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Consistent Pauli reduction on group manifolds

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ABSTRACT

We prove an old conjecture by Duff, Nilsson, Pope and Warner asserting that the NS–NS sector of supergravity (and more general the bosonic string) allows for a consistent Pauli reduction on any d-dimensional group manifold G, keeping the full set of gauge bosons of the $G \times G$ isometry group of the bi-invariant metric on G. The main tool of the construction is a particular generalised Scherk–Schwarz reduction ansatz in double field theory which we explicitly construct in terms of the group's Killing vectors. Examples include the consistent reduction from ten dimensions on $S^3 \times S^3$ and on similar product spaces. The construction is another example of globally geometric non-toroidal compactifications inducing non-geometric fluxes.

1. Introduction

Although the idea of Kaluza–Klein theories originated in the 1920s [1,2], it was with the advent of higher-dimensional supergravities and string theory that the need for developing schemes for obtaining lower-dimensional theories by means of dimensional reduction became compelling. The original idea of Kaluza [1], subsequently developed by Klein [2], was straightforwardly extended from a circle reduction to a reduction on a d-dimensional torus. By this means, for example, four-dimensional ungauged $\mathcal{N} = 8$ supergravity was constructed, by reducing eleven-dimensional supergravity on a 7-torus [3,4]. A key feature in this, and most other, dimensional reductions is that one truncates the infinite "Kaluza Klein towers" of lower-dimensional fields that result from the generalised Fourier expansions of the higher-dimensional fields to just a finite subset, typically, but not always, just the massless fields.

Since the reduction is being applied to a highly non-linear theory, the question then arises as to whether the truncation to a finite subset of the fields is a consistent one. One way to formulate the question is whether in the full lower-dimensional theory, prior to the truncation, the equations of motion of the fields to be truncated are satisfied when one sets these fields to zero. The potential danger is that non-linear products of the fields that are being retained might act as sources for the fields that are to be truncated.

In the case of a circle or toroidal reduction, the consistency of the truncation is guaranteed by a simple group-theoretic argument. The fields that are retained are all the singlets under the $U(1)^d$ isometry of the d-torus, while all the fields that are set to zero are non-singlets (i.e. they are charged under the $U(1)$ factors). It is evident, by charge conservation, that no powers of neutral fields can act as sources for charged fields, and so the consistency is guaranteed.

A more general class of dimensional reductions was described by DeWitt in 1963 [5]. In these, one takes the internal d-dimensional space to be a compact group manifold G, equipped with its bi-invariant metric. The isometry group of this metric is $G_L \times G_R$, where G_L denotes the left action of the group G and G_R denotes the right action. If all the towers of lower-dimensional fields were retained in a reduction on the group manifold G, then the massless sector would include the Yang–Mills gauge bosons of the isometry group, $G_L \times G_R$. However, in the DeWitt reduction only the gauge bosons of G_R (or, equivalently and alternatively, G_L) are retained. To be precise, the lower-dimensional fields that are retained in the truncation are all those that are singlets under G_L. There is now again a simple group-theoretic argument that demonstrates the consistency of the DeWitt reduction: The fields that are being truncated are all those that are non-singlets under G_L. It is evident that no non-linear powers of the G_L-singlets that are retained can act as sources for the fields that are being set to zero, and so the truncation must be consistent.

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A much more subtle situation arises if one tries to make more general kinds of dimensional reduction that are not of the toroidal or DeWitt type. One of the earliest, and most important, examples is the 7-sphere reduction of eleven-dimensional supergravity. The massless sector of the reduced four-dimensional theory contains the fields of maximal $\mathcal{N}=8$ gauged $SO(8)$ supergravity [6, 7], but there is no obvious reason why it should be consistent to set the massive towers of fields to zero. In particular, one can easily see that if a generic theory is reduced on S^7 (or indeed any other sphere), then a quadratic product formed from the $SO(8)$ gauge bosons will act as a source for certain massive spin-2 fields. This sets off a chain reaction that then requires an infinity of four-dimensional fields to be retained. A first indication that something remarkable might be occurring in the case of eleven-dimensional supergravity and the S^7 reduction was found in [8], where it was shown that a conspiracy between contributions in the reduction ansatz for the eleven-dimensional metric and the 4-form field strength resulted in an exact cancellation of the potentially-troublesome source term for the massive spin-2 fields that was mentioned above. Subsequent work by de Wit and Nicolai in the 1980s [9], with more recent refinements [10–12], has established that the truncation to the massless $\mathcal{N}=8$ gauged $SO(8)$ supergravity is indeed consistent. There are a few other examples of supergravity sphere reductions that also admit analogous remarkable consistent truncations.

Dimensional reductions on a d-dimensional internal manifold M_d with isometry group G that admit a consistent truncation to a finite set of fields that includes all the gauge bosons of the Yang–Mills group G were called Pauli reductions in [13]. (The idea of such reductions was first proposed, but not successfully implemented, by Pauli in 1953 [14–16].) It was also observed in [13] that in addition to the necessary condition for consistency that was first seen in [8], which was essentially the absence of a cubic coupling of two gauge bosons to the massive spin-2 modes in the untruncated lower-dimensional theory, a rather different necessary condition of group-theoretic origin could also be given. Namely, one can consider first the (trivially consistent) truncation of the theory when reduced instead on the torus T^d. The resulting lower-dimensional theory will have a (non-compact) group S of global symmetries, with a maximal compact subgroup K. If the higher-dimensional theory were to admit a consistent Pauli reduction on the manifold M_d then it must be possible to obtain that theory, with its Yang–Mills gauge group G, by gauging the theory obtained instead in the T^d reduction. (Conversely, by scaling the size of the M_d reduction manifold to infinity, the gauged theory should limit to the ungauged one.) This will only be possible if the isometry group G of the manifold M_d is a subgroup of the maximal compact subgroup K of the global symmetry group S of the T^d reduction.

