phi_I, \tag{26}$$

which characterize the adiabatic fluctuations.

Moreover, we usually define another useful gauge-invariant variable, curvature perturbation on uniform-density hypersurface [41], with its expression as follows,

$$\zeta \equiv -\Phi - H \frac{\delta\rho}{\dot{\rho}}. \tag{27}$$

On large scales, we have $\mathcal{R} \simeq -\zeta$ in a spatially flat universe. Thus, both two quantities can be used to describe adiabatic fluctuations. If the matter content of a cosmological system is made of multiple components, we can define the curvature perturbation associated with each individual energy density components, which are given by

$$\zeta_I = -\Phi - H \frac{\delta\rho_I}{\dot{\rho}_I}. \tag{28}$$

Since in a system with multiple matter components, there are non-vanishing entropic pressure perturbations even with every component being adiabatic. So we can describe the entropy perturbations by using the following expressions,

$$\mathcal{S}_{IJ} = 3(\zeta_I - \zeta_J), \tag{29}$$

which are so called relative entropy perturbations.

Furthermore, we can define the adiabatic unit vector e_σ^I as follows,

$$e_\sigma^I = \sqrt{P_{I,X_I}} \frac{\dot{\phi}_I}{\dot{\sigma}}. \tag{30}$$

To take a further step, we have

$$\delta\sigma = \sqrt{P_{I,X_I}} \delta\phi_I e_\sigma^I, \tag{31}$$

and thus we can see that this decomposition is no longer on an orthonormal basis.

In order to make the analysis more explicitly, we consider an example of two k-essence fields. In this case the adiabatic perturbation and entropy perturbation can be given by

$$\delta\sigma = \cos\theta \sqrt{P_{1,X_1}} \delta\phi_1 + \sin\theta \sqrt{P_{2,X_2}} \delta\phi_2, \tag{32}$$

$$\delta s = -\sin\theta \sqrt{P_{1,X_1}} \delta\phi_1 + \cos\theta \sqrt{P_{2,X_2}} \delta\phi_2, \tag{33}$$

and the angle is defined by

$$\tan\theta = \frac{\sqrt{P_{2,X_2}} \dot{\phi}_2}{\sqrt{P_{1,X_1}} \dot{\phi}_1}. \tag{34}$$

Using the above decomposition, we now obtain the formal expressions of dimensionless curvature and isocurvature perturbation variables,

$$\mathcal{R} \simeq H \frac{\delta\sigma}{\dot{\sigma}}, \qquad \mathcal{S} = H \frac{\delta s}{\dot{\sigma}}, \tag{35}$$

on the spatially flat slices. Note that, as $\mathcal{S}$ is not directly observable during inflation, what we are interested in is its spectral index but not the amplitude. Therefore, its normalization is quite arbitrary. We take such a particular choice in Eq. (35) since it has been widely used in usual double-field inflation as shown in Refs. [42–45].

4. A model of two DBI type actions

In this section, we study a specific inflation model involving multiple sound speeds. The model was originally proposed in Ref. [24], where the model is constructed by multiple DBI type actions and its general feature has been studied under certain approximations. Now we focus on a concrete model which only involves two DBI type actions, with

$$P_I(X_I, \phi_I) = \frac{1}{f(\phi_I)} \left[1 - \sqrt{1 - 2f(\phi_I)X_I} \right] - V_I(\phi_I), \tag{36}$$

with $I = 1, 2$. This model might be viewed as an effective description of D-brane dynamics (for example, see Refs. [46]). Considering a system constructed by two D3-branes in a background metric field with negligible covariant derivatives of the field strengths and assuming that these two branes are falling into their own throats, this system can be described by the above action which has a stringy origin as shown in Ref. [47].

In this model, the scalar ϕ_I describes the position of the brane. If we consider the branes are falling into the AdS-like throats and neglect the backreaction of the branes upon the background geometry, the warping factor usually takes the form

$$f(\phi_I) = \frac{\lambda_I}{\phi_I^4}. \tag{37}$$

This assumption can be satisfied when the contribution of the background flux is much larger than that from the branes.

