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A benchmark approach of counterparty credit exposure of Bermudan option under Lévy Process: the Monte Carlo-COS Method

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Abstract

An advanced method, which we call Monte Carlo-COS method, is proposed for computing the counterparty credit exposure profile of Bermudan options under Lévy process. The different exposure profiles and exercise intensity under different measures, $\mathbb{P}$ and $\mathbb{Q}$, are discussed. Since the COS method [1] delivers accurate Bermudan prices, and no change of measure [2] needed to get the $\mathbb{P}$-probability distribution, the exposure profile produced by the Monte Carlo-COS algorithm can be used as a benchmark result, E.g., to analyse the reliability of the popular American Monte Carlo method [3, 4, 5]. The efficient calculation of expected exposure (EE) [6] can be further applied to the computation of credit value adjustment (CVA) [6].

Keywords: counterparty credit risk, Monte Carlo-COS method, Bermudan option, Lévy process, American Monte Carlo method, credit value adjustment;

1. Introduction

The computation of counterparty credit exposure of exotic instruments with no analytical solution is a challenging problem. According to Basel II and Basel III, counterparty credit risk is the risk that a counterparty in a derivatives transaction will default prior to the expiration of the instrument and will not therefore make the current and future payments required by the contract. For quantification of counterparty credit risk of exotic instruments with no analytical solution, such as calculation of potential future exposure (PFE), expected exposure (EE), and credit value adjustment (CVA), an efficient computation method for counterparty credit exposure is required.

In this paper, we propose an advanced approach, which we call Monte Carlo-COS method (MCCOS), to give an accurate result of the exposure profile (See definition 2.4) of a single asset Bermudan option under Lévy process. Different from the American Monte Carlo method[1] [3, 4, 5], in the Monte Carlo-COS method, one can calculate the exposure profile without using any change of measure. Combined with the computational advantage of COS method on accuracy and speed of option pricing, the exposure profile produced by the Monte Carlo-COS method can serve as a "benchmark" for analysing the reliability of the American Monte Carlo method.

The literature on the subject is quite rich. Canabarro and Duffie [7] and Duffie and Singleton [8] discuss techniques for measuring and pricing counterparty credit risk; Lomibao and Zhu [9] present a "direct jump to

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[1]we call the Longstaff-Schwartz method, stochastic mesh method and other methods which are used for pricing Bermudan option and American option as American Monte Carlo algorithm.

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simulation date" method, and derive analytic expressions to calculate the exposure on a number of path-dependent instruments, except Bermudan option and American option; In Pykhtin and Zhu [10, 11], the modeling framework for counterparty credit exposure is proposed.

Based on this modeling framework, the American Monte carlo method is proposed for exposure calculation in some literatures. In Schöftner [5] a modified least squares Monte Carlo algorithm is applied; Cesari [4] combines the bundling technique [12] with Longstaff-Schwartz method for exposure calculation; Ng [13] applies the stochastic mesh method to the credit exposure calculation. However, the exposure distribution under real-world measure $\mathbb{P}$ is not presented.

The paper is structured in the following way. Section 2 provides the definition of the exposure profiles of counterparty credit exposure, and describes the modeling approach for exposure calculation of exotic options. Section 3 shows the connection between dynamic programming and exposure calculation. Section 4 explains the application of Monte Carlo-COS method to get a benchmark result for the Bermudan option. Section 5 gives the numerical experiments and analyses the difference of exposure profile and exercise intensity under different measures. Section 6 concludes the presented approach to calculate the exposure profiles.

2. Option Price Distribution and Counterparty Credit Exposure

In this section, we give the definition of counterparty credit exposure and introduce the modeling framework for calculation of exposure profile of exotic options.