A generic theory will not satisfy the above necessary condition for admitting a consistent Pauli reduction. For example, pure Einstein gravity in $(n+d)$ dimensions gives rise, after reduction on T^d, to an n-dimensional theory with $S=GL(d,\mathbb{R})$ global symmetry, whose maximal compact subgroup is $K=SO(d)$. By contrast, the isometry group of the d-sphere is $G=SO(d+1)$, which is thus not contained within K. The situation is very different if we consider certain supergravity theories, such as eleven-dimensional supergravity. If it is reduced on T^7 the resulting four-dimensional ungauged theory has an enhanced $E_{7(7)}$ global symmetry, for which the maximal compact subgroup is $K=SU(8)$. This is large enough to contain the $G=SO(8)$ isometry group of the 7-sphere, and thus this necessary condition for consistency of the truncation in the S^7 reduction is satisfied.

It is evident from the above discussion that if an $(n+d)$-dimensional theory is to admit a consistent Pauli reduction on S^d, in which all the Yang–Mills gauge bosons of the isometry group $SO(d+1)$ are retained, then the theory must have some special features that lead to its T^d reduction yielding a massless truncation with some appropriate enhancement of the generic $GL(d,\mathbb{R})$ global symmetry group. Similarly, one may be able to rule out other putative consistent Pauli reductions by analogous arguments.

This brings us to the topic of the present paper. It was observed in [17] that in a reduction of the $(n+d)$-dimensional bosonic string on a group manifold G of dimension d, the potentially dangerous trilinear coupling of a massive spin-2 mode to bilinears built from the Yang–Mills gauge bosons of $G_L \times G_R$ was in fact absent. On that basis, it was conjectured in [17] that there exists a consistent Pauli reduction of the $(n+d)$-dimensional bosonic string on a group manifold G of dimension d, yielding a theory in n dimensions containing the metric, the Yang–Mills gauge bosons of $G_L \times G_R$, and d^2+1 scalar fields which parameterise $\mathbb{R}\times SO(d,d)/(SO(d)\times SO(d))$. Further support for the conjectured consistency was provided in [13], where it was observed that the $K=SO(d)\times SO(d)$ maximal compact subgroup of the enhanced $O(d,d)$ global symmetry of the T^d reduction of the bosonic string is large enough to contain the $G_L \times G_R$ gauge group as a subgroup.

In this paper, we shall present a complete and constructive proof of the consistency of the Pauli reduction of the bosonic string on the group manifold G. Our construction makes use of the recent developments realising non-toroidal compactifications of supergravity via generalised Scherk–Schwarz-type reductions [18] on an extended spacetime within duality covariant reformulations of the higher-dimensional supergravity theories [19–28]. In this language, consistency of a truncation ansatz translates into a set of differential equations to be satisfied by the group-valued Scherk–Schwarz twist matrix U encoding all dependence on the internal coordinates. Most recently, this has been put to work in the framework of exceptional field theory in order to derive the full Kaluza–Klein truncation of IIB supergravity on a 5-sphere to massless $\mathcal{N}=8$ supergravity in five dimensions [29,30]. In this paper, we explicitly construct the $SO(d,d)$ valued twist matrix describing the Pauli reduction of the bosonic string on a group manifold G in terms of the Killing vectors of the group manifold. We show that it satisfies the relevant consistency equations, thereby establishing consistency of the truncation. From the Scherk–Schwarz reduction formulas we then read off the explicit truncation ansätze for all fields of the bosonic string. We find agreement with the linearised ansatz proposed in [17] and for the metric we confirm the non-linear reduction ansatz conjectured in [13].

Our solution for the twist matrix straightforwardly generalises to the case when G is a non-compact group. In this case, the construction describes the consistent reduction of the bosonic string on an internal manifold M_d whose isometry group is given by the maximally compact subgroup $K_L \times K_R \subset G_L \times G_R$. The truncation retains not only the gauge bosons of the isometry group, but the gauge group of the lower-dimensional theory enhances to the full non-compact $G_L \times G_R$. At the scalar origin, the gauge group is broken down to its compact part. This is a standard scenario in supergravity with non-compact gauge groups: for the known sphere reductions the analogous generalisations describe the compactification on hyperboloids $H^{p,q}$ and lower-dimensional theories with $SO(p,q)$ gauge groups [31,32,26,33].

The paper is organised as follows. In section 2 we briefly review the $O(d,d)$ covariant formulation of the low-energy effective action of the $(n+d)$-dimensional bosonic string. In section 3 we review how this framework allows the reformulation of consistent truncations of the original theory as generalised Scherk–Schwarz reductions on the extended space–time. We spell out the consistency equations for the Scherk–Schwarz twist matrix and construct an explicit solution in terms of the Killing vectors of the bi-invariant metric on a d-dimensional group manifold G. For compact G, the construction results in the Pauli reduction of the bosonic string on G to a lower-dimensional theory with gauge group $G\times G$. For non-compact G, the construction gives rise to a consistent truncation on an internal space M_d whose isometry group is given by two copies of the maximally compact subgroup $K\subset G$. Again, the gauge group of the lower-dimensional theory is $G\times G$. In section 4, we work out the complete non-linear reduction ansatz for the higher-dimensional fields, i.e. metric, two-form and dilaton. We discuss our findings in section 5, in particular the examples of consistent truncations of ten-dimensional $N=1$ supergravity down to four dimensions on products of spheres and hyperboloids.