4.1. Background equations

After having introduced the model, now we can study its background dynamics in the frame of FRW metric. For the scalars, the sound speeds are given by

$$c_{sI} = \sqrt{1 - 2f(\phi_I)X_I}, \tag{38}$$

and there is $P_{I,X_I} = 1/c_{sI}$. The above equation yields

$$|\dot{\phi}_I| = \phi_I^2 \left(\frac{1 - c_{sI}^2}{\lambda_I} \right)^{\frac{1}{2}}. \tag{39}$$

Since in the current Letter we focus our interests on the relativistic limit with small sound speeds, then $|\dot{\phi}_I| \simeq \phi_I^2/\sqrt{\lambda_I}$.

Specifically, we consider the case of IR type potential with the form of

$$V_I = V_{0I} - \frac{1}{2} m_I^2 \phi_I^2. \tag{40}$$

The first part of the potential V_{0I} origins from the anti-brane tension from other throats. In IR DBI inflation [23], D-branes roll towards the tip of the throats, thus the potential contains terms like tachyon. Moreover, due to the warping factor $f(\phi_I)$, those scalars are able to stay near the top of their potentials, and so we have $H^2 \simeq \frac{1}{3} \sum_I V_{0I}$. In the following, we would like to investigate the background in details.

In order to obtain a semi-analytic solution, we would like to take a useful assumption with $s_I \equiv \frac{\dot{c}_{sI}}{H c_{sI}}$ being small numbers, and consider the relativistic limit of the branes.

A similar case of a single field has been studied in [23]. However, in our model the total Lagrangian is constructed by two DBI fields, where each one field contribute one Lagrangian and has its own sound speed, respectively. Therefore, we actually have two sound speeds. Recall the generalized Klein–Gordon equations (7), they can be reexpressed as follows,

$$\frac{d}{dt}\left(\frac{\dot{\phi}_I}{c_{sI}}\right) + 3H\frac{\dot{\phi}_I}{c_{sI}} + \frac{f_{,I}}{f^2}(1 - c_{sI}) - \frac{f_{,I}\dot{\phi}_I^2}{2fc_{sI}} + V_{,I} = 0. \tag{41}$$

Under the relativistic limit of the scalars we can have an ansatz, which takes the following form,

$$\phi_I = -\frac{\sqrt{\lambda_I}}{t}\left(1 - \frac{\alpha_I}{(-t)^{p_I}} + \cdots\right), \tag{42}$$

where we set $t \to -\infty$ at the beginning of inflation.[2] Therefore, to insert the ansatz into the above equation, then we find the leading terms in Eq. (41) come from the second term

$$\frac{3H\sqrt{\lambda_I}}{\sqrt{2\alpha_I(p_I - 1)}(-t)^{2-\frac{p_I}{2}}}, \tag{43}$$

and the potential term which is equal to

$$\frac{\sqrt{\lambda_I}m_I^2}{t}. \tag{44}$$

The others are suppressed by $\frac{1}{Ht}$ which is negligible in inflation (where $|Ht| \gg 1$ or equivalently $\phi_I \ll \sqrt{\lambda_I}H$), and this requirement is consistent with the assumption that the scalars lie on the top of potential during inflation. Finally, by matching the leading terms, we get $p_I = 2$ and $\alpha_I = \frac{9H^2}{2m_I^4}$, and so the solutions of the scalars are given by

$$\phi_I \simeq -\frac{\sqrt{\lambda_I}}{t}\left(1 - \frac{9H^2}{2m_I^4 t^2} + \cdots\right). \tag{45}$$

Making use of the solutions, we have the sound speeds

$$c_{sI} \simeq -\frac{3H}{m_I^2 t}, \tag{46}$$

to the leading order, and so can check that the approximations with $s_I \ll 1$ are consistent with the equations of motion when $t \to -\infty$.