2.1. Exposure definition

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, let T be a fixed positive number, and let $\mathcal{F}_t$, $0 \leq t \leq T$, be a filtration of sub-σ-algebras of $\mathcal{F}$. We define the value of a derivative security under the risk-neutral measure $\mathbb{Q}$ [14] over time as a stochastic process $V(t), 0 \leq t \leq T$, which is driven by the stochastic process of risk factors $X(t), 0 \leq t \leq T$, such as stock prices, foreign exchange rates, and interest rates[6]. We call $(t, X(t))$ the state of the economy at time t. Denote the derivative's discounted net cashflow between t and T as $C_{ASHFLOWS}(t, T)$ (i.e., all of the cashflows are discounted back to time t), then $V(t) = \mathbb{E}^{\mathbb{Q}}[C_{ASHFLOWS}(t, T)|\mathcal{F}_t]$. We use the notation from [6] and give the definition of counterparty credit risk measures as follows:

Definition 2.1. *The credit exposure, E_t, of a derivative security at time t to a counterparty is defined as the non-negative value of the risk-neutral expected discounted value of future cashflows, i.e.,*

$$E_t = \max(V(t), 0) = V(t)^+, 0 \leq t \leq T \tag{1}$$

Definition 2.2. *The potential future exposure (PFE) at time t as seen from time zero is defined as*

$$PFE_{\alpha,t} = inf\{x : \mathbb{P}(E_t \leq x) \geq \alpha)\}, 0 \leq t \leq T \tag{2}$$

where α is the given confidence level, and $\mathbb{P}$ is the real-world measure.

Definition 2.3. *The expected exposure (EE) at time t as seen from time zero, which is used in computing credit value adjustment (CVA), is given by:*

$$EE_t = \mathbb{E}^{\mathbb{P}}[E_t], 0 \leq t \leq T \tag{3}$$

here the expectation is taken under the real-world measure $\mathbb{P}$.

Definition 2.4. *The exposure profile of counterparty credit exposure is defined as the the graph of $PFE_{\alpha,t}$ or EE_t, as a function of t.*

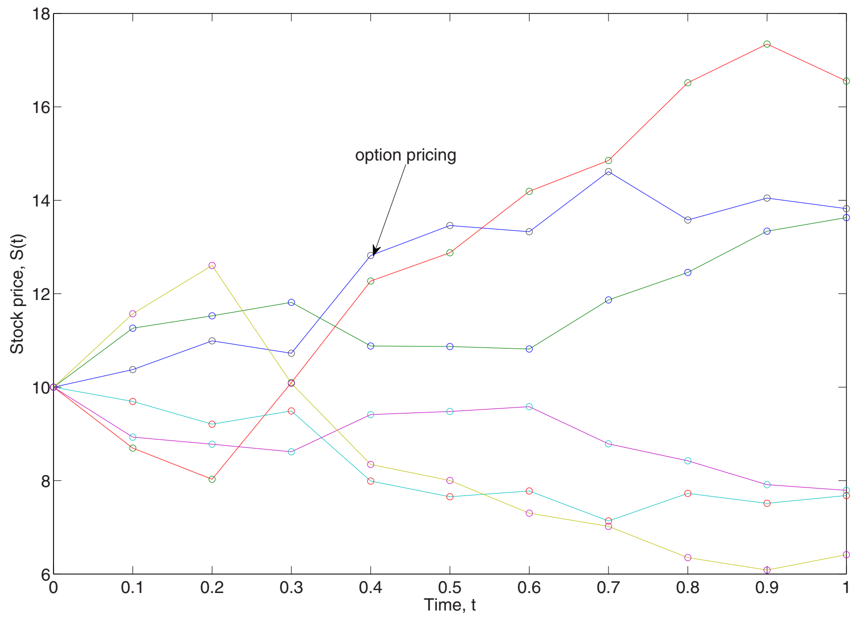

Fig. 1. Modeling framework.

2.2. *Exposure valuation: the modeling framework*

The main problem to calculate $PFE_{\alpha,t}$ in (2) and EE_t in (3) is to calculate the probability distribution of E_t (or $V(t)$) under the real-world measure $\mathbb{P}$. The exact probability distribution, which usually has no explicit solution, can be approximated by an empirical distribution of the sample results of E_t (or $V(t)$) on each simulated state $(t, X(t))$.

