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Phantom and non-phantom dark energy: The cosmological relevance of non-locally corrected gravity

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ABSTRACT

In this Letter we have investigated the cosmological dynamics of non-locally corrected gravity involving a function of the inverse d'Alembertian of the Ricci scalar, $f(\Box^{-1}R)$. Casting the dynamical equations into local form, we derive the fixed points of the dynamics and demonstrate the existence and stability of a one parameter family of dark energy solutions for a simple choice, $f(\Box^{-1}R) \sim \exp(\alpha\Box^{-1}R)$. The effective EoS parameter is given by, $w_{\rm eff} = (\alpha-1)/(3\alpha-1)$ and the stability of the solutions is guaranteed provided that $1/3 < \alpha < 2/3$. For $1/3 < \alpha < 1/2$ and $1/2 < \alpha < 2/3$, the underlying system exhibits phantom and non-phantom behavior respectively; the de Sitter solution corresponds to $\alpha = 1/2$. For a wide range of initial conditions, the system mimics dust like behavior before reaching the stable fixed point. The late time phantom phase is achieved without involving negative kinetic energy fields. A brief discussion on the entropy of de Sitter space in non-local model is included.

1. Introduction

At present, there are two major theoretical approaches to late time acceleration of universe which is supported by different data sets of observations. The standard lore is related to the modification of right-hand side of the Einstein equations supplementing the stress energy tensor by a *dark energy* component [1]. Recently, serious attempts have been made to modify the geometry itself or the original Einstein–Hilbert action. A large number of papers are currently devoted to the investigations of $f(R)$ gravities (see review [2] and references therein). These theories are motivated by phenomenological considerations. The problems faced by these theories can be circumvented in specific models [3]. It is really interesting that these models do not reduce to *cosmological constant* Λ in the low curvature regime and thus can be distinguished from the latter. However, in any proposal on modification of gravity at large scales, it becomes mandatory to check whether the local gravity constraints are satisfied. The latter can be achieved for specific $f(R)$ gravity models provided one invokes the chameleon scenario [3,4] *à la* Greek epicycle.

It is also of crucial importance to explore the possibility of obtaining late time acceleration from a fundamental theory. Thus the string curvature corrections to gravity and their cosmological relevance is a subject of current interest. Attempts have recently been made to derive current acceleration using the Gauss–Bonnet (GB) term and higher order curvature invariants coupled to a dynamically evolving scalar field [5]. The model with GB invariant exhibits a remarkable property that it does not disturb the scaling regime and can give rise to late time transition from matter regime to late time acceleration (see Ref. [6] and references therein). Unfortunately, the coupling of GB term to scalar field gets large at late times; the nucleosynthesis also imposes stringent constraints on these models. Inclusion of higher order corrections introduces further technical complications [7].

Most of the proposals aimed to describe the late time cosmic evolution are faced with one or the other problem. Recently, an interesting idea of using non-locally corrected Einstein theory was put forward in [8,9]. These corrections typically involve combinations of inverse of d'Alembertian of the Ricci scalar and might be induced by quantum loops and (or) stringy considerations. Being non-local in character, these extra terms added to Einstein–Hilbert action can lead to late time acceleration as a time delayed effect avoiding the fine tuning problem [8]. The non-local dynamics can be cast in a local form by introducing a number of auxiliary fields [9].

In this Letter, we consider the simplest form of non-local corrections to gravity and investigate the underlying cosmological dynamics in details. We explore the possibility of a matter like

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regime which can finally mimic (phantom) dark energy and in particular the de Sitter solution, at late times. The entropy of de Sitter space in non-local gravity is also briefly discussed.

2. Non-local cosmology and its late time attractors

In what follows we consider the local form of non-locally corrected gravity. Let us consider the following simple example of the non-local action [8]

$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2}\left(1+f\left(\Box^{-1}R\right)\right)+\mathcal{L}_{\mathrm{matter}}\right\}, \tag{1}$$

where f is some function of d'Alembertian and denoted by $\Box$. The action would lead to non-local equations of motion which are difficult to investigate. The above action can be cast into local form by introducing two scalar fields ϕ and ξ [9]

$$\begin{aligned}S&=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\phi)\right)+\xi(\Box\phi-R)\right\}+\mathcal{L}_{\mathrm{matter}}\right]\\&=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\phi)\right)-\partial_\mu\xi\partial^\mu\phi-\xi R\right\}+\mathcal{L}_{\mathrm{matter}}\right].\end{aligned} \tag{2}$$

The equivalence of actions (1) and (2) can easily be checked. Indeed, by varying the action over ξ, we obtain

$$\Box\phi=R\quad\text{or}\quad\phi=\Box^{-1}R, \tag{3}$$

which after substitution into (2), gives the original action (1).