4.2. Quantum fluctuations and power spectrum

Now let us study the dynamics of quantum fluctuations in this model. To do so, we go back to the second order action (20) directly and define the new variables which are canonically normalized with conformal time. These variables are defined as

$$v_I = a\frac{\sqrt{P_{I,X_I}}}{c_{sI}}\delta\phi_I, \tag{47}$$

for the two scalars. Thus the dominant terms of second order action can be given as follows,

$$S_2 \supseteq \int d\tau\, dx^3 \sum_{I=1}^{2}\left[v_I'^2 - c_{sI}^2\partial_i v_I \partial_i v_I + \frac{z_I''}{z_I}v_I^2\right], \tag{48}$$

under an assumption of weakly coupling between two fields. In the above action, we have introduced some background-dependent functions

$$z_I = a\frac{\dot{\phi}_I\sqrt{P_{I,X_I}}}{c_{sI}H}. \tag{49}$$

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[2] In stringy configuration, the flux–antibrane annihilation in the multiple throats naturally provides an attractive point for small field inflation with the branes generating at the tips of the throats through tunneling from an eternal inflation [22,48].

In the inflationary background, the equations of motion describing these two canonical perturbation variables in Fourier space are given by

$$v_{Ik}'' + \left(c_{sI}^2 k^2 - \frac{z_I''}{z_I}\right)v_{Ik} = 0, \qquad \frac{z_I''}{z_I} \simeq \frac{2}{\tau^2}. \tag{50}$$

One can see that, for each field fluctuation, there is a corresponding sound horizon respectively. This scenario is quite different from the case considered in Ref. [29] where there is only sound speed characterize the propagation of the adiabatic mode.

To complete the quantization of the field fluctuations, we can decompose the variables as

$$v_I(\tau, \vec{x}) = \int \frac{dk^3}{(2\pi)^{\frac{3}{2}}}\left[a_{I\vec{k}}v_{Ik} + a_{I-\vec{k}}^\dagger v_{I-k}^*\right]e^{i\vec{k}\vec{x}}, \tag{51}$$

where the operators a_I and $a_I^\dagger$ are annihilation and creation operators, which satisfy the following commutation relation

$$[a_{I\vec{k}}, a_{I\vec{k}'}^\dagger] = \delta(\vec{k} - \vec{k}'). \tag{52}$$

Moreover, the normalization conditions require

$$v_{Ik}v_{Ik}^{*\prime} - v_{Ik}^* v_{Ik}' = i. \tag{53}$$

Eventually, we choose that all the modes of perturbations behave as in the adiabatic Minkowski vacuum initially, and thus obtain the solutions

$$v_{Ik} = \frac{e^{-ic_{sI}k\tau}}{\sqrt{2c_{sI}k}}\left(1 - \frac{i}{c_{sI}k\tau}\right). \tag{54}$$

These solutions imply that the power spectrum of the field fluctuations are of the value,

$$\delta\phi_I = \sqrt{P_{\delta\phi_I}} \simeq \frac{H_{*I}}{2\pi}, \tag{55}$$

after they escape out of their own sound horizons [49]. The subscript '$*I$' denotes the sound horizon crossing time for the field perturbation $\delta\phi_I$.

Since in inflation the Hubble parameter is almost unchanged, and the amplitudes of field perturbations are also nearly conserved due to the feature of nearly scale-invariance, we can conclude that the amplitudes of all field fluctuations take almost the same value outside of their sound horizons. Due to this quantity, we can take the maximum of those sound horizons as the final freezing scale for all the field fluctuations, which corresponds to the perturbation mode with the largest sound speed. For example, if we take $m_1 < m_2$ in our model which gives $c_{s1} > c_{s2}$ according to Eq. (46), then the critical sound horizon takes $\frac{c_{s1}}{H}$.

Based on the above analysis and making use of Eqs. (35), (55) and the background solutions, one obtains the curvature perturbation at the *maximum sound horizon* crossing time

$$\mathcal{R} \simeq \frac{\mathcal{N}^2}{2\pi\sqrt{\lambda_I}}\left(1 + \frac{27H^4}{2m_1^2 m_2^2 \mathcal{N}^2}\right), \tag{56}$$

where we have considered the next-to-leading correction and introduced the efolding number $\mathcal{N} \equiv \int H\, dt$. The leading term of entropy perturbation can be resolved as well,

$$\mathcal{S} \simeq \frac{27H^4(m_1^2 - m_2^2)}{4\pi\sqrt{\lambda_I}m_1^3 m_2^3} + O(\mathcal{N}^{-2}), \tag{57}$$

which is proportional to a parameter, *relative sound speed* Δc_s, at the crossing time, with its definition in form of

$$\Delta c_s = c_{s1} - c_{s2} \simeq \frac{3H(m_1^2 - m_2^2)}{m_1^2 m_2^2 t}. \tag{58}$$