Assuming one has a model describing the stochastic process of risk factors $X(t)$, $0 \le t \le T$, which is already calibrated to the market data at time zero, then two basic steps are involved in the modelling framework [3, 4]:

1. Simulate the model under the real-world measure $\mathbb{P}$ (i.e., the market price of risk has to be incorporated into the model) to get the scenarios of risk factors $X(t)$, $t \in [0, T]$, see figure 1.
2. Calculate the option price for every simulated state $(t, X(t))$, under the risk-neutral measure $\mathbb{Q}$. The option can be seen as a newly issued one from the future state $(t, X(t))$, with time to maturity $T - t$.

3. Dynamic Programming and Exposure Calculation

In contrast to European options, which can only be exercised at maturity, a Bermudan option can be exercised at a fixed set of exercise opportunities, $\mathcal{T} = \{t_1, ..., t_M\}$,$0 = t_0 \le t_1, t_M = T$. Assume the exercise dates are equally spaced, i.e., $t_i - t_{i-1} = \Delta t, i = 1, ...M$. If the option is exercised at t_i, the option holder gets the exercise value $h(t_i, S_{t_i})$.

To determine $V_0(S_0)$, the Bermudan option value at initial time 0, with initial stock price S_0, one needs to solve the following dynamic programming recursion:

$$V_M(S_M) = \max(h(t_M, S_M), 0) \tag{4}$$
$$c(t_{m-1}, S_{m-1}) = \exp(-r\Delta t)\mathbb{E}^{\mathbb{Q}}[V_m(S_m)|\mathcal{F}_{t_{m-1}}], m = M, M-1, ..., 1 \tag{5}$$
$$V_{m-1}(S_{m-1}) = \max\{h(t_{m-1}, S_{m-1}), c(t_{m-1}, S_{m-1})\} \tag{6}$$
$$V_0(S_0) = c(t_0, S_0) \tag{7}$$

where we use the simplified notation X_m for X_{t_m}. we assume a constant interest rate r, so $\exp(-r\Delta t)$ denotes the discount factor for time interval Δt. c is the continuation value of the option and V the value of the option immediately before the exercise opportunity. As indicated in (4), the continuation value c at terminal time t_M equals 0.

Note that t_0 is not included in the exercise dates. If one issues a new Bermudan option from an intermediate state (t_{m-1}, S_{m-1}), with possible exercise dates $[t_m, ..., t_M]$ (Here t_{m-1} is not an exercise date.), then the price of this new option is equal to the continuation value $c(t_{m-1}, S_{m-1})$ in (5) [15]. Based on this observation, we can calculate the credit exposure for each exercise date, $\mathcal{T} = \{t_1, ..., t_M\}$, as a by-product of the option pricing procedure, which therefore yields estimated distributions of credit exposure, on each possible exercise date.

In an ordinary option pricing procedure of American Monte Carlo method, such as LSM, the stock price S_t is usually simulated under the risk-neutral measure $\mathbb{Q}$. However, in risk management, industries are more interested in values under the real-world measure $\mathbb{P}$, i.e., asset price processes evolve in the real-world measure $\mathbb{P}$. In [4, 5], the authors use the change of measure method to get the $\mathbb{P}$-distribution. In contrast to the American Monte Carlo method used in [4, 5], in the Monte Carlo-COS method, one can efficiently compute the option prices on all the grid points which are simulated under measure $\mathbb{P}$, without using any change of measure. The algorithm is explained in the following section.

4. A Benchmark Approach: The Monte Carlo-COS Method

The Monte Carlo-COS method is based on the work of [4, 1]. We assume the underlying stochastic process is a Lévy process.

