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Procedia Engineering 144 (2016) 959 – 966

Procedia Engineering

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12th International Conference on Vibration Problems, ICOVP 2015

Vibrations of a composite beam under thermal and mechanical loadings

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Abstract

Dynamics of a composite beam subjected to thermal and mechanical loadings is presented in the paper. The extended Timoshenko beam model takes into account shear and inertia of the cross-section and nonlinear longitudinal displacement caused by mechanical and thermal loadings. It has been shown that thermal and mechanical fields are fully coupled and the heat pulse may change the transient dynamics of the system. For the case of elevated temperature and a steady state problem the model is reduced to nonlinear ordinary differential equations of motion. It has been shown that the elevated ambient temperature may drastically change its response.

Peer-review under responsibility of the organizing committee of ICOVP 2015

Keywords: Non-linear vibrations; composites; thermal loading; bifurcation

1. Introduction

In a real environment, composite structures are subjected to varied dynamical and thermal conditions. The bases of the thermoelasticity are presented in [1,2]. It has been clearly shown that in order to explain the dynamics of thermoelastic structures the mathematical models have to consider coupled thermo-mechanical fields [3,4]. Although the temperature and elastic deformations are in fact coupled [3,4] for relatively thin structures it is acceptable to assume that the temperature distribution is independent of the deformation or that the structure gets the elevated temperature instantly. This approach is widely used in papers [5–7] to model the beam or plates dynamics under steady state conditions. In [5,8] thermo-mechanical, geometrically nonlinear vibrations of beams are studied considering reduced models of the structures and selected heat distributions. The authors found a very reach nonlinear dynamic behaviour [8] of the system including, periodic, quasi-periodic and chaotic oscillations with a very important temperature influence. Furthermore, in papers [9,10] it is demonstrated that the varied ambient temperature may change the system

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Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.123

response and the method of delamination detection may not work properly. The delamination may be masked by the elevated temperature.

In this paper we study dynamics of a Timoshenkos beam model which is close to the two-dimensional theory and is of practical importance for composite structures. The proposed model is nonlinear and accounts the shear deformation, rotary inertia and furthermore, the geometrically nonlinear longitudinal displacements which are a source of nonlinearity. This extended mathematical model of the Timoshenko beam has the form of four partial differential equations. The full coupled problem is solved considering a heat impact and transient response of the structure. In next step the problem is reduced to a steady state in order to check the influence of ambient temperature on steady state oscillations.

2. Mathematical model

The model of the geometrically nonlinear version of the Timoshenko beam theory is used to describe the beam motion. The equations describing the coupled thermo-elastic vibrations of beam with a rectangular cross-section and the heat propagation can be expressed by the following equations [3,4]

$$\begin{aligned}
&\frac{c_p}{\lambda_T}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} - \frac{\alpha_T E T_0}{\lambda_T}\left[-z\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial x}\right) + \alpha_T\frac{\partial T}{\partial t} + \frac{\partial w}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial x}\right)\right]\\
&EF\frac{\partial^2 u}{\partial x^2} = -EF\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + Eb\alpha_T\frac{\partial \gamma_T}{\partial x}\\
&EI\frac{\partial^2 \psi}{\partial x^2} - kGF\left(\frac{\partial w}{\partial x} - \psi\right) + \alpha_T bE\frac{\partial \chi_T}{\partial x} - c_2\frac{\partial \psi}{\partial t} - \rho F\frac{\partial^2 \psi}{\partial t^2} = 0\\
&kGF\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \psi}{\partial x}\right) + EF\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \alpha_T h^{-1}\gamma_T\right]\frac{\partial^2 w}{\partial x^2} - c_1\frac{\partial w}{\partial t} - \rho F\frac{\partial^2 w}{\partial t^2} = p(x,t)
\end{aligned} \tag{1}$$

where $\chi_T = \int_{-h/2}^{h/2} T(x,z,t)\, z dz, \quad \gamma_T = \int\limits_{-h/2}^{h/2} T(x,z,t) dz.$

In above equations E is the Young modulus and G is the shear modulus and α_T is the coefficient of thermal expansion. $F = bh$ is the area of the beam cross-section, $I = bh^3/12$, l is the beam length, $T(x,y,z)$ is current temperature, T_0 is the initial constant temperature, λ_T is the thermal conductivity and c_p is the heat capacity per unit volume, $w(x,t)$ is the transverse displacement, $\psi(x,t)$ is the rotation angle. c_1 and c_2 are damping coefficients which are assumed to be proportional to the mass terms ρF and ρI, respectively. Coefficient k is the shear correction factor. In the mathematical model it is accepted that longitudinal inertia effect can be neglected.