2. $O(d,d)$ covariant formulation of the $(n+d)$-dimensional bosonic string

Our starting point is the $(n+d)$-dimensional bosonic string (or NS–NS sector of the superstring)

$$S=\int dX^{n+d}\sqrt{|\hat{G}|}\,e^{-2\phi}\Big(R+4\,\hat{G}^{\hat\mu\hat\nu}\partial_{\hat\mu}\phi\partial_{\hat\nu}\phi-\frac{1}{12}\,H^{\hat\mu\hat\nu\hat\rho}H_{\hat\mu\hat\nu\hat\rho}\Big)\,, \tag{1}$$

with dilaton ϕ and three-form field strength $H_{\hat\mu\hat\nu\hat\rho}\equiv 3\,\partial_{[\hat\mu}C_{\hat\nu\hat\rho]}$. As described in the introduction, the conjecture of [17] states this theory admits a consistent Pauli reduction to n dimensions on a d-dimensional group manifold G retaining the full set of $G_L\times G_R$ non-abelian gauge fields, according to the isometry group of the bi-invariant metric on G. In the following, for the explicit reduction formulas we will use the metric in the Einstein frame

$$G_{\hat\mu\hat\nu}\equiv e^{-4\beta\phi}\,\hat{G}_{\hat\mu\hat\nu}\,, \tag{2}$$

with $\beta=1/(n+d-2)$, and split coordinates according to

$$\{X^{\hat\mu}\}\to\{x^\mu,y^m\}\,,\qquad \mu=0,\dots,n-1\,,\quad m=1,\dots,d\,. \tag{3}$$

The key tool in the following construction is double field theory (DFT) [34–37], the duality covariant formulation of the bosonic string. Most suited for our purpose, is the reformulation of the action (1) in which an $O(d,d)$ subgroup of the full duality group is made manifest [38]. This is obtained by Kaluza–Klein decomposing all fields according to n external and d internal dimensions (keeping the dependence on all $(n+d)$ coordinates) and rearranging the various components into $O(d,d)$ objects, in terms of which the action (1) can be rewritten in the form

$$\begin{aligned}S=\int dx^n dY^{2d}\sqrt{|\mathrm{g}|}\,e^{-2\Phi}\Big(&\widehat{\mathcal{R}}+4\,\mathrm{g}^{\mu\nu}D_\mu\Phi D_\nu\Phi\\&-\frac{1}{12}\mathcal{H}^{\mu\nu\rho}\mathcal{H}_{\mu\nu\rho}+\frac{1}{8}\,\mathrm{g}^{\mu\nu}D_\mu\mathcal{H}^{MN}D_\nu\mathcal{H}_{MN}\\&-\frac{1}{4}\,\mathcal{H}_{MN}\mathcal{F}^{\mu\nu M}\mathcal{F}_{\mu\nu}{}^N+\frac{1}{4}\,\mathcal{H}^{MN}\partial_M\mathrm{g}^{\mu\nu}\,\partial_N\mathrm{g}_{\mu\nu}+\mathcal{R}(\Phi,\mathcal{H})\Big)\,.\end{aligned} \tag{4}$$

Formally, this theory lives on an extended space of dimension $(n+2d)$ with coordinates $\{x^\mu,Y^M\}$, with all fields subject to the section constraint $\partial^M\otimes\partial_M\equiv 0$ which effectively removes the d non-physical coordinates. Fundamental $SO(d,d)$ indices M, N are lowered and raised with the $SO(d,d)$ invariant metric η_{MN} and its inverse. Moreover, $\mathcal{H}_{MN}$ is a symmetric $SO(d,d)$ group matrix, $\mathcal{H}_{\mu\nu\rho}$ and $\mathcal{F}_{\mu\nu}{}^M$ are the non-abelian field strengths of an external two-form $\mathcal{B}_{\mu\nu}$ and vector $\mathcal{A}_\mu{}^M$, respectively, and $\mathcal{R}(\Phi,\mathcal{H})$ is the scalar DFT curvature [37]. All derivatives and field strengths in (4) are covariantised with respect to generalised diffeomorphisms on the extended space. Specifically,

$$\begin{aligned}D_\mu\Phi&=\partial_\mu\Phi-\mathcal{A}_\mu{}^M\partial_M\Phi+\frac{1}{2}\,\partial_M\mathcal{A}_\mu{}^M\,,\\D_\mu\mathcal{H}_{MN}&=\partial_\mu\mathcal{H}_{MN}-\mathcal{A}_\mu{}^K\partial_K\mathcal{H}_{MN}\\&\quad-2\,\partial_{(M}\mathcal{A}_\mu{}^K\mathcal{H}_{N)K}+2\,\partial^K\mathcal{A}_{\mu\,(M}\mathcal{H}_{N)K}\,,\\\mathcal{F}_{\mu\nu}{}^M&=\partial_\mu\mathcal{A}_\nu{}^M-\partial_\nu\mathcal{A}_\mu{}^M-\big[\mathcal{A}_\mu,\mathcal{A}_\nu\big]_C^M-\partial^M\mathcal{B}_{\mu\nu}\,,\\\mathcal{H}_{\mu\nu\rho}&=3\,D_{[\mu}\mathcal{B}_{\nu\rho]}+3\,\mathcal{A}_{[\mu}{}^N\partial_\nu\mathcal{A}_{\rho]N}-\mathcal{A}_{[\mu N}\big[\mathcal{A}_\nu,\mathcal{A}_{\rho]}\big]_C^N\,,\end{aligned} \tag{5}$$

in terms of the Courant bracket $[\cdot,\cdot]_C$, see [38] for details.