The local equations of motion can now be derived by varying with respect to the metric and the auxiliary fields ϕ and ξ. Variation of (2) with respect to $g_{\mu\nu}$ gives

$$\begin{aligned}0=&\frac{1}{2}g_{\mu\nu}\left\{R\left(1+f(\phi)-\xi\right)-\partial_\rho\xi\partial^\rho\phi\right\}\\&-R_{\mu\nu}\left(1+f(\phi)-\xi\right)+\frac{1}{2}(\partial_\mu\xi\partial_\nu\phi+\partial_\mu\phi\partial_\nu\xi)\\&-(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)\left(f(\phi)-\xi\right)+\kappa^2T_{\mu\nu}.\end{aligned} \tag{4}$$

The variation with respect to ϕ leads to

$$0=\Box\xi+f'(\phi)R. \tag{5}$$

For obvious reason, we shall specialize to FRW background with the metric

$$ds^2=-dt^2+a(t)^2\sum_{i=1,2,3}\left(dx^i\right)^2, \tag{6}$$

in which case the scalar fields ϕ and ξ are functions of time alone. In the FRW background, the time–time and the space–space components of Eq. (4) yield the modified Hubble equation and equation for acceleration

$$\begin{aligned}0=&-3H^2\left(1+f(\phi)-\xi\right)-3H\left(f'(\phi)\dot\phi-\dot\xi\right)\\&+\frac{1}{2}\dot\xi\dot\phi+\kappa^2\rho,\end{aligned} \tag{7}$$

$$\begin{aligned}0=&\left(2\dot H+3H^2\right)\left(1+f(\phi)-\xi\right)+\frac{1}{2}\dot\xi\dot\phi\\&+\left(\frac{d^2}{dt^2}+2H\frac{d}{dt}\right)\left(f(\phi)-\xi\right)+\kappa^2p,\end{aligned} \tag{8}$$

where ρ and p refer to the energy density and pressure of the background fluid and satisfy the usual continuity equation

$$\dot\rho+3H(1+w)\rho=0 \tag{9}$$

with w being the equation of state parameter of the background matter. The scalar field equations assume the following form

$$0=\ddot\phi+3H\dot\phi+6\dot H+12H^2, \tag{10}$$

$$0=\ddot\xi+3H\dot\xi-\left(6\dot H+12H^2\right)f'(\phi). \tag{11}$$

For the sake of simplicity, we shall use the exponential form of $f(\phi)$

$$f(\phi)=f_0e^{\alpha\phi}, \tag{12}$$

where f_0 and α are constants. We shall now cast the underlying dynamical equations in an autonomous form and look for dark energy solution as stable fixed points of the dynamics. We use the following notations (henceforth, we use the unit $\kappa^2=1$)

$$N=\ln a,\qquad x=\frac{\dot\phi}{H},\qquad y=\frac{\dot\xi}{Hf(\phi)},\qquad z=\frac{\kappa^2\rho}{H^2f(\phi)} \tag{13}$$

to arrive at the autonomous form of evolution equations

$$\frac{dx}{dN}=-3x-12-\frac{\dot H}{H^2}(x+6), \tag{14}$$

$$\frac{dy}{dN}=-3y+12\alpha-\alpha xy-\frac{\dot H}{H^2}(y-6\alpha), \tag{15}$$

$$\frac{dz}{dN}=-\left(3+3w+\alpha x+2\frac{\dot H}{H^2}\right)z, \tag{16}$$

and the constraint equation

$$\frac{\xi-1}{f(\phi)}=1-\frac{1}{6}xy+\alpha x-y-\frac{1}{3}z. \tag{17}$$

Differentiating Eq. (7) with respect to time and using the continuity equation (9), we can express $\dot H/H^2$ in the term of the autonomous variables as

$$\frac{\dot H}{H^2}=\frac{24\alpha+4\alpha x-\alpha^2x^2-(4+x)y-(1+w)z}{2[(1+\frac{x}{6})(-6\alpha+y)+\frac{z}{3}]}. \tag{18}$$