Assuming that the heat flow $q(x,t)$ acts on the upper beam surface, and the lower surface and the edges of the beam are subjected to convective heating (cooling) the boundary conditions for the equation describing the heat propagation are:

$$\begin{aligned}
&\frac{\partial T}{\partial z} = \begin{cases} -\lambda_T^{-1} q(x,t) & if \quad t \le t_p \\ d_t(T_e - T) & if \quad t > t_p \end{cases} \qquad for\, z = h/2\\
&\frac{\partial T}{\partial z} = \lambda_T^{-1} d_t(T_e - T) \;\; for: \;\; z = -h/2,\; x = 0,\; x = l
\end{aligned} \tag{2}$$

where d_t is the heat transfer characteristic. When d_t is equal to zero the surface is heat isolated and, when d_t tends to infinity the temperature of the surface gets instantly value T_0.

In the analysis we assume boundary conditions for:

- the clamped beam

$$u(0,t) = u(l,t) = w(0,t) = w(l,t) = 0 \text{ and } \frac{\partial \psi(0,\mathrm{t})}{\partial \mathrm{x}} = \frac{\partial \psi(\mathrm{l},\mathrm{t})}{\partial \mathrm{x}} = 0\,,$$

- the simply supported beam

$$w(x,0) = 0,\; \dot{w}(x,0) = 0,\; \psi(x,0) = 0,\; \dot{\psi}(x,0) = 0,\; T(x,z,0) = T_e,\; x \in [0,l],\; z \in [-h/2, h/2]\,,$$

and initial conditions defined as

$$w(x,0)=0,\ \dot{w}(x,0)=0,\ \psi(x,0)=0,\ \dot{\psi}(x,0)=0,\ T(x,z,0)=T_e,\ x\in[0,l],\ z\in[-h/2,h/2]\ .$$

3. Governing equations

The dimensionless variables are used in the further analysis and model reduction. We introduce dimensionless variables:

$$\bar{w}=w/l\,,\quad \bar{u}=u/l\,,\quad \theta=(T-T_0)/T_0\,,\quad \bar{x}=x/l\,,\quad \bar{z}=z/h\,,\quad \bar{t}=tc/l\,,\quad c^2=E/\rho\,.$$

Thus, transverse vibrations of the beam can be written in the following form:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2}=G_u+G_u^T\\
&\frac{\partial^2\psi}{\partial x^2}+\beta\alpha\left(\frac{\partial w}{\partial x}-\psi\right)-d_2\frac{\partial\psi}{\partial t}-\frac{\partial^2\psi}{\partial t^2}=G_1^T,\\
&\beta\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial\psi}{\partial x}\right)-d_1\frac{\partial w}{\partial t}-\frac{\partial^2 w}{\partial t^2}=-\overline{p}+G_2^T+G_2^L
\end{aligned}\tag{3}$$

where $\overline{p}=pl/EF$, $\alpha=12l^2/h^2$, $\beta=kG/E$, $d_1=c_1l^2/EF$, $d_2=c_2l^2/EI$, $G_u=-\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}$, $G_u^T=\alpha_T T_0\frac{\partial\gamma_\theta}{\partial x}$ and by G_2^L, G_1^T and G_2^T denote the components of the vectors $\mathbf{G}^L(0,G_2^L)$ and $\mathbf{G}^T(G_2^L,G_1^T)$ which may be called nonlinear force vectors due to the finite displacements and non-uniform temperature distribution. They have the form:

$$\begin{aligned}
&G_2^L(x,t)=-\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\frac{\partial^2 w}{\partial x^2}\\
&G_1^T=\alpha_T T_0\alpha\frac{\partial\chi_\theta}{\partial x}\,,\ G_2^T=\alpha_T T_0\gamma_\theta\frac{\partial^2 w}{\partial x^2}\,,\ \chi_\theta=\int_{-1/2}^{1/2}\theta(x,z,t)z\,dz\,,\ \gamma_\theta=\int_{-1/2}^{1/2}\theta(x,z,t)\,dz
\end{aligned}\tag{4}$$

The approach proposed is based on the successive solution of the equations for the mechanical vibrations of the beam and the heat transfer. The equation of the heat propagation is discretized by a finite difference method and for the equations representing mechanical vibrations a modal coordinate transformation is applied. For details of the applied algorithm one can refer to [3].