The section constraint $\partial^M\otimes\partial_M\equiv 0$ is solved by splitting the internal coordinates according to

$$\{Y^M\}\to\{y^m,y_m\}\,, \tag{6}$$

in a light-cone basis where

$$\eta_{MN}\equiv\begin{pmatrix}0&\delta_m{}^n\\\delta^m{}_n&0\end{pmatrix}\,, \tag{7}$$

and restricting the dependence of all fields to the physical coordinates y^m by imposing $\partial^m\equiv 0$, thereby reducing the extended space–time in (4) back to $(n+d)$ dimensions. Upon breaking the DFT field content accordingly, and rearranging of fields, the $O(d,d)$ covariant form (4) then reproduces the bosonic string (1). The precise dictionary can be straightforwardly worked out by matching the gauge and diffeomorphism transformations of the various fields. For the DFT p-forms and metric this yield

$$\begin{aligned}&\mathcal{A}_\mu{}^m=A_\mu{}^m\equiv G^{mn}G_{\mu n}\,,\quad \mathcal{A}_{\mu\,m}=-\left(C_{\mu\,m}-A_\mu{}^nC_{nm}\right),\\&\mathcal{B}_{\mu\nu}=C_{\mu\nu}+2A_{[\mu}{}^mC_{\nu]\,m}+A_{[\mu}{}^mA_{\nu]}{}^nC_{mn}+A_{[\mu}{}^mA_{\nu]m}\,,\\&\mathrm{g}_{\mu\nu}=e^{4\beta\phi}\left(G_{\mu\nu}-A_\mu{}^mA_\nu{}^nG_{mn}\right).\end{aligned} \tag{8}$$

The dictionary for the DFT scalar fields is most conveniently obtained by comparing the transformation of the DFT vector fields under generalised external diffeomorphisms to the transformations in the original theory (1) and yields

$$\begin{aligned}&\mathcal{H}^{mn}=e^{-4\beta\phi}\,G^{mn}\,,\qquad \mathcal{H}_m{}^n=e^{-4\beta\phi}\,G^{nk}C_{km}\,,\\&\mathcal{H}_{mn}=e^{-4\beta\phi}\,G^{kl}C_{km}C_{ln}+e^{4\beta\phi}\,G_{mn}\,,\\&e^{\Phi}=e^{\frac{\beta}{\gamma}\phi}\left(\det G_{mn}\right)^{-1/4}\,,\end{aligned} \tag{9}$$

with $\gamma=\frac{1}{n-2}$. With the dictionary (8), (9), and imposing $\partial^m\equiv 0$, the $O(d,d)$ covariant action (4) reduces to the original action (1) of the bosonic string. The reduction ansatz on the other hand will be most compactly formulated in terms of the $O(d,d)$ objects.

3. Generalised Scherk–Schwarz ansatz and consistency equations

An important property of the $O(d,d)$ covariant form of the action (4) is the fact that particular solutions and truncations of the theory take a much simpler form in terms of the $O(d,d)$ objects $\mathcal{A}_\mu{}^M$, $\mathcal{H}_{MN}$, etc., as opposed to the original fields of the bosonic string (1). In particular, consistent truncations to n dimensions can be described by a generalised Scherk–Schwarz ansatz in which the dependence on the compactified coordinates Y^M is carried by an $SO(d,d)$ matrix $U_M{}^A$ and a scalar function ρ, according to [19,20][1]

$$\begin{aligned}&\mathcal{H}_{MN}=U_M{}^A(y)M_{AB}(x)U_N{}^B(y)\,,\qquad e^{\Phi}\;=\;\rho^{(n-2)/2}(y)\,e^{\varphi(x)}\,,\\&\mathcal{A}_\mu{}^M=(U^{-1})_A{}^M(y)\,A_\mu{}^A(x)\,,\qquad \mathcal{B}_{\mu\nu}\;=\;B_{\mu\nu}(x)\,,\\&\mathrm{g}_{\mu\nu}=e^{4\gamma\varphi(x)}\,g_{\mu\nu}(x)\,.\end{aligned} \tag{10}$$

[1] Since with (4) we use DFT in its split form with internal and external coordinates, the reduction ansatz (10) resembles the corresponding ansatz in exceptional field theory [26] for the p-forms and metric.

Here, $A_\mu{}^M$ and $B_{\mu\nu}$ are the gauge vectors and two-form of the reduced theory. The symmetric $SO(d,d)$ group valued matrix $M_{AB}(x)$ can be thought of as parameterising the coset space $SO(d,d)/(SO(d)\times SO(d))$, and together with $e^{\varphi}(x)$ carries the d^2+1 scalar fields of the reduced theory. The ansatz (10) describes a consistent truncation of (4), provided $U_M{}^A$ and ρ satisfy the consistency equations

$$\eta_{D[A}\,(U^{-1})_B{}^M(U^{-1})_{C]}{}^N\partial_M U_N{}^D = f_{ABC} \;=\; \text{const.}\,, \tag{11}$$

$$\rho^{-1}\,\partial_M\rho = -\gamma\,(U^{-1})_A{}^N\partial_N U_M{}^A\,, \tag{12}$$

with the $SO(d,d)$ invariant constant matrix η_{AB} and $\gamma=\frac{1}{n-2}$. If $U_M{}^A$ and ρ in addition depend only on the physical coordinates on the extended space (6)

$$\partial^m U_M{}^A = 0 \;=\; \partial^m\rho\,, \tag{13}$$

the ansatz (10) likewise describes a consistent truncation of the original theory (1). As a consequence of this section condition, the Jacobi identity is automatically satisfied for f_{ABC} upon using its explicit expression (11)

$$[X_A, X_B] = -X_{AB}{}^C X_C \tag{14}$$

where we have introduced the generalised structure constant $X_{AB}{}^C = f_{[ABD]}\eta^{DC}$. Then, for a given solution of (11), (12), the explicit reduction formulas for the original fields are obtained by combining (10) with the dictionary (8), (9), as we will work out shortly.