For a Bermudan option, regression-based approximation methods, such as the LSM method, are used to approximate the following conditional expectation on possible exercise dates:

$$c(t_{m-1}, S_{m-1}(p)) = \exp(-r\Delta t)\mathbb{E}^{\mathbb{Q}}[V_m(S_m)|\mathcal{F}_{t_{m-1}}], \tag{8}$$

with $p = 1, ..., P$ the simulated sample paths. If we define $x = \log(S_{m-1}(p)/K)$, $y = \log(S_m/K)$, with K the strike price, and denote $\widetilde{V}_t(y) = V_t(K\exp(y)) = V_t(S_t)$, then it can be represented as,

$$c(t_{m-1}, x) = \exp(-r\Delta t)\mathbb{E}^{\mathbb{Q}}[\widetilde{V}_m(y)|x] = \exp(-r\Delta t)\int_{\mathbb{R}} \widetilde{V}_m(y) f(y|x)\mathrm{d}y, \tag{9}$$

where $f(y|x)$ is the probability density function of y given x under risk-neutral measure $\mathbb{Q}$.

An alternative way for efficient calculation of (9) is by numerical integration, particularly we choose the COS method developed in [1] as the main component of our algorithm.

Different from the option pricing problem in [1], for the exposure profile problem, the option price on every grid point simulated under measure $\mathbb{P}$ has to be calculated. And the early exercise event has to be taken into account for each simulated path, since the option price should be floored to zero after exercise event happens. This is done by finding the earliest exercise time, τ^p, for each path p and set the value after τ^p into zero.

There are three main components in the Monte Carlo-COS method for exposure profile calculation:

1. Scenario generation for the future economic state under measure $\mathbb{P}$;
2. Instrument valuation of all the simulated grid points by COS method;
3. Exposure profile calculation.

4.1. Fourier cosine expansion

In this section, we explain the COS method for instrument valuation of all the simulated grid points. The following proposition[1] gives another representation of (9), based on Fourier cosine expansion:

Proposition 4.1. *Let the underlying stochastic process of stock price S_t be Lévy process, then the continuation value at grid point $(t_{m-1}, S_{m-1}(p))$, $c(t_{m-1}, S_{m-1}(p))$, can be approximated by,*

$$\hat{c}(t_{m-1}, x) = \exp(-r\Delta t)\sum_{k=0}^{N-1}{}' Re\{\varphi_{levy}(\frac{k\pi}{b-a}; \Delta t)\exp(-ik\pi\frac{x-a}{b-a})\}V_k(t_m) \tag{10}$$

where $\varphi_{levy}(\omega;\Delta t) = \phi_{levy}(\omega;0,\Delta t)$*, and* ϕ_{levy} *is the characteristic function of Lévy process.* $V_k(t_m)$ *is the Fourier-cosine series coefficients of* $\widetilde{V}_m(y)$ *on* $[a,b]$*,*

$$V_k(t_m) = \frac{2}{b-a}\int_a^b \widetilde{V}_m(y)cos(k\pi\frac{y-a}{b-a})\mathrm{d}y \tag{11}$$

Here $[a,b]$ *is the truncation rage of the integration of risk-neutral evaluation formula in* (9)*.* $c(t_{m-1}, S_{m-1}(p))$ *is equivalent to the value of Bermudan option newly issued at grid point* $(t_{m-1}, S_{m-1}(p))$*, with maturity time* t_M *and possible exercise dates,* $t_m, ..., t_M$*.*

Proof. The main proof can be found in [1]. □

4.2. Recovery of $V_k(t_m)$

To compute (10), one needs to know the Fourier cosine coefficients, $V_k(t_m)$, given in (11). The derivation of an induction formula for $V_k(t_m)$ of Bermudan option, backwards in time, was the basis of the work in [1]. It is briefly explained here.

First, the early exercise point, $x^*(t_m)$, at time t_m, which is the point where the continuation value equals the payoff, i.e., $c(x^*(t_m), t_m) = g(x^*(t_m))$, is determined by Newton's method.