4. Numerical sulution of a coupled problem

A beam made of composite material is studied numerically in order to check the applied approaches and to estimate the influence of the geometrical nonlinearities, temperature and mechanical loading parameters on the response of the beam. Numerical calculations are performed for a clamped symmetric cross–ply laminated composite beam composed of 20 layers $[(0/90)_{10}]_S$ with physical data presented in Table 1.

For the case of the coupled thermo-mechanical vibrations of the beam the mechanical load is considered to be uniformly distributed along the beam length and it acts according to the law $p=p_0 sin(\omega_e t)$ where ω_e denotes the excitation frequency. It is accepted that the frequency of excitation is equal to the first natural frequency of the beam, i.e. $\omega_e=\omega_1$.

The heat pulse acting on the upper beam surface is supposed to be distributed along the beam length according to the sinus rule and its amplitude follows rule:

$$\begin{aligned}
&q(x,t)=\begin{cases}q_0\,(1-t/t_0)\sin(\pi x/l) & \text{for}\quad 0\le t\le t_0\\ 0 & \text{for}\quad t>t_0\end{cases}\\
&q(x,t)=\begin{cases}2q_0\,(t/t_0)\sin(\pi x/l) & \text{for}\quad 0\le t\le t_0/2\\ 2q_0\,(1-t/t_0)\sin(\pi x/l) & \text{for}\quad t_q/2\le t\le t_0\\ 0 & \text{for}\quad t>t_0\end{cases}\\
&q(x,t)=\begin{cases}q_0\sin(\pi x/l) & \text{for}\quad 0\le t\le t_0\\ 0 & \text{for}\quad t>t_0\end{cases}
\end{aligned}\tag{5}$$

Table 1. Physical parameters of the symmetric cross-ply laminated beam.

Physical parameter	Value (unit)
Length	$l = 0.5\ (m)$
1 layer thickness	$h_i = 0.25\ (mm)$
beam thickness	$h = 0.005\ (mm)$
Youngs moduli	$E_1 = 56, E_2 = 16\ (GPa)$
Effective Youngs modul	$E_{ef} = 41.92\ (GPa)$
Poissons ratio	$\nu = 0.269\ (-)$
Coefficient of thermal expansion	$\alpha_T = 13.2 \times 10^{-6}\ (K^{-1})$
Heat capacity per unit volume	$c_p = 2484000\ (N/m^2/K)$
Thermal conductivity	$\lambda_T = 207\ (N/s/K)$
Mass density	$\rho = 2052\ (kg/m^3)$