In order to explicitly solve the generalised Scherk–Schwarz consistency conditions (11)–(13), let us first note that with the index split (6), and the parameterisation

$$\begin{aligned} U_M{}^A \;&=\; \eta^{AB}\left\{\mathcal{Z}_{B\,m}, \mathcal{K}_B{}^m\right\}\,,\\ (U^{-1})_A{}^M \;&=\; \left\{\mathcal{K}_A{}^m, \mathcal{Z}_{A\,m}\right\}\,, \end{aligned} \tag{15}$$

of the $SO(d,d)$ matrix, equation (11) turns into

$$\begin{aligned} \mathcal{L}_{\mathcal{K}_A}\mathcal{K}_B{}^m &= -X_{AB}{}^C\,\mathcal{K}_C{}^m\,,\\ \mathcal{L}_{\mathcal{K}_A}\mathcal{Z}_{B\,m} + \mathcal{K}_B{}^n\left(\partial_m\mathcal{Z}_{A\,n}-\partial_n\mathcal{Z}_{A\,m}\right) &= -X_{AB}{}^C\,\mathcal{Z}_{C\,m}\,. \end{aligned} \tag{16}$$

The $SO(d,d)$ property of $U_M{}^A$ translates into

$$2\,\mathcal{K}_{(A}{}^m\mathcal{Z}_{B)\,m} = \eta_{AB} \;\equiv\; \begin{pmatrix} 0 & \delta_a{}^b\\ \delta^a{}_b & 0\end{pmatrix}\,. \tag{17}$$

In the following, we will construct an explicit solution of (16), (17) in terms of the Killing vectors of the bi-invariant metric on a d-dimensional group manifold G. For compact G, the resulting reduction describes the Pauli reduction of the bosonic string on G. For non-compact G, this describes a consistent truncation on an internal space M_d with isometry group given by two copies of the maximally compact subgroup $K\subset G$. Specifically, we choose the $\mathcal{K}_A$ as linear combinations of the $G_L\times G_R$ Killing vectors $\{L_a^m, R_a^m\}$, in the following way

$$\mathcal{K}_A{}^m \equiv \{L_a{}^m + R_a{}^m,\, L^{a\,m} - R^{a\,m}\}\,, \tag{18}$$

with their algebra of Lie derivatives given by

$$\mathcal{L}_{L_a}L_b = -f_{ab}{}^c\,L_c\,,\qquad \mathcal{L}_{L_a}R_b = 0\,,\qquad \mathcal{L}_{R_a}R_b = f_{ab}{}^c\,R_c\,, \tag{19}$$

in terms of the structure constants $f_{ab}{}^c$ of $\mathfrak{g}\equiv\mathrm{Lie}\,G$, and with indices $a,b,\ldots$, raised and lowered by the associated Cartan–Killing form $\kappa_{ab}\equiv f_{ac}{}^d f_{bd}{}^c$. Moreover, the bi-invariant metric on the group manifold can be expressed by

$$\tilde{G}^{mn} \equiv -4\,L_a{}^m L^{a\,n} \;=\; -4\,R_a{}^m R^{a\,n}\,. \tag{20}$$

With (19), the ansatz (18) solves the first equation of (16), with structure constants $X_{AB}{}^C$ given by

$$\begin{aligned} X_{abc} &= f_{abc}\,,\quad & X_a{}^{bc} \;&=\; f_a{}^{bc}\,,\\ X^a{}_b{}^c &= f^a{}_b{}^c\,,\quad & X^{ab}{}_c \;&=\; f^{ab}{}_c\,, \end{aligned} \tag{21}$$

and all other entries vanishing. Indeed, these structure constants are of the required form $X_{AB}{}^C = f_{[ABD]}\eta^{DC}$, cf. (14). We may define the $G_L\times G_R$ invariant Cartan–Killing form of the algebra (14)

$$\kappa_{AB} \equiv \frac{1}{2}X_{AC}{}^D X_{BD}{}^C \;=\; \begin{pmatrix}\kappa_{ab} & 0\\ 0 & \kappa^{ab}\end{pmatrix}\,, \tag{22}$$

such that the Killing vectors (18) satisfy

$$\kappa^{AB}\,\mathcal{K}_A{}^m\mathcal{K}_B{}^n \;=\; -\,\tilde{G}^{mn}\,,\qquad \eta^{AB}\,\mathcal{K}_A{}^m\mathcal{K}_B{}^n = 0\,, \tag{23}$$

and moreover $\kappa^{AB}\eta_{AB}=0$.

In order to solve the second equation of (16), with the same structure constants (21), we start from the ansatz[2]

$$\mathcal{Z}_{A\,m} = -\,\kappa_A{}^B\,\mathcal{K}_{B\,m} + \mathcal{K}_A{}^n\,\tilde{C}_{nm}\,. \tag{24}$$

Here, the space–time index in the first term has been lowered with the group metric $\tilde{G}_{mn}$ from (20), and $\tilde{C}_{mn}=\tilde{C}_{[mn]}$ represents an antisymmetric 2-form, such that the $SO(d,d)$ property (17) is identically satisfied. With this ansatz for $\mathcal{Z}_{A\,m}$, the second equation of (16) turns into

$$\begin{aligned} &\kappa_A{}^C\mathcal{K}_B{}^n\left(\partial_n\mathcal{K}_{C\,m}-\partial_m\mathcal{K}_{C\,n}\right) - 3\,\mathcal{K}_A{}^k\,\mathcal{K}_B{}^n\,\partial_{[k}\tilde{C}_{mn]}\\ &\quad = 2\,\eta^{DE}\,X_{A(E}{}^C\,\kappa_{B)C}\,\mathcal{K}_{D\,m}\,. \end{aligned} \tag{25}$$