Second, based on $x^*(t_m)$, $V_k(t_m)$ is split into two parts: one on the interval $[a, x^*(t_m)]$, and another on $(x^*(t_m), b]$, i.e.,

$$V_k(t_m) = \begin{cases} C_k(a, x^*(t_m), t_m) + G_k(x^*(t_m), b), & call, \\ G_k(a, x^*(t_m)) + C_k(x^*(t_m), b, t_m), & put, \end{cases}$$

for $m = M-1, ..., 1$, and at $t_M = T$,

$$V_k(t_M) = \begin{cases} G_k(x^*(0, b), & call, \\ G_k(x^*(a, 0), & put. \end{cases}$$

Here C_k and G_k are the Fourier coefficients for the continuation value and payoff function, respectively, which read,

$$G_k(x_1, x_2) = \frac{2}{b-a}\int_{x_1}^{x_2} g(x)cos(k\pi\frac{x-a}{b-a})\mathrm{d}x,$$

and

$$C_k(x_1, x_2, t_j) = \frac{2}{b-a}\int_{x_1}^{x_2} c(x, t_j)cos(k\pi\frac{x-a}{b-a})\mathrm{d}x.$$

For $k = 0, 1, ..., N-1$ and $m = 1, 2, ..., M$, $G_k(x_1, x_2)$ has analytical solution, and the challenge is to compute the C_k efficiently. The following proposition from [1] claims that $C_k(x_1, x_2, t_m)$, $k = 0, 1, ..., N-1$, can be recovered from $V_l(t_{m+1})$, $l = 0, 1, ..., N-1$.

Proposition 4.2. *For* $m = M$*,* $V_k(x_1, x_2, t_m)$ *(and* $C_k(x_1, x_2, t_m)$*) has analytical solution; for* $m = M-1, ..., 1$*,* $G_k(x_1, x_2)$ *has analytical solution, and* $C_k(x_1, x_2, t_m)$ *can be approximated by* $\widehat{C}_k(x_1, x_2, t_m)$*, i.e.,*

$$\widehat{C}_k(x_1, x_2, t_m) = \begin{cases} \exp(-r\Delta t)Re\{\sum_{l=0}^{\prime N-1} \varphi_{levy}(\frac{l\pi}{b-a};\Delta t)V_l(t_{m+1}).\mathcal{M}_{k,l}(x_1, x_2)\} & m = M-1 \\ \exp(-r\Delta t)Re\{\sum_{l=0}^{\prime N-1} \varphi_{levy}(\frac{l\pi}{b-a};\Delta t)\widehat{V}_l(t_{m+1}).\mathcal{M}_{k,l}(x_1, x_2)\} & m = M-2, ..., 1 \end{cases}$$

with $\mathcal{M}_{k,l}(x_1, x_2)$ *defined as*

$$\mathcal{M}_{k,l}(x_1, x_2) = \frac{2}{b-a}\int_{x_1}^{x_2} \exp(il\pi\frac{x-a}{b-a})cos(k\pi\frac{x-a}{b-a})\mathrm{d}x,$$

and $i = \sqrt{-1}$ *being the imaginary unit.* $\widehat{V}_l(t_{m+1})$ *is the approximation of* $V_l(t_{m+1})$ *by replacing* $C_k(x_1, x_2, t_{m+1})$ *with* $\widehat{C}_k(x_1, x_2, t_{m+1})$*.*

Proof. The derivation of the result can be found in [1]. □

4.3. Application for exposure calculation

Denote the truncation interval for grid point $(t_{m-1}, S_{m-1}(p))$ by $[a_{m-1,p}, b_{m-1,p}]$, $m = 1, ..., M, p = 1, ..., P$, where

$$a_{m-1,p} = \xi_1 - L\sqrt{\xi_2 + \sqrt{\xi_4}} + \log(S_{m-1}(p)/K)$$
$$b_{m-1,p} = \xi_1 + L\sqrt{\xi_2 + \sqrt{\xi_4}} + \log(S_{m-1}(p)/K)$$

with $L \in [6, 12]$ depending on a user-defined tolerance level, TOL, and $\xi_1, ..., \xi_4$ being the cumulants of Lévy process[2], with time interval Δt. The error in the pricing formula connected to the size of the domain decreases exponentially with L, and in most cases, as shown in [1], with $L = 10$ the option price converges well for most Lévy processes.