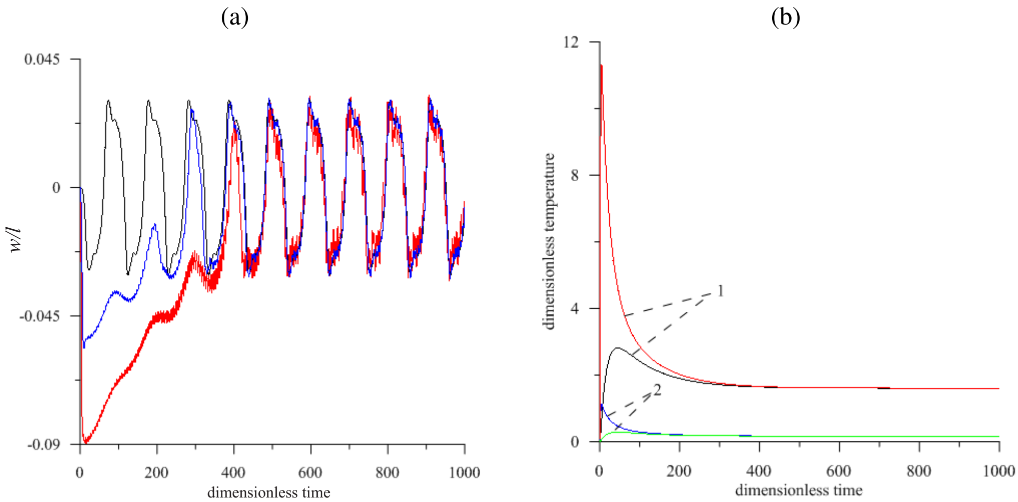

Fig. 1. (a) Time history diagram of the response of the beam centre for different heat impacts; $p = 0.1 \times 10^5\ N/m$, $\bar{\omega}_e = 0.06$, $\bar{T}_{duration} = 18$, $q = 5 \times 10^4\ kW/m^2$ - red colour, $q = 1 \times 10^4\ kW/m^2$ - blue color, $q = 0$ - black colour. (b) The temperature propagation in time for two cases (upper and lower surface of the beam centre). 1- q=$5 \times 10^4\ kW/m^2$ - red colour, 2 - q=$1 \times 10^4\ kW/m^2$ - blue colour.

The left and right edges of the clamped beams are heat insulated and at the lower surface of the beam its supposed a convective heat exchange. Five layers along beam thickness are used to discretize the equation for the heat propagation in z direction.

The time-history diagram of the response of the beam centre subjected to harmonic loading with $p = 10^4\ N/m$ and $\omega_e = 542.38\ rad/s$ is shown in Fig. 1(a) (note that the first eigenfrequency of the beam is $\omega_1 = 583.44\ rad/s$). The beam is subjected to a very short pulse $\overline{T}_{duration} = 18$. The pulse in the considered case follows the low given with Eqn. 5a. The response of unheated beam is plotted with black colour. As can be seen the heat pulse lead to an essential increasing of the amplitude of the vibrations. Furthermore, due to the heat impact the beam buckles. It takes 10 times of the heat impacts duration the beam to return to regular vibrations due to the mechanical loading in the case of $q = 1 \times 10^4\ \ kW/m^2$ and more than 20 times $\overline{T}_{duration}$ in case of $q = 5 \times 10^4\ \ kW/m^2$.

The temperature propagation in time at two adjacent layers along the beam thickness: $z = h/2$ and $z = h/4$ is shown in Fig. 1(b). In case $q = 1 \times 10^4\ kW/m^2$ the temperatures at these layers equalize for $\bar{t} = 100$ and for the case $q = 5 \times 10^4\ kW/m^2$ at $\bar{t} = 300$.

The responses of the beam subjected to mechanical harmonic loading and three heat pulses with three different durations are presented in Fig. 2(a). Additionally the response of the unheated beam is also plotted in this figure. It is seen that for the selected value of q, the very short duration of the heat impact does not influence essentially the response. The amplitude of the vibration increases during the heat impact but then the beam continues to vibrate as an unheated beam. For the higher values of the duration, the elevated temperature manages to propagate along larger

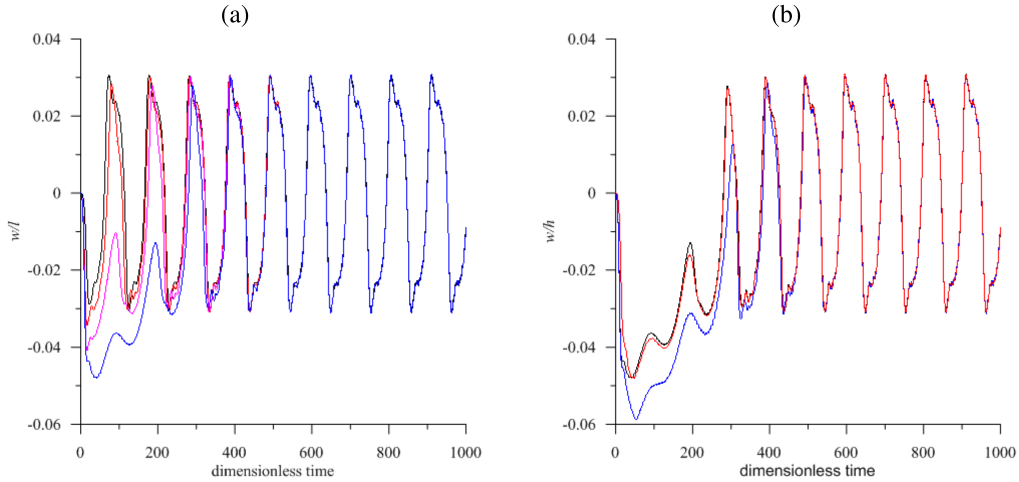

Fig. 2. (a) Influence of the duration of the pulse heating; black colour no pulse heating, red colour $\overline{T}_{duration} = 18$, magenta $\overline{T}_{duration} = 54$, blue colour - $\overline{T}_{duration} = 150$. $q = 0.1 \times 10^4\ kW/m^2$. (b) Influence of the pulse shape of the thermal impact on the response; black color decreasing triangular pulse (Eqn. 5a), red color - isosceles triangular pulse (Eqn. 5b), blue color rectangular pulse (Eqn. 5c); $\overline{T}_{duration} = 150$, $q = 0.1 \times 10^4\ kW/m^2$.