Concerning the result (57), what we have obtained is the dominant term which comes from the cancellation between the first and second term in Eq. (33) in virtue of the values of field fluctuations. This result is reliable if we only consider the dominant term and have neglected the correlations between two fields inside Hubble radius, which has been intensively discussed in the literature. However, a more careful derivative shows that the subdominant term, which comes from the correlations of two fields, would become important at specific cases, for example when $m_1 = m_2$ in our model, and so would lead to an average of the square of the isocurvature perturbation not vanishing even for $m_1 = m_2$. As shown in Eq. (57), this correlation term is proportional to $\mathcal{N}^{-2}$ where $\mathcal{N}$ is the efolding number. Moreover, we would like to emphasize that, since entropy perturbation is not directly detectable, we do not care about its amplitude too much but its spectral index instead.

Besides, we can see that, if there is only one single field, the entropy perturbation vanishes. Even for the case of double fields, the entropy perturbation is still not large in usual DBI inflation, since its amplitude is suppressed by the relative sound speed for which we have expanded the detailed formalism in our concrete model. However, one may notice that a relatively small mass term may uplift the entropy perturbations. For example, if we take $m_1 \ll m_2$, the amplitude of entropy perturbation takes an approximate form as $\mathcal{S} \sim \frac{H^4}{m_1^3 m_2}$ which can be very large due to an enough light mass m_1. This feature might be very important and could be applied in a curvaton model with DBI type actions which will be discussed in near future [50].

Moreover, the leading term of the curvature perturbation is consistent with the result obtained in Ref. [23], and the second order contribution is suppressed by a square of the efolding number and so we can neglect it in usual case. One should keep in mind that, if we tune one of the masses to be small enough, the second order term could also dominate over. Furthermore, we give the spectral tilts as follows,

$$n_{\mathcal{R}} - 1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k} \simeq -\frac{4}{\mathcal{N}}, \tag{59}$$

$$n_{\mathcal{S}} - 1 \equiv \frac{d \ln P_{\mathcal{S}}}{d \ln k} \simeq -8\epsilon, \tag{60}$$

in which we have the following relation

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{\lambda_I (m_1^2 + m_2^2)}{3\mathcal{N}^3 M_p^2}, \tag{61}$$

in our model. The dependence of the spectral indices on the e-folding number $\mathcal{N}$ for different background values are plotted in Fig. 1.

From Fig. 1 we can read that, both the spectral index of curvature perturbation and that of entropy perturbation are roughly scale-invariant, but with their tilts a little red. Moreover, the deviation of the entropy perturbation from a scale-invariant spectrum is smaller than that of the curvature perturbation at the regime of large efolding numbers, while larger at small efolding numbers. According to the recent cosmological observation, for instance WMAP five year data [3], we have a constraint on the amplitude of the curvature perturbation $\mathcal{R} \lesssim 4.8 \times 10^{-5}$. So if we choose the background parameters $\lambda_I = 10^{14}$, the best fit value for efolding number is $\mathcal{N} \simeq 55$ which satisfies the current cosmological observations very well.
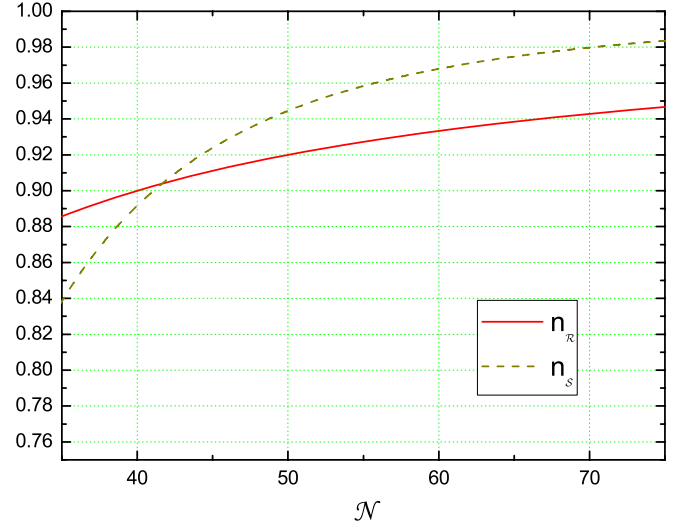