First we shall analyze the dynamics in the absence of the background fluid. In this case, the system (14)–(15) possesses two critical points, $(x_c,y_c)=(0,0)$ and $(x_c,y_c)=(-2/\alpha,6\alpha/(2-3\alpha))$. The stability of the critical points can easily be analyzed. The eigenvalues of the stability matrix for the autonomous system corresponding to the first critical point $(x_c,y_c)=(0,0)$ are given by, $\{-1,-1\}$ making it a stable node. From Eq. (18), we easily find the effective EoS parameter at the critical point under consideration

$$w_{\mathrm{eff}}=-1-\frac{2}{3}\frac{\dot H}{H^2}\bigg|_{(0,0)}=\frac{1}{3}. \tag{19}$$

For the fixed point $(x_c,y_c)=(-2/\alpha,6\alpha/(2-3\alpha))$, using Eq. (18), we find

$$w_{\mathrm{eff}}=\frac{\alpha-1}{3\alpha-1}. \tag{20}$$

In this case, the eigenvalues of the stability matrix are given by, $(3\alpha-2)/(3\alpha-1)$, $(3\alpha-2)/(3\alpha-1)$. Thus the fixed point is stable provided that $1/3<\alpha<2/3$ leading to dark energy solutions, $-\infty<w_{\mathrm{eff}}<-1/3$; the stable de Sitter solution is obtained for $\alpha=1/2$, see Fig. 1. In Fig. 2, we have plotted the phase portrait demonstrating the stability of both the critical points. Without the loss of generality, we have assumed both ϕ and $\dot\phi$ to be negative which corresponds to a particular type of boundary condition in Eq. (3) [8]. Inclusion of matter in the system makes the analysis cumbersome in general. However, one can easily obtain the fixed point for $\dot H=0$

$$x_c=-4,\qquad y_c=-\frac{12\alpha}{4\alpha-3},\qquad z_c=-\frac{8\alpha(2\alpha-1)}{1+w}. \tag{21}$$

In this case, the eigenvalues of the stability matrix are

$$\{-3,4\alpha-3,4\alpha-3-3w\} \tag{22}$$

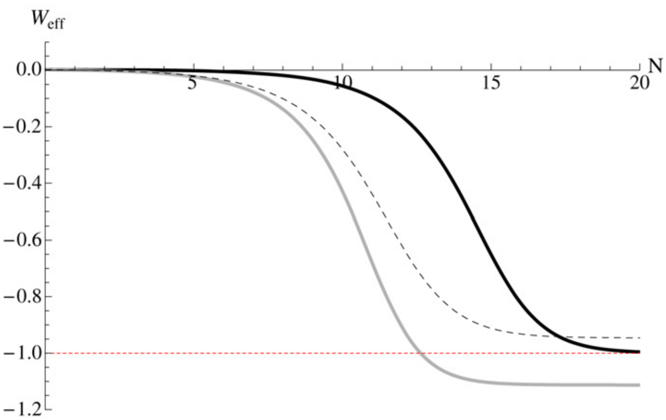

Fig. 1. The evolution of $w_{\rm eff}$ from the different initial conditions, dark black line corresponds to $\alpha = 0.5$, $x = -1.99$, $y = 5322.02$, gray to $\alpha = 0.487$, $x = -1.99$, $y = 1012.06$ and dashed to $\alpha = 0.507$, $x = -1.99$, $y = 1013.06$. At the fixed point $w_{\rm eff} = -1$, -1.11 and -0.95 respectively.

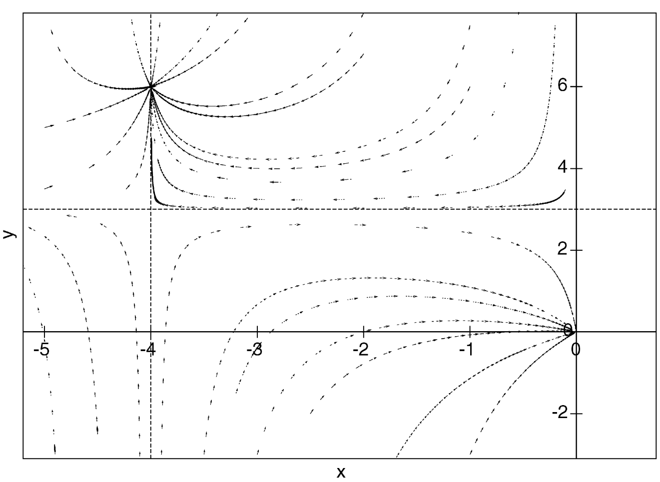

Fig. 2. The phase portrait of the system in absence of matter for $\alpha = 1/2$. The phase space splits into two disjoint regions: all the trajectories starting from $y > 3$ converge to de Sitter where as they approach $(0, 0)$ for $y < 3$ and $x > -4$.

with a restriction that $\alpha = 3(1 + w)/4$. In this case, the fixed point is stable provided that $w < 0$.