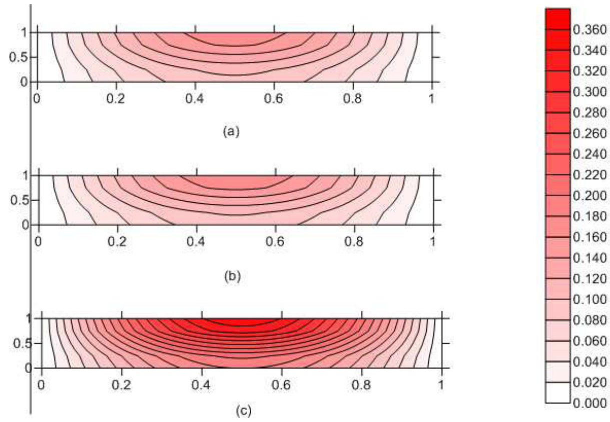

Fig. 3. Temperature distribution at $\bar{t} = 200$ for decreasing triangular (a), isosceles triangular (b) and rectangular pulses (c).

parts of the beam and the amplitudes become larger, the beam buckles and return to the original equilibrium state after longer time.

The shape of the heat pulse may influence the response of the beam as well (Fig. 2b). The areas which the heat pulse formed in the case of pulse (5a) and (5b) are equal and, because of this, the responses of the beam in these two cases are very similar. The pulse with rectangular shape according to Eqn.(5c) forms a larger area and this naturally leads to vibrations with larger amplitudes and longer time necessary for the beam to return to vibration around the original unheated equilibrium state.

The distribution of the temperature field at particular moment of time for the same cases is shown in Fig. 3. The usage of same colour scale for the three contour maps clearly shows that the pulse (5c) leads to higher temperatures along bigger parts of the beam.

5. Steady state vibrations in elevated temperature

Assuming that the distribution of temperature along x and z axis is constant: $\theta(x, z) = const.$, we get: $\chi_\Theta = 0$, $\gamma_\Theta = \Delta T$, where ΔT is a difference between the reference temperature and the current temperature. According to papers [5,8] inertia of the longitudinal displacement can be neglected and the model can be simplified to two partial differential equations having the dimensionless form:

$$\begin{aligned} &\frac{\partial^2\psi}{\partial x^2} + \beta\alpha\left(\frac{\partial w}{\partial x} - \psi\right) - d_2\frac{\partial\psi}{\partial t} - \frac{\partial^2\psi}{\partial t^2} = 0 \\ &\beta\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial\psi}{\partial x}\right) - d_1\frac{\partial w}{\partial t} - \frac{\partial^2 w}{\partial t^2} = -p + G_2^T + G_2^L \end{aligned} \tag{6}$$

Applying the Galerkin procedure the set of PDEs is reduced into ODEs. The vibration modes are taken from linear eigenvalue problem which, in the studied case, is solved for a simply supported beam. Assuming solution in a series of product of time and space dependent functions

$$w(x,t) = \sum_{n=1}^{N_f} w_n(x)q_n(t), \quad \psi(x,t) = \sum_{n=1}^{N_f} \psi_n(x)q_n(t) \tag{7}$$

next substituting Eqn. (7) into (6), multiplying each of equations by a proper mode, adding them by sides and integrating over the beam length we get a set of nonlinear ODEs

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \int_0^1 \left[p(x,t)w_n(x) - G_2^L(x,t)w_n(x) - G_2^T(x,t)w_n\right] dx\,. \tag{8}$$

Functions included on the right hand side are defined as

$$\begin{aligned} &G_2^L = K\sum_{n=1}^{N_f} q_n(t)\frac{d^2 w_n}{dx^2}, \quad G_2^T = \alpha_T\Delta T\sum_{n=1}^{N_f} q_n(t)\frac{d^2 w_n}{dx^2}, \\ &K = -\int_0^1 \sum_{n=1}^{N_f}\sum_{j=1}^{N_f} \xi q_n\frac{dw_n(\xi)}{d\xi} q_j\frac{d^2 w_j(\xi)}{d\xi^2} d\xi, \quad p = p(x,t)\;. \end{aligned} \tag{9}$$