Fig. 1. The plot of spectral indices as functions of the e-folding number $\mathcal{N}$ for curvature and entropy perturbations. The red solid line denotes the spectral index of curvature perturbation; the yellow dashed line denotes the spectral index of entropy perturbation. The background parameters are taken as: $\lambda_1 = \lambda_2 = 10^{14}$, $m_1 = 10^{-6}$, $m_2 = 5 \times 10^{-6}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

5. Non-Gaussianities

In the above section we have calculated the primordial fluctuations in linear order. If our model indeed makes sense to the physics of the early universe, it would be necessary to extend the theoretical framework beyond the leading order. In particular, the information of non-Gaussianity provides a potentially powerful discriminant between numerous models describing early universe and have attracted considerable interests. Non-Gaussianities in usual single field inflation models were considered in [51–55] and in more detail in [56] and it was found that non-Gaussianities would be small. Later, large non-Gaussianity of specific shape [57] which is of equilateral type can be obtained in single DBI inflation [58], but its value of local type is still very small [59]. The non-Gaussianity of local type can be sizable in bounce cosmology as studied by Ref. [60]. We refer Ref. [61] for a comprehensive review on this issue and Refs. [62] for some recent developments.

In this section we use $\delta\mathcal{N}$ formalism [63–65] to study the non-Gaussianities in details. To start, we define the power spectrum $P_{\mathcal{R}}$ and bispectrum $B_{\mathcal{R}}$ as follows,

$$\langle \mathcal{R}_{k_1} \mathcal{R}_{k_2} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) P_{\mathcal{R}}(k_1), \tag{62}$$

$$\langle \mathcal{R}_{k_1} \mathcal{R}_{k_2} \mathcal{R}_{k_3} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3), \tag{63}$$

and then these two spectra can be related in terms of the nonlinearity parameter f_{NL},

$$B_{\mathcal{R}}(k_1, k_2, k_3) = \frac{3}{10}(2\pi)^4 \frac{\sum k_i^3}{\prod k_i^3} P_{\mathcal{R}}^2 f_{\text{NL}}(k_1, k_2, k_3), \tag{64}$$

in momentum space.

The concept of $\delta\mathcal{N}$ formalism identifies the curvature perturbation with the perturbation of local expansion $\mathcal{R} = \delta\mathcal{N}$, and so the curvature perturbation can be expanded as follows,

$$\mathcal{R} = \sum_I \mathcal{N}_{,I} \delta\phi_I + \frac{1}{2} \sum_{IJ} \mathcal{N}_{,IJ} \delta\phi_I \delta\phi_J + \cdots. \tag{65}$$

If we have calculated the three point correlators of field fluctuations, the non-linearity parameter can obtained by making use of above equations.

5.1. Equilateral type

Specifically, now we make a rough study on the three point correlator and non-Gaussianity of equilateral type. A simple way to investigate the three point correlator is to perturb the second order Lagrangian as shown in Eq. (20). With an assumption of weak coupling between the fields, we perturb the sound speeds in the quadratic Lagrangian and then obtain the Lagrangian with the leading order terms up to cubic parts,

$$\mathcal{L}_3 \supseteq \sum_I \frac{a^3}{2c_{sI}^5 \dot{\phi}_I} \left[\delta\dot{\phi}_I^3 - \frac{c_{sI}^2}{a^2} \delta\dot{\phi}_I (\nabla\delta\phi_I)^2 \right]. \tag{66}$$

Correspondingly, the dominant terms in the interaction Hamiltonian in Fourier space are given by

$$H_{\rm int} \supseteq \int dk^3 \left[-\sum_I \frac{a^3}{2c_{sI}^5 \dot{\phi}_I} \left(\delta\dot{\phi}_I^3 + \frac{c_{sI}^2}{a^2} k^2 \delta\dot{\phi}_I \delta\phi_I^2 \right) \right]. \tag{67}$$