Despite the fact that it is extremely difficult to treat this system analytically, several exact solutions may be found. For instance, in case of the de Sitter solution, Eq. (10) can be solved as

$$\phi = -4H_0 t - \phi_0 e^{-3H_0 t} + \phi_1, \tag{23}$$

where ϕ_0 and ϕ_1 are the constants of integration. For simplicity, we assume that $\phi_0 = \phi_1 = 0$. Using Eq. (11), we get the expression for ξ,

$$\xi = \frac{3f_0}{4\alpha - 3} e^{-4\alpha H_0 t} - \frac{\xi_0}{3H_0} e^{-3H_0 t} - \xi_1. \tag{24}$$

Here ξ_0 and ξ_1 are constants. In the absence of matter, $\rho = 0$, the de Sitter space corresponds to $\alpha = 1/2$ and $\xi_1 = -1$. When $\rho \neq 0$, there exists a de Sitter solution provided we choose

$$\alpha = \frac{3}{4}(1 + w), \qquad f_0 = \frac{\kappa^2 \rho_0}{3H_0^2(1 + 3w)}, \qquad \xi_1 = -1. \tag{25}$$

In the presence of matter with $w \neq 0$, one may find a de Sitter solution even for a more complicated choice of $f(\phi)$

$$f(\phi) = f_0 e^{\phi/2} + f_1 e^{3(w+1)\phi/4}. \tag{26}$$

The solution is given by

$$\phi = -4H_0 t, \qquad \xi = 1 + 3f_0 e^{-2H_0 t} + \frac{f_1}{w} e^{-3(w+1)H_0 t},$$
$$\rho = -\frac{3(3w + 1)H_0^2 f_1}{\kappa^2} e^{-3(1+w)H_0 t}. \tag{27}$$

Let us now briefly point out some generalities of our formulation. The above discussion of non-local cosmology is based upon the simplest version of non-locality. Indeed, we could start from a general non-local action:

$$S = \int d^4x \sqrt{-g} \big[F\big(R, \Box R, \Box^2 R, \ldots, \Box^m R, \Box^{-1} R,$$
$$\Box^{-2} R, \ldots, \Box^{-n} R\big) + \mathcal{L}_{\rm matter} \big] \tag{28}$$

with m and n being the positive integers. By introducing scalar fields A, B, χ_k, η_k $(k = 1, 2, \ldots, m)$, and ξ_l, ϕ_l $(l = 1, 2, \ldots, n)$, we may rewrite the action (28) in the following form:

$$S = \int d^4x \sqrt{-g} \Bigg[BR - BA + F(A, \eta_1, \eta_2, \ldots, \eta_m, \phi_1, \phi_2, \ldots, \phi_n)$$
$$+ \partial_\mu \chi \partial^\mu A + \sum_{k=2}^{m} \partial_\mu \chi_k \partial^\mu \eta_{k-1} + \sum_{l=1}^{n} \partial_\mu \xi_l \partial^\mu \phi_l$$
$$+ \sum_{k=1}^{m} \chi_1 \eta_1 + A\xi_1 + \sum_{l=2}^{n} \xi_l \phi_{l-1} + \mathcal{L}_{\rm matter} \Bigg]. \tag{29}$$

The variations over A, B χ_k, ξ_l leads to the following equations

$$0 = R - A = \eta_1 - \Box A = \eta_k - \Box \eta_{k-1} = \Box \phi_1 - A$$
$$= \Box \phi_l - \phi_{l-1} \quad (k = 2, \ldots, m, \; l = 2, \ldots, n) \tag{30}$$

which gives $\eta_k = \Box^k R$ and $\Box^l \phi_l = R$ and we find that the actions (29) and (28) are equivalent.