The right-hand side of (25) vanishes by invariance of the Cartan–Killing form κ_{AB}. From (23), one derives the following identity

$$\partial_{[m}\mathcal{K}_{A\,n]} = X_{AC}{}^B\kappa^{CD}\mathcal{K}_{B\,m}\mathcal{K}_{D\,n}\,, \tag{26}$$

for the derivative of the Killing vectors. Inserting this relation in (25) gives

$$3\,\mathcal{K}_A{}^k\partial_{[k}\tilde{C}_{mn]} = 2\,X_A{}^{BC}\mathcal{K}_{B\,m}\mathcal{K}_{C\,n}\,, \tag{27}$$

where we have used $\kappa_A{}^E X_{ED}{}^C\kappa^{DB} = X_A{}^{BC}$. We note that both sides of this equation vanish under projection with $\eta^{DA}\mathcal{K}_{A\,p}$ as a consequence of (23). Projecting instead with $\kappa^{DA}\mathcal{K}_{A\,p}$, equation (27) reduces to an equation for $\tilde{C}_{mn}$

$$3\,\partial_{[k}\tilde{C}_{mn]} = \tilde{H}_{kmn} \;\equiv\; -\,2X^{ABD}\kappa_D{}^C\mathcal{K}_{A\,k}\mathcal{K}_{B\,m}\mathcal{K}_{C\,n}\,. \tag{28}$$

Explicitly, the flux $\tilde{H}_{kmn}$ takes the form

$$\tilde{H}_{kmn} = -16\,f^{abc}\,L_{a\,k}L_{b\,m}L_{c\,n} \;=\; -\,16\,f^{abc}\,R_{a\,k}R_{b\,m}R_{c\,n}\,, \tag{29}$$

and can be integrated since $\partial_{[k}\tilde{H}_{lmn]}=0$, due to the Jacobi identity on f_{abc}. We have thus solved the second equation of (16).

With (18), (24), the remaining consistency equation (12) reduces to

$$\begin{aligned} (n-2)\,\mathcal{K}_A{}^m\partial_m\log\rho = \partial_m\mathcal{K}_A{}^m \;&=\; -\,\tilde{\Gamma}_{mn}{}^m\mathcal{K}_A{}^n\,,\\ \Longrightarrow\qquad \rho \;&=\; (\det\tilde{G}_{mn})^{-\gamma/2}\,. \end{aligned} \tag{30}$$

We have thus determined the $SO(d,d)$ matrix $U_M{}^A$ and the scalar function ρ solving the system (11), (12) in terms of the Killing vectors on a group manifold G, and a two-form determined by (28). The resulting structure constants are given by (21) such that the gauge group of the reduced theory is given by $G_L\times G_R$.

[2] Let us stress that our notation is such that adjoint G indices $a,b,\ldots$ are raised and lowered with the Cartan–Killing form κ_{ab}, whereas fundamental $SO(d,d)$ indices $A,B,\ldots$ are raised and lowered with the $SO(d,d)$ invariant metric η_{AB} from (17) and *not* with the G-dependent Cartan–Killing form κ_{AB} from (22).

4. Reduction ansatz and reduced theory

We now have all the ingredients to read off the full non-linear reduction ansatz of the bosonic string (1). Combining the DFT reduction formulas (10) with the dictionary (8), (9), and the explicit expressions (18), (24) for the Scherk–Schwarz twist matrix, we obtain

$$\begin{aligned} ds^2 = \Delta^{-2\gamma}(x,y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu \\ + G_{mn}(x,y)\left(dy^m + \mathcal{K}_A{}^m(y) A_\mu^A(x) dx^\mu\right) \\ \times \left(dy^n + \mathcal{K}_B{}^n(y) A_\nu^B(x) dx^\nu\right) , \end{aligned} \tag{31}$$

for the metric in the Einstein frame, with $G_{mn}(x,y)$ given by the inverse of

$$G^{mn}(x,y) = \Delta^{2\gamma}(x,y)\, \mathcal{K}_A{}^m(y)\mathcal{K}_B{}^n(y) e^{4\gamma\varphi(x)} M^{AB}(x) . \tag{32}$$

The dilaton and the original two-forms are given by

$$\begin{aligned} e^{4\beta\phi} &= \Delta^{2\gamma}(x,y)\, e^{4\gamma\varphi(x)} \\ C_{mn} &= \tilde{C}_{mn}(y) + \Delta^{2\gamma}(x,y)\, \kappa_A{}^D \mathcal{K}_{D\,m} \mathcal{K}_B{}^p \\ &\quad \times G_{pn}(x,y)\, e^{4\gamma\varphi(x)} M^{AB}(x) , \\ C_{\mu\,m} &= \Big(\kappa_A{}^D \mathcal{K}_{D\,m} + \Delta^{2\gamma}(x,y)\, \kappa_C{}^E \mathcal{K}_A{}^n \mathcal{K}_{E\,n} \mathcal{K}_D{}^p \\ &\quad \times G_{pm}(x,y)\, e^{4\gamma\varphi(x)} M^{CD}(x)\Big) A_\mu{}^A(x) , \\ C_{\mu\nu} &= B_{\mu\nu}(x) - \kappa_B{}^C \mathcal{K}_A{}^m \mathcal{K}_{C\,m} A_{[\mu}{}^A(x) A_{\nu]}{}^B(x) \\ &\quad - \Delta^{2\gamma}(x,y)\, \kappa_C{}^E \mathcal{K}_B{}^n \mathcal{K}_{E\,n} \mathcal{K}_D{}^p \mathcal{K}_A{}^m \\ &\quad \times G_{pm}(x,y)\, e^{4\gamma\varphi(x)} M^{CD}(x)\, A_{[\mu}{}^A(x) A_{\nu]}{}^B(x) , \end{aligned} \tag{33}$$

where we have introduced the function $\Delta^2(x,y) \equiv \det(\tilde{G}_{mn}(y))^{-1} \cdot \det(G_{mn}(x,y))$. In these expressions, all space–time indices on the Killing vectors $\mathcal{K}_A{}^m$ are raised and lowered with the metric $\tilde{G}_{mn}(y)$ from (20), rather than with the full metric $G_{mn}(x,y)$. For the group manifold $G = SU(2)$, the construction describes the S^3 reduction of the bosonic string, for which the full reduction ansatz has been found in [39]. For general compact groups, the reduction ansatz for the internal metric (32) was correctly conjectured in [13].[3]