Then we decompose the field fluctuations in canonical quantization process with creation and annihilation operators defined in Eq. (52),

$$\delta\phi_{Ik}(t) = u_I(\vec{k})a_{Ik} + u_I^*(-\vec{k})a_{I-k}^{\dagger},$$
$$u_I(\vec{k}) = \frac{H}{\sqrt{2k^3}}(1 + ic_{sI}k\tau)e^{-ic_{sI}k\tau}. \tag{68}$$

Since we have obtained the interaction Hamiltonian and the modes of the field fluctuations, now we are able to calculate the three point correlator, which takes,[3]

$$\langle \delta\phi_{Ik_1} \delta\phi_{Ik_2} \delta\phi_{Ik_3} \rangle = -i \int dt \, \langle [\delta\phi_{Ik_1} \delta\phi_{Ik_2} \delta\phi_{Ik_3}, H_{\rm int}] \rangle. \tag{69}$$

From Eqs. (67) and (68), we can see that $H_{\rm int} \sim \sum_I 1/c_{sI}^2$. Note that this is consistent with the result in usual single DBI inflation which is proportional to $1/c_s^2$ [59].

Furthermore, from Eq. (72), we can read the partial derivative of the efolding number with respect to each field. Substituting this equation into Eq. (65) and using the expression (64), we finally obtain an approximate form of $f_{\rm NL}$ of equilateral type as follows,

$$f_{\rm NL}^{\rm equil} \sim \frac{(c_{s1} + c_{s2})(c_{s1}^5 + c_{s2}^5)}{c_{s1}^2 c_{s2}^2 (c_{s1}^2 + c_{s2}^2)^2}. \tag{70}$$

This result is also consistent with the case of single DBI inflation [59]. Moreover, if we take c_{s1} larger than c_{s2}, then the non-linearity parameter takes $f_{\rm NL} \sim 1/c_{s2}^2$. Therefore, we can conclude the non-Gaussianity of equilateral type is sensitive to the smallest sound speed. (To make a comparison, the linear curvature perturbation strongly depend on the largest sound speed, which corresponds to the maximum sound horizon.)

5.2. Local type

To take a further step, we consider non-Gaussianity of local type in the model we have studied in previous section.

As analyzed previously, for all field fluctuations, their amplitudes take almost the same value $\delta\phi_I \simeq \frac{H}{2\pi}$ after they exit their own sound horizons. Thus the non-linearity parameter of local type is given by

$$f_{\rm NL} \simeq \frac{5}{6} \frac{\sum_{JK} \mathcal{N}_{,J}\mathcal{N}_{,K}\mathcal{N}_{,JK}}{(\sum_I \mathcal{N}_{,I}\mathcal{N}_{,I})^2}. \tag{71}$$

Moreover, from the formula of curvature perturbation (35), we can read the following relations,

$$\mathcal{N}_{,1} \simeq \frac{\sqrt{\lambda_I}H\phi_1^2}{\phi_1^4 + \frac{c_{s1}}{c_{s2}}\phi_2^4}, \qquad \mathcal{N}_{,2} \simeq \frac{\sqrt{\lambda_I}H\phi_2^2}{\phi_2^4 + \frac{c_{s2}}{c_{s1}}\phi_1^4}. \tag{72}$$

Moreover, we define a parameter

$$q \equiv \frac{\phi_2}{\phi_1}, \tag{73}$$

which represents the ratio between two scalars. Using this parameter we can simply describe two background evolutions. One is $q \sim 1$, which denotes that both two fields are rolling down along their potentials synchronously as considered in previous sections; the other is $q \gg 1$, which describes that the heavy field ϕ_2 enters its warping throat and provides first few efolds for inflation and then the light field ϕ_1 starts the second episode of inflation. The latter case provides a specific realization of cascade inflation [70–75].