In the further analysis we take just one mode reduction therefore, in such a case we get one nonlinear ODE

$$\ddot{q}_1 + 2\xi_1\omega_1\dot{q}_1 + \omega_1^2 q_1 + C_{1,111}q_1^3 + C_1^T\Delta T q_1 = p_1\sin\omega t\,. \tag{10}$$

6. Efect of elevated temperature

Numerical calculations are performed for a symmetric cross-ply laminated composite beam composed of 20 layers $[(0/90)_{10}]_S$ with physical data presented in Table 1. Applying the procedure described in previous chapter we obtain dimensionless coefficients of nonlinear equation (10)

$$C_{1,111} = 0.412607,\; C_1^T = -1.19911\times10^{-5},\; \omega_1 = 0.0284864,\; \zeta_1 = 0.017552 \tag{11}$$

Amplitude p_1 and frequency ω of external load are varied in order to demonstrate essential nonlinear phenomena around the first resonance zone.

The resonance curves corresponding to steady state solutions calculated for two different ambient temperatures $\Delta T = 20$ (black line) and $\Delta T = 50$ (red line) are presented in Fig. 4. Mechanical loading is fixed as $p_1 = 2\times10^{-6}$ and excitation frequency varied (the first dimensionless natural frequency equals $\omega = 0.0284864$). We may notice the nonlinear stiffening effect and that the elevated temperature has increased amplitudes and shifted the resonance curve into the lower frequencies direction Fig. 4(a). Furthermore, the small peak on the resonance curve occurred for very low frequencies. The effect of temperature for fixed frequency $\omega = 0.029$ and varied amplitude p_1 is presented in Fig. 4(b). Again nonlinear phenomenon is observed by existence of multi-solution region with unstable branches (dashed line). Again the increased temperature (red line) increases the amplitude of vibrations and also enlarges the zone of triple solution. It is worth noting that above certain threshold about $p_1 = 1\times10^{-4}$ the solution becomes instable (dashed line).

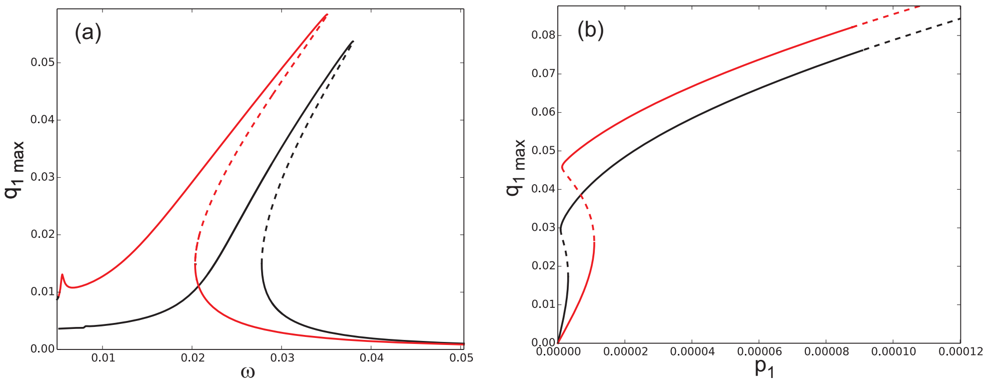

Fig. 4. (a) Resonance curves around the first natural frequency for excitation amplitude $p_1 = 2 \times 10^{-6}$ and (b) response of the beam against amplitude p_1 and fixed excitation frequency $\omega = 0.029$ (b) computed for $\Delta T = 20$ - black line, $\Delta T = 50$ - red line.