In order to compare our formulas to the linearised result given in [17], we first note that for compact G, we may normalise the Cartan–Killing form as $\kappa_{AB} = -\delta_{AB}$, such that the background (at $M_{AB}(x) = \delta_{AB}$) is given by

$$\mathring{G}_{mn} = \tilde{G}_{mn} , \qquad \mathring{C}_{mn} = \tilde{C}_{mn} , \qquad \mathring{\phi} = 0 . \tag{34}$$

We then linearise the reduction formulas (31)–(33) around the scalar origin

$$M_{AB}(x) = \delta_{AB} + m_{AB}(x) + \dots , \tag{35}$$

and (back in the string frame) obtain

$$\begin{aligned} \hat{G}_{mn}(x,y) &= \tilde{G}_{mn}(y) + \hat{h}_{mn}(x,y) + \dots , \\ C_{mn}(x,y) &= \tilde{C}_{mn}(y) + \hat{k}_{mn}(x,y) + \dots , \end{aligned} \tag{36}$$

[3] The translation uses an explicit parameterisation of the $SO(d,d)$ matrix M_{AB} in a basis where η_{AB} is diagonal, as

$$\tilde{M}_{AB} = \begin{pmatrix} (1+PP^t)^{1/2} & P \\ P^t & (1+P^tP)^{1/2} \end{pmatrix} ,$$

in terms of an unconstrained $d \times d$ matrix $P_a{}^b$.

with

$$\begin{aligned} \hat{h}_{mn}(x,y) &= -m_{AB}(x)\, \mathcal{K}^A{}_m(y)\, \mathcal{K}^B{}_n(y) , \\ \hat{k}_{mn}(x,y) &= m_{AB}(x)\, \kappa^{AD} \mathcal{K}_{D\,m}(y)\, \mathcal{K}^B{}_n(y) , \end{aligned} \tag{37}$$

as well as

$$\phi = \varphi(x) + \frac{1}{4}\tilde{G}^{mn}\hat{h}_{mn} + \dots , \tag{38}$$

for the dilaton, where we have used the linearisation $\Delta(x,y) = 1 + \frac{1}{2}\tilde{G}^{mn}\hat{h}_{mn} - 2d\beta\phi + \dots$. Parameterising the scalar fluctuations (35) as

$$m_{AB} \equiv \begin{pmatrix} a & -b \\ b & -a \end{pmatrix}_{AB} , \tag{39}$$

with symmetric a and antisymmetric b, in accordance with the $SO(d,d)$ property of M_{AB}, we finally obtain the fluctuations

$$\begin{aligned} \hat{h}_{mn} + \hat{k}_{mn} &= S^{ab}(x) L_{a\,n}(y) R_{b\,m}(y) , \\ \phi &= \varphi(x) + \frac{1}{4} S^{ab}(x) L_a{}^m(y) R_{b\,m}(y) , \end{aligned} \tag{40}$$

with $S^{ab} \equiv 4\left(a^{ab} + b^{ab}\right)$. These precisely reproduces the linearised result given in [17].

After the full non-linear reduction (31)–(33), the reduced theory is an n-dimensional gravity coupled to a 2-form and $2d$ gauge vectors with gauge group $G_L \times G_R$. The (d^2+1) scalar fields couple as an $\mathbb{R} \times SO(d,d)/(SO(d) \times SO(d))$ coset space sigma model, and come with a scalar potential [40,41]

$$\begin{aligned} V(x) = \frac{1}{12} e^{4\gamma\varphi(x)} X_{AB}{}^C X_{DE}{}^F M^{AD}(x) \\ \times \left(M^{BE}(x) M_{CF}(x) + 3\,\delta_C^E \delta_F^B\right) , \end{aligned} \tag{41}$$

with the structure constants $X_{AB}{}^C$ from (21). Due to the dilaton prefactor, this potential cannot support (A)dS geometries, but only Minkowski or domain wall solutions.

Let us finally comment on adding a cosmological term $e^{4\beta\phi}\Lambda$ in the higher-dimensional theory (1). E.g. for the bosonic string such a term would arise as conformal anomaly in dimension $n+d \neq 26$. In the Einstein frame, the modified action takes the form

$$\begin{aligned} S = \int dX^{n+d} \sqrt{|G|}\Big(R + 4\, G^{\hat\mu\hat\nu} \partial_{\hat\mu}\phi \partial_{\hat\nu}\phi \\ - \frac{1}{12} e^{-8\beta\phi} H^{\hat\mu\hat\nu\hat\rho} H_{\hat\mu\hat\nu\hat\rho} + e^{4\beta\phi}\Lambda\Big) , \end{aligned} \tag{42}$$

with constant Λ. With the $O(d,d)$ dictionary (9), it follows that the effect of this term in the $O(d,d)$ covariant action (4) is a similar term

$$\mathcal{L}_c = \sqrt{|g|}\, e^{-2\Phi} \Lambda , \tag{43}$$

manifestly respecting $O(d,d)$ covariance. The presence of this term thus does not interfere with the consistency of the truncation ansatz and simply results in a term

$$\mathcal{L}_c = \sqrt{|g|}\, e^{4\gamma\varphi} \Lambda , \tag{44}$$

in the reduced theory, as already argued in [17,39].