Substituting the above relations into Eq. (71) and using the definition (73), we can obtain an approximate form of non-Gaussianity of local type directly,

$$f_{\rm NL}^{\rm local} \sim \frac{q}{\mathcal{N}_1}, \tag{74}$$

where $\mathcal{N}_1$ denotes the efolds contributed by the field ϕ_1. From this result, one can see that the non-Gaussianity of local type is suppressed by the efolding number. For example, we consider two fields are rolling down in the same rate and thus there is $\mathcal{N} \simeq 60$, we obtain $f_{\rm NL}^{\rm local} \sim O(10^{-2})$. In this case the deviation of curvature perturbation from Gaussian distribution is very small, and also consistent with the result in single DBI inflation. However, if we let the light field only provides the last several efolds during which there is $q \gg 1$, it gives $f_{\rm NL}^{\rm local} \propto q$ which could be very large. We understand its physics in the following. Initially, the heavy field ϕ_2 dominates the background evolution, and so the perturbations from the other field ϕ_1 contribute on iso-curvature modes. Later the first period of inflation ceases with a cutoff $\phi_2 \sim M_p$ and then ϕ_1 dominates over, and this process converts entropy perturbations into adiabatic. In this process there is usually a non-linear growth of curvature perturbations outside sound horizon. This mechanism has been widely applied in curvaton [76–81], and Ekpyrotic models [82–85].

6. Conclusion and discussions

In this Letter, we have studied an inflation model involving multiple sound speeds. This scenario can be embedded into inflation models with multiple components, for example a model of N-flation, which in usual can avoid some difficulties of single field inflation models, and so is regarded as an attractive implementation of inflation. In recent years, there have been many works around the issue of N-flation, such as Refs. [86–92], and we refer to Refs. [93,94] for recent reviews. However, in all these pioneer works, there is only one sound speed involved. In the model we proposed, due to a number of sound speeds, it may lead to plentiful interesting phenomena. In the current Letter, we only focus

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[3] For simplicity, we have assumed there is no coupling between two fields and calculate the three-point function of each field. The studies on N-flation with single DBI action were intensively studied in Refs. [27,66–69].

on the curvature and entropy perturbations and has already found that the decomposition on these two modes in usual N-flation models cannot be applied in our model.

Specifically, we have studied a new model in which a collection of two DBI fields drives inflation simultaneously. This model was originally proposed in Ref. [24], which has discovered that some non-perturbative effects are involved when we study background evolution and curvature perturbation. In the current Letter, we considered a tachyonic potential and provided detailed calculations on perturbations. To linear order, we find that the perturbations do not get freezed until the modes exit the maximum of all the sound horizons, and so the linear perturbations depend on the largest sound speed. Moreover, both the curvature perturbations and entropy perturbations are nearly scale-invariant with red tilts. However, their spectral indices are different. For curvature perturbations, the tilt is suppressed by the efolding number as $1/\mathcal{N}$; while the tilt of the spectral index of entropy perturbations is suppressed by efolding number as $1/\mathcal{N}^3$.

Furthermore, we have investigated the non-Gaussianities of equilateral type and local type in this model. By calculating the three point correlators, our results show that, for the non-Gaussianity of equilateral type is much sensitive to the smallest sound speed which takes the form $f_{NL} \sim 1/c_s^2$. Besides, since our model is constructed by double fields, there is entropy perturbations generated during inflation. Therefore, when the entropy perturbations contribute to curvature perturbations at late times of inflation, they can lead to a sizable non-Gaussianity of local type. However, if both two field evolve in the same rate during inflation, the non-Gaussianity of local type is still suppressed by slow roll parameter.

We close by making a comparison between our work and Ref. [66] in the limit where the sound speeds of the adiabatic and entropy modes are the same. In the current Letter we roughly focus our interests on three points, curvature perturbation, entropy perturbation, and non-Gaussianities. In the limit with two sound speeds being the same, the curvature perturbations in our model coincide with those obtained in [66]. However, the entropy perturbations are different. As we have explained in the main context, since we focus on the examples that the two scalar fluctuations are nearly uncorrelated, the dominant term of entropy perturbation comes from their cancellation. If we take the same value for two sound speeds, it requires $m_1 = m_2$ in our model and so corresponds to the case that the correlation term dominates over in the expression of entropy perturbation. In this situation the behavior of entropy perturbation would reduce to the result obtained in Ref. [66]. The study of this scenario will appear in the near future. Finally, by making a comparison directly, we can find the form of equilateral non-Gaussianity is consistent with that in Ref. [66] when we take two sound speeds be the same.

Acknowledgements

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