Let us now consider a scale transformation given by

$$g_{\mu\nu} \to \frac{1}{2B} g_{\mu\nu}. \tag{31}$$

Then the action (29) is transformed into the action in the Einstein frame:

$$S = \int d^4x \sqrt{-g} \Bigg[\frac{R}{2} - \frac{3}{2B^2} \partial_\mu B \partial^\mu B$$
$$+ \frac{1}{2B} \Bigg(\partial_\mu \chi \partial^\mu A + \sum_{k=2}^{m} \partial_\mu \chi_k \partial^\mu \eta_{k-1} + \sum_{l=1}^{n} \partial_\mu \xi_l \partial^\mu \phi_l \Bigg)$$
$$+ \frac{1}{4B^2} \Bigg(-BA + F(A, \eta_1, \ldots, \eta_m, \phi_1, \ldots, \phi_n)$$
$$+ \sum_{k=1}^{m} \chi_1 \eta_1 + A\xi_1 + \sum_{l=2}^{n} \xi_l \phi_{l-1} \Bigg) + \mathcal{L}^A_{\rm matter} \Bigg]. \tag{32}$$

Here $\mathcal{L}^A_{\rm matter}$ can be obtained by scale transforming of the metric tensor in $\mathcal{L}_{\rm matter}$ by (31). The above action can be regarded as a non-linear σ model with potential coupled to gravity. Action (1) can be thought as a particular case of the general non-local action. In this case, however, the continuity equation (9) is no longer valid; additional terms on the right-hand side of this equation are generated which are responsible for inducing interaction of matter with background [4]. This aspect is crucial for the discussion of local gravity constraints on the models based on the modified theories of gravity.

3. The entropy of de Sitter space

In this section, we shall investigate the properties of entropy in de Sitter space in non-local modified gravity with the cosmological constant and matter.

The starting point is the trace of equation of motion of the non-local model, which reads, for a constant curvature solution and in the presence of a cosmological constant Λ and matter, as

$$\big(f(\phi)+1-\xi\big)R = 4\Lambda-\kappa^2T+6f'(\phi)R+3f''(\phi)\partial_\rho\phi\partial^\rho\phi+\partial_\rho\xi\partial^\rho\phi, \quad (33)$$

with T the stress tensor trace. Let us suppose that spherically-symmetric, static, constant curvature solution exists

$$ds^2=-A(r)\,dt_s^2+A(r)^{-1}\,dr^2+r^2\,d\Sigma^2. \quad (34)$$

There is a horizon if $A(r_H)=0$ with $A'(r_H)\neq 0$ and $r_H>0$. There is also entropy associated with this horizon, entropy that may be evaluated using Wald's method. For the local action, one has

$$\mathcal{S}_{\mathrm{BH}}=\frac{A_H}{4G}\big[1-\xi+f(\phi)\big]_{r_H}, \quad (35)$$

with all the fields being evaluated on the horizon and $A_H=4\pi r_H^2$. Thus, in general, there is a violation of the "Area Law".

Within the above black hole like metric, equation $\Box\phi=R$ and equation for the field ξ read for time dependent and spherically symmetric fields as

$$-\frac{\partial_{t_s}^2\phi}{A}+\frac{1}{r^2}\partial_r\big(r^2A\partial_r\phi\big)=R,$$
$$-\frac{\partial_{t_s}^2\xi}{A}+\frac{1}{r^2}\partial_r\big(r^2A\partial_r\xi\big)=-f'(\phi)R. \quad (36)$$

One may look for the solution of the first equation in the form $\phi=-4H_0t_s-2\ln A(r)$. As a result, it follows that de Sitter space $A_{\mathrm{dS}}(r)=1-Rr^2/12$ is a solution. Then, it is easy to show that the solution for ξ reads

$$\xi(r,t_s)=B+\frac{3f_0}{(4\alpha-3)}\big(1-H_0^2r^2\big)^{-2\alpha}\mathrm{e}^{-4\alpha H_0t_s}. \quad (37)$$

As a result, the entropy factor becomes

$$1-\xi+f(\phi)=1-B+\frac{(4\alpha-6)f_0}{(4\alpha-3)}\big(1-H_0^2r^2\big)^{-2\alpha}\mathrm{e}^{-4\alpha H_0t_s}. \quad (38)$$

As a consequence, for $\alpha>0$, the field ξ and the entropy are, in general, divergent on the horizon, while, as soon as $\alpha<0$, the ξ field and the de Sitter entropy are finite and independent on the time t_s on the horizon. However, there exists a positive value, namely $\alpha=3/2$, which renders the de Sitter entropy finite. We will see the physical significance of these two choices later.