The common truncation interval for all the grid points is chosen as $[a, b]$ in the following way,

$$a = \min\{a_{m-1,p} : m = 1, ..., M, p = 1, ..., P\},$$
$$b = \max\{b_{m-1,p} : m = 1, ..., M, p = 1, ..., P\}.$$

Consider the sample vector at time t_{m-1},

$$\mathbf{SV}_{m-1} = [S_{m-1}(1), ..., S_{m-1}(P)].$$

For a vector $\mathbf{xv}_{m-1} = [\log(S_{m-1}(1)/K), ..., \log(S_{m-1}(P)/K)]$, the COS formula (10) can be written as a vector form,

$$\hat{c}(t_{m-1}, \mathbf{xv}_{m-1}) = \exp(-r\Delta t) \sum_{k=0}^{N-1}{}' Re\{\varphi_{levy}(\frac{k\pi}{b-a}; \Delta t) \exp(-ik\pi \frac{\mathbf{xv}_{m-1} - a}{b-a})\} V_k(t_m) \tag{12}$$

which is particularly useful for exposure calculation of all the grid points in a sample vector.

According to the proposition (4.2), for the case of Lévy process, the Fourier cosine coefficients, $V_k(t_m)$, $k = 0, 1, ..., N-1$, can be recovered from $V_l(t_{m+1})$, $l = 0, 1, ..., N-1$, without knowing the option price for each time step. Once the Fourier cosine coefficients for each time step is calculated, one just inserts them into formula (12) to get the continuation value (or the Bermudan option price) of all the grid points, i.e., $\hat{c}(t_{m-1}, \mathbf{xv}_{m-1})$.

4.4. The Monte Carlo-COS algorithm

We list the Monte Carlo-COS algorithm for exposure profile calculation of Bermudan option as follows,

1. Simulate P paths for the stock price, S_t, under the real-world measure $\mathbb{P}$.
2. Calculate the common truncation interval for all of the simulated grid points, $[a, b]$.
3. For each time step, calculate the Fourier cosine coefficients, $V_k(t_m)$, $k = 0, 1, ..., N-1, m = 1, ..., M$.
4. At terminal date $t_M = T$, set
$$V_M(S_M(p)) = \max(h(t_M, S_M(p)), 0)$$
for $p = 1, ..., P$, and define the stopping time $\tau_M = T$.
5. Apply backward induction, i.e., $m \to m-1$ for $m = M, ..., 1$,
   (a) Calculate the continuation value, $\hat{c}(t_{m-1}, S_{m-1}(p))$, by inserting the Fourier cosine coefficients into formula (12).
   (b) Define a new stopping time according to the stopping rule for Bermudan option,
$$\tau^p_{m-1} = \min\{k \in \{m-1, ..., M\} | h(t_k, S_k(p)) \geq c(t_k, S_k(p))\}$$

[2]For example, if the stochastic process is geometric Brownian motion, then $\xi_1 = (\mu - \frac{1}{2}\sigma^2)\Delta t$, $\xi_2 = \sigma^2 \Delta t$, $\xi_4 = 0$, with μ the drift coefficient, and σ the diffusion coefficient.