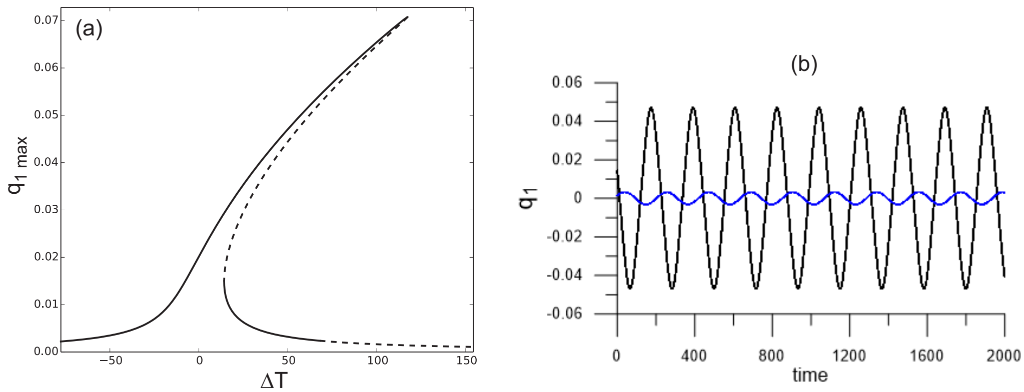

Fig. 5. (a) Response of the beam against temperature and fixed mechanical excitation with amplitude $p_1 = 2 \times 10^{-6}$ and frequency $\omega = 0.029$; (b) time histories for $\Delta T = 50$ and various initial conditions

The influence of the beam response against the temperature, varied from about $\Delta T = -100$ till $\Delta T = +150$ is presented in Fig. 5. The shape of the curve reminds a shape of a resonance curve with the essential difference that the lower branch above temperature $\Delta T = 75$ becomes unstable. The elevated temperature may increase vibration amplitude or may transits the system from single solution to multi-solution zone. Such a case we can observe, for example, if $\Delta T = 50$ - we get three solutions, with two of them stable. Time histories of small amplitude (in blue) and large amplitude (in back) vibrations are presented in Fig. 5(b).

The increased ambient temperature together with mechanical loading may change the beam dynamics drastically. Such a scenario is presented in bifurcation diagram versus amplitude of excitation p_1 and elevated temperature $\Delta T = 100$. This diagram is obtained by direct numerical simulation of Eqn. (10) for seven various initial conditions. The transient response above 500 periods has been rejected. We see that the response of the beam is complex with two zones (black regions) in which oscillations have complex nature. This irregular dynamics and transition to it will be discussed in a separate work.

7. Conclusions

The large amplitude vibrations of moderately thick beams subjected to mechanical load and thermal flow acting for a short period on the upper surface of the structures are studied in this paper, considering the coupled thermo-mechanical model. The case when the mechanically loaded beam is at elevated temperature, without considering the heat propagation, is studied separately. The results obtained show that thermal loads may change significantly the structures dynamic behavior because they introduce stresses due to thermal expansion. The obtained results from

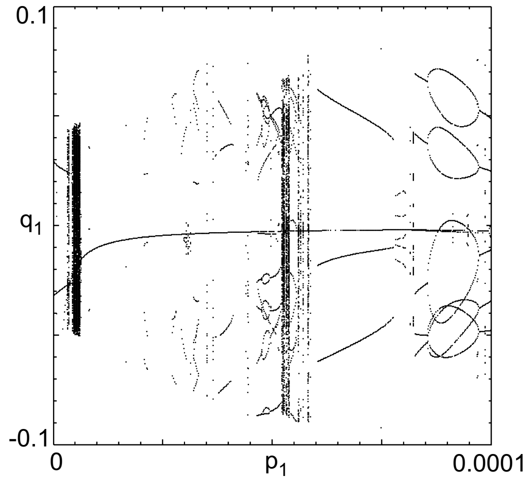

Fig. 6. Bifurcation diagram of the beam displacement against amplitude of periodic excitation p_1 with fixed frequency $\omega = 0.029$ and elevated temperature $\Delta T = 100$.

coupled thermomechanical equations are very different from the case when the structure is considered instantly heated. Due to the in-plane forces introduced by the thermal terms in the equations, a beam can buckle and vibrate around a new equilibrium state, changing dramatically beam response. The elevated temperature may transit the system from stable to unstable zones or instead of single solution we can get multi solutions with small and large amplitudes.

These analyses are expected to have an application for improving the modeling of structures working in heavy environment conditions in many high technological areas. They can be implemented in the structural design in aerospace or civil engineering, machinery, high speed vehicles, etc.

Acknowledgements

The first author would like to acknowledge the partial support received from the Polish NSF Grant S-38/M/2015, the second author the partial support from Bulgarian NSF Grant DUNK-01/3 and the third author the financial support from grant DEC-2012/07/B/ST8/03931 from the Polish National Science Centre.

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