Furthermore, in order to satisfy Einstein equations, the choice made for $f(\phi)$ has to satisfy the trace constraint (33), where the stress tensor of the matter and cosmological constant contribution are present. A direct computation shows that the trace constraint is satisfied as soon as $1-B=\Lambda/3H_0^2$ and the stress tensor trace is not zero and is given by $T=T_0A_{\mathrm{dS}}(r)^{-2\alpha}\mathrm{e}^{-4H_0\alpha t_s}$ where T_0 depends on α. Again, this trace diverges on the horizon as soon as $\alpha>0$. Note that in the case $\alpha=\frac{3}{2}$, the entropy of the de Sitter solution reads $\mathcal{S}_{\mathrm{dS}}=(A_H/4G)(\Lambda/3H_0^2)$. For vanishing cosmological constant we have found a de Sitter solution with finite entropy which is vanishing! However, recall the Wald method gives the entropy modulo a constant.

In the FRW space–time, the solutions we have obtained lead to

$$1-\xi+f(\phi)=\frac{\Lambda}{3H_0^2}+\frac{\kappa^2(1-w)\rho_0}{3H_0^2w(1+3w)}\mathrm{e}^{-3H_0(1+w)t}, \quad (39)$$

while, equation of motion for ξ gives

$$1-\xi+f(\phi)=1-B+f_0\frac{4\alpha-6}{4\alpha-3}\mathrm{e}^{-4\alpha H_0t}. \quad (40)$$

A comparison with $w\neq 1$ gives

$$1-B=\frac{\Lambda}{3H_0^2},\qquad 4\alpha=3(1+w),\qquad f_0=-\frac{k^2\rho_0}{3H_0^2(1+3w)}, \quad (41)$$

while if $w=1$, we have f_0 undetermined. As a check, we may pass to the static de Sitter patch by transforming the time coordinate $t=t_s+(1/2H_0)\ln(1-H_0^2r^2)$, and note that one gets Eq. (38) again.

Thus, the existence conditions found for the de Sitter entropy require a non-vanishing positive cosmological constant and matter. In general, this matter, in order to have finite entropy, should be "phantom" matter, since the condition $\alpha<0$ is equivalent to $1+w<0$. However, as already noted in the static patch, if we have matter such as $w=1$, namely stiff matter $p=\rho$, important in the early universe, the time dependence of the entropy drops out, in agreement with the result obtained in the static gauge. One can also see that entropy divergences in the static gauge correspond to the (non-physical) time-dependence of entropy in the cosmological gauge and thus this situation seems problematic. This consideration indicates that perhaps even the definition of entropy for non-local gravity should be reconsidered. It is also interesting that AdS black hole solution for the theory under consideration does not exist, while de Sitter solution exists.

4. Conclusions

In this Letter we have investigated the cosmological relevance of non-local corrections to gravity of the type $f(\Box^{-1}R)\sim\exp(\alpha\Box^{-1}R)$ which can be motivated by non-perturbative quantum effects. Casting the equations in local form we analyzed the underlying dynamics and explored the possibility of finding stable dark energy solutions. We have shown that the system might mimic the matter dominated regime for a long time and ultimately switches over to dark energy solutions at late times, thereby alleviating the fine tuning problem. The class of the dark energy solutions is parameterized by α which ranges as $1/3<\alpha<2/3$. The corresponding effective EoS parameter varies as, $-\infty<w_{\mathrm{eff}}<-1/3$. In case of $\alpha>1/2$, we have non-phantom dark energy; the de Sitter solution is obtained for $\alpha=1/2$. It is remarkable that there exists a range of the parameter, namely, $1/3<\alpha<1/2$ for which the model leads to phantom dark energy solutions which are attractors of the system. It should be emphasized that the observed acceleration can be achieved for natural values of $\alpha\sim 1$. Secondly, the model under consideration does not involve negative kinetic energy fields. Unlike standard phantom theories giving rise to transient phantom phase, the non-locally corrected gravity leads to a class of phantom energy solutions which are the late time attractors of the dynamics.

It is also important to note that even if more precise observational data define the EoS parameter to be slightly different from -1, there exists a possibility of realizing such a scenario in non-local gravity as the effective quintessence or phantom cosmology.

Interestingly, the existence of de Sitter entropy in the non-local model requires a non-vanishing positive cosmological constant and phantom matter.

Last but not the least, we should remember that the low energy modifications of gravity crucially differ from the corresponding local versions. In general, any large scale modification of gravity faces two major challenges: The first is related to the presence of either the ghost or the tachyon modes in these models; secondly, the solar physics imposes stringent constraints on these models. Thus it would really be interesting to carry out these investigations for non-local theories discussed in this Letter; we defer this analysis to our future work. It would also be interesting to investigate the complicated versions of non-local gravity mentioned at the end of Section 2.

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