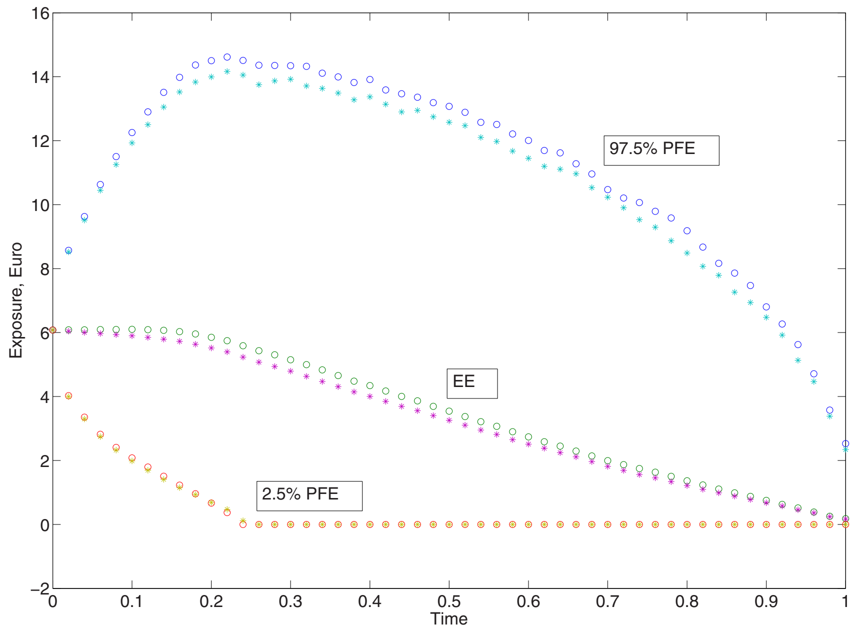

Fig. 2. The exposure profiles of Bermudan option under different measures, i.e., $\mathbb{Q}$ (o) and $\mathbb{P}$ (*).

(c) For each sample path $p = 1, ..., P$, set

$$V_{m-1}(S_{m-1}(p)) = \max(h(t_{m-1}, S_{m-1}(p)), c(t_{m-1}, S_{m-1}(p)))$$

and $V_t(S_t(p)) = 0$ for $t > \tau^p_{m-1}$.

6. Calculate the exposure at initial time, $V_0(S_0) = c(0, S_0)$.
7. Set $E^p_{t_m} = \max(V_m(S_m(p)), 0)$ for the credit exposures.
8. The measure $\mathbb{P}$-exposure profiles of PFE_{α,t_m} and EE_{α,t_m} can be calculated directly by the empirical distribution of $E^p_{t_m}$. Since the scenario is simulated under measure $\mathbb{P}$, no change of measure needed.

Remark 4.1. *Once the COS method is extended into the* 2 *dimension case [16] or more, the MCCOS algorithm can be extended straightforwardly into the multi-asset case.*

5. Numerical Experiments: Exposure Profiles under Different Measures

In this section, we investigate the difference between the exposure profiles calculated under different measures, i.e., $\mathbb{Q}$ and $\mathbb{P}$. For comparison, we take the same parameters as in [5] for the Bermudan option, with initial price $S_0 = 100$, strike price $K = 100$, constant interest rate $r = 0.05$, real world drift $\mu = 0.1$, volatility $\sigma = 0.2$ and 50 exercise dates. The underlying stochastic process is geometric Brownian motion process (GBM). We take 18,000 paths and 50 time steps for the underlying value. Only the exposures on possible exercise dates are considered.

We investigate the exposure profiles calculated under different measures by two settings:

1. $\mathbb{Q}$-exposure profile, i.e., the stock prices are simulated under measure $\mathbb{Q}$. The exposure profiles are obtained based on the $\mathbb{Q}$-probability distribution of credit exposure.

Time	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mathbb{P}$	5.8983	5.5188	4.7929	4.0037	3.2563	2.5100	1.8140	1.2148	0.6762	0.1654
$\mathbb{Q}$	6.1020	5.8501	5.1485	4.3417	3.5437	2.7390	1.9942	1.3643	0.7519	0.1799

Table 1. Expected Exposure (EE) calculated under measure $\mathbb{P}$ and $\mathbb{Q}$.