5. Conclusions

We have in this paper given a complete and constructive proof of the consistency of the Pauli reduction of the low-energy effective action of the bosonic string on the group manifold G, proving the conjecture of [17]. The construction is based on the $O(d,d)$ covariant reformulation of the original theory in which the consistent truncations of the latter are rephrased as generalised Scherk–Schwarz reductions on an extended spacetime. We have explicitly constructed the relevant $SO(d,d)$ valued twist matrix, carrying the dependence on the internal variables, in terms of the Killing vectors of the group manifold G. From the twist matrix, we have further read off the full non-linear reduction ansätze for all fields of the bosonic string. The construction is another example of the power of the generalised Scherk–Schwarz reductions on extended spacetime and hints towards a more systematic understanding of the conditions under which consistent Pauli reductions are possible. In this respect, it would be very interesting to classify the possible solutions of the system of equations (16) encoding the consistent reduction.

For a compact group manifold G, the obtained twist matrix describes the consistent Pauli reduction of the bosonic string on G. Interestingly, the construction straightforwardly generalises to the case when G is a non-compact group. In this case, the resulting twist matrix is still built from the Killing vectors on G, but describes the consistent reduction of the bosonic string on an internal manifold M_d whose metric is read off from (32) as

$$\mathring{G}^{mn} = (\det \mathring{H}/\det \tilde{G})^{\beta}\, \mathring{H}^{mn}\,, \qquad \text{with}$$
$$\mathring{H}^{mn} \equiv \mathcal{K}_A{}^m(y)\mathcal{K}_B{}^n(y)\,\delta^{AB}\,, \tag{45}$$

and $\tilde{G}_{mn}$ defined in (20) as the bi-invariant metric on G. It follows that the isometry group of this background metric $\mathring{G}_{mn}$ is the maximally compact subgroup $K_L \times K_R \subset G_L \times G_R$. The truncation in this case retains not only the gauge bosons of the isometry group, but the gauge group of the lower-dimensional theory enhances to the full non-compact $G_L \times G_R$. At the scalar origin, the non-compact gauge group is broken down to its compact part $K_L \times K_R$. This is a standard scenario in supergravity. For the known sphere reductions the corresponding generalisation describes the compactification on non-compact hyperboloidal spaces $H^{p,q}$ inducing lower-dimensional theories with $SO(p,q)$ gauge groups [31,32,26,33]. Similar to (45), the background 2-form $\mathring{C}_{mn}$ is read off from (33) and in this case differs from $\tilde{C}_{mn}$ by a contribution from the second term.

In general, the background geometry (34) or (45) does not provide a solution to the higher-dimensional field equations. This corresponds to the fact that the scalar potential (41) in general does not possess a stationary point at the scalar origin. However, a quick computation shows that at the origin $M_{AB} = \delta_{AB}$, the potential (41) is always stationary with respect to variation of the parameters of M_{AB}, such that there is a ground state with running dilaton $\varphi(x)$. Via (31)–(33), this domain wall solution is uplifted to the higher-dimensional theory.

A necessary and sufficient condition for the existence of a ground state with constant dilaton is $V|_{M_{AB}=\delta_{AB}} = 0$, i.e. necessarily a Minkowski vacuum. Evaluating the scalar potential at the origin translates this condition into

$$0 \stackrel{!}{=} V\Big|_{M_{AB}=\delta_{AB}} = \frac{2}{3}\left(2\,n_{\text{non-cp}} - n_{\text{cp}}\right)\,, \tag{46}$$

with n_{cp}, $n_{\text{non-cp}}$ denoting the number of compact and non-compact generators of G, respectively. A number of groups satisfy this condition

$$G = SO(1,5)\,,\ \ SO(5,20)\,,\ \ SO(20,76)\,,\ \ \dots\,,$$
$$G = SU(1,4)\,,\ \ SU(4,15)\,,\ \ SU(15,56)\,,\ \ \dots\,,$$
$$G = E_{6(-26)}\,,\ \text{with compact } F_4\,, \tag{47}$$

thus allowing for a consistent truncation of the bosonic string around $\text{Mink}_n \times M_{\dim G}$, the latter equipped with the metric (45). While these examples are presumably more of a mathematical curiosity, a group of more physical relevance is the choice

$$G = SO^*(4)\ \equiv\ SO(3)\times SO(2,1)\,, \tag{48}$$

satisfying the condition (46). With this group, the above construction describes the consistent truncation of ten-dimensional $N = 1$ supergravity down to four dimensions on the manifold $S^3 \times H^{2,2}$ giving rise to a half-maximal $SO(4) \times SO(2,2)$ gauged theory in four dimensions with Minkowski vacuum. The form of the vacuum resembles the Minkowski vacua found in [42] with uplift to eleven dimensions. It would be very interesting to embed this vacuum into the maximal theories allowing for Minkowski vacua [43,44] whose respective gauge groups $SO^*(8)$ and $SO(4) \times SO(2,2) \ltimes T^{16}$ indeed contain two copies of (48). The embedding of the maximal theory in higher dimensions may then be addressed similar to the construction of this paper within the proper full exceptional field theory [45].

Among the interesting examples with running dilaton, our construction includes the consistent truncation on $S^3 \times S^3$ corresponding to the compact choice $G = SO(4)$. In this case, the above construction gives the consistent embedding of $SO(4)^2$-gauged half-maximal supergravity into ten dimensions, extending the construction of [46], in which the scalar sector was truncated to the dilaton. Again, it would be interesting to embed this truncation into the maximal theory. It is likely that different embeddings into IIA and IIB may give rise to inequivalent maximal four-dimensional gaugings, as observed for the IIA/IIB S^3 reductions to seven dimensions [47].

Let us finally mention that the presented construction provides another example of globally geometric non-toroidal compactifications inducing non-geometric fluxes. In the language of [48], the structure constants (21) induced by this reduction combine a 3-form flux H_{abc} with non-geometric $Q_a{}^{bc}$ flux. However, despite their non-geometric appearance, the fluxes satisfy the condition $f_{KMN}f^{KMN} = 0$, necessary for a potential geometric origin [49], which we have provided here. For the compactifications on S^3 and $S^3 \times S^3$, this scenario has been discussed in [50,51], see also [52].

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