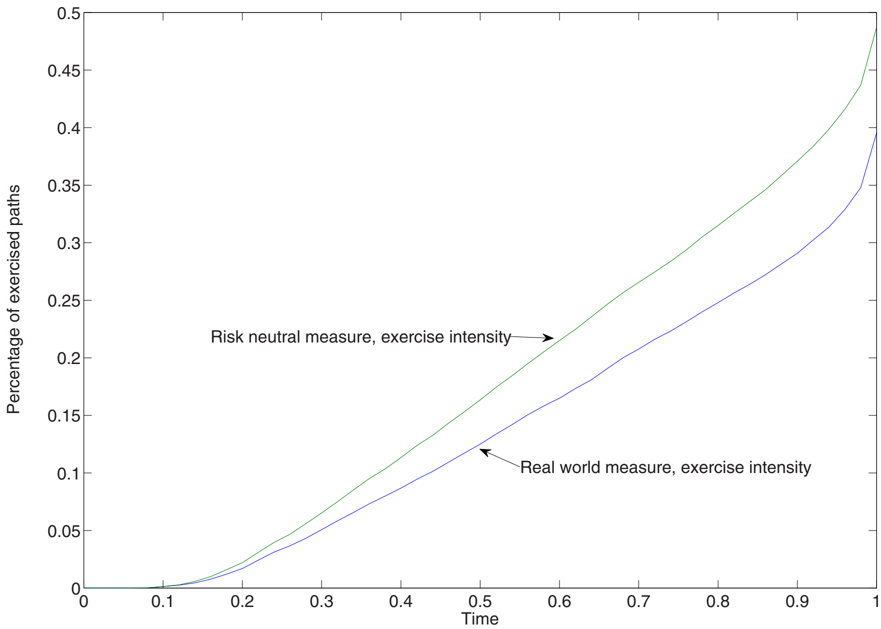

Fig. 3. The exercise intensity of Bermudan option under different measures.

2. $\mathbb{P}$-exposure profile, i.e., the stock prices are simulated under measure $\mathbb{P}$. The exposure profiles are obtained based on the $\mathbb{P}$-probability distribution of credit exposure.

The difference between $\mathbb{Q}$-exposure profile and $\mathbb{P}$-exposure profile is illustrated in figure 2. Note that in this parameter setting, $\mu > r$, and we find the $\mathbb{P}$-exposure profiles are lower than $\mathbb{Q}$-exposure profiles. The initial prices V_0 for both settings coincide, because the risk-neutral pricing formula is independent of different measures.

When $\mu > r$, at each time step t, the stock price S_t simulated under measure $\mathbb{P}$ tends to be higher than S_t simulated under measure $\mathbb{Q}$. For a Bermudan put option issued at time t, with maturity T and initial stock price S_t, a higher initial stock price S_t (i.e., simulated under measure $\mathbb{P}$) leads to a lower option price, thus a lower $\mathbb{P}$-exposure profile.

Table 1 provides the number of expected exposure calculated under different measures, which can be further applied to computation of credit value adjustment (CVA).

Figure 3 shows the percentage of paths that have already been exercised at time t. In the example, the exercise intensity under measure $\mathbb{Q}$ is higher than that under measure $\mathbb{P}$. This significantly influences the future exposure values, since after exercise, the contract does not exist any more and exposure is floored to zero.

Although it is exercised more often under measure $\mathbb{Q}$ than that under measure $\mathbb{P}$ (figure 3), the $\mathbb{Q}$-exposure profile is still higher than the $\mathbb{P}$-exposure profile (figure 2).

6. Conclusion

This paper proposes an advanced method, named Monte Carlo-COS method to calculate the exposure profile of single asset Bermudan options that has no analytical solution, under Lévy process. The result can serve as a benchmark for analysing the error in American Monte Carlo method [3, 4, 5]. The difference of exposure profiles and exercise intensity under different measures(i.e., $\mathbb{P}$ and $\mathbb{Q}$) is also discussed.

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