Calculation of high-temperature insulation parameters and heat transfer behaviors of multilayer insulation by inverse problems method

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Abstract In the present paper, a numerical model combining radiation and conduction for porous materials is developed based on the finite volume method. The model can be used to investigate high-temperature thermal insulations which are widely used in metallic thermal protection systems on reusable launch vehicles and high-temperature fuel cells. The effective thermal conductivities (ECTs) which are measured experimentally can hardly be used separately to analyze the heat transfer behaviors of conduction and radiation for high-temperature insulation. By fitting the effective thermal conductivities with experimental data, the equivalent radiation transmittance, absorptivity and reflectivity, as well as a linear function to describe the relationship between temperature and conductivity can be estimated by an inverse problems method. The deviation between the calculated and measured effective thermal conductivities is less than 4%. Using the material parameters so obtained for conduction and radiation, the heat transfer process in multilayer thermal insulation (MTI) is calculated and the deviation between the calculated and the measured transient temperatures at a certain depth in the multilayer thermal insulation is less than 6.5%.

1. Introduction

Thermal insulation is a subject of great interest to the new generation of reusable launch vehicles and thermal protection systems, 1 which can sustain severe heating during the process of aerodynamic reentry when the surface temperature is high. The heat transfer inside thermal insulators is composed of conduction, natural convection and radiation. Convection can be neglected 2,3 in porous media at high temperatures and
atmospheric pressure. Previous studies show that radiation is the dominant mode of heat transfer when the temperature is higher than 573 K. Heat transfer through fibrous media has displayed that radiation accounts for 40%-50% of the total heat transfer inside light-weight fibrous thermal insulations at moderate temperatures. The complex coupling of conductivity, convection and radiation, especially the radiation, makes the analysis and the design of thermal insulations difficult.

Heat transfer through fibrous and multilayer insulations has been investigated by various researchers, both experimentally and analytically during the last 30 years. Two main problems exist in the characterization of the thermal insulations. As the effective thermal conductivity (ETC) measured by experimental apparatus cannot be directly used to analyze the thermal behavior of high-temperature insulation, parameters measured need to be separated into two parts: radiation and thermal conductivity parameters. And the low value of effective thermal conductivity makes it difficult to obtain thermal conductivity at a certain temperature. Lee and Cunnington provided a comprehensive review of heat transfer in generally porous materials. Walter et al.7 used the first-principle approach to analyze combined heat transfer in a highly porous silica insulation material called LI900. The two flux approximation was frequently used to describe the thermal behavior of fibrous insulation. Tong et al.9,9 used the two flux model which assumed a linearized anisotropic scattering to model heat transfer inside fibrous thermal insulation and compared the calculated values with experimental data up to 450 K at 105 Pa. Zhao et al.10–12 used the two flux model that assumed a modified factor of extinction and an equivalent albedo of scattering to model the heat transfer of fibrous insulation used in thermal protection. Daryabeigi modeled heat transfer in aluminia fibrous insulation to predict effective thermal conductivities at gas pressures 10–2 Pa and 10–5 Pa and at temperatures up to 1273 K. The model was based on a modified two flux approximation assuming anisotropic scattering and gray medium. The two flux approximation contained such parameters as the specific extinction coefficient, albedo of scattering, backscattering fraction, solid conduction exponent term, etc. Generally, these parameters were obtained by experimental and numerical methods. For multilayer thermal insulations (MTIs), Spinnler et al.4,14 modeled heat transfer in multilayer thermal insulations using a radiation scaling method. The model was used to calculate the effective thermal conductivities at temperatures between 473 K and 1273 K and make optimization of multilayer thermal insulation. Li and Cheng developed a model using an energy balance equation, which was concerned with radiation emissivity, perforation coefficient and neglected the radiation flux in spacer materials, for steady temperature calculation of the insulation layer in a multilayer perforated insulation material at 300 K and made optimum design of the multilayer perforated insulation material.

In this work, we established a model based on the finite volume method to investigate the thermal behaviors of both the fibrous insulation and multilayer thermal insulation, which contains the following thermal parameters: the effective conductivity of the solid and gas, equivalent radiation transmittance, absorptivity and reflectivity. These thermal parameters can be used to study the thermal behavior of high-temperature insulations, and to solve the difficulty of analyzing thermal behavior with effective thermal conductivity. The relationship between temperature and thermal conductivity of the solid and gas was also investigated to calculate the conductivity at a certain temperature. Experimental and numerical methods were combined to optimize the parameters of high-temperature insulation by an inverse problems method. Finally, the calculated results were compared with the measured data under different conditions.

2. Experimental apparatus

In this work, the effective thermal conductivity of thermal insulation was measured according to YB/T4130-2005 in Luoyang Institute of Refractories Research, China. The effective thermal conductivity was measured under atmospheric pressure with the nominal hot temperatures set at 473 K, 573 K, 673 K, 773 K, 873 K, 973 K, 1073 K, 1173 K, 1273 K, 1373 K and 1473 K. The uncertainty of the measured effective thermal conductivity was approximately 8%.

In order to study the transient thermal behavior of multilayer thermal insulation, a graphite heater (20 WM electric arc wind tunnel) was used to provide a time-dependent surface temperature. The multilayer thermal insulation was composed of 5 mm calcium silicate and aluminum silicate fibers (CA) and 10 mm nanoporous silica fibers (NP2) with carbon screens per 2 mm. The multilayer thermal insulation was placed between a septum plate and a water-cooled plate. Thermocouples were used to measure the front and back surface transient temperatures as well as the internal temperature responses of multilayer thermal insulation at the depth of 5 mm as shown in Section 6.

3. Theoretical analysis

Heat transfer through high-temperature insulation is composed of conduction, natural convection and radiation. Stark and Frick pointed out that natural convection heat transfer can be neglected through porous media. Therefore, the governing conservation of the energy equation for one dimensional heat transfer inside thermal insulations which combines conduction and radiation can be described as:

\[ \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) - q_r \]

subject to the following initial and boundary conditions:

\[
\begin{align*}
T(x, 0) &= T_0(x) \\
T(0, t) &= T_1(t) \\
T(L, t) &= T_2(t)
\end{align*}
\]

where \( \rho \) is density; \( c \) is specific heat capacity; \( k_c \) is the effective conductivity of the solid and gas; \( q_r \) is the radiation heat flux;
The finite volume method is used to solve the governing conservation of the energy equation. This means that the thermal insulation material is divided into \( N \) thin, gray, isothermal layers. For layer \( i \), the energy equation is composed of six parts (see Fig. 1): \( q_{le}^i, q_{re}^i \) are the heat fluxes conducted from \( i - 1 \) direction and \( i + 1 \) direction, respectively; \( q_{fe}^i, q_{ge}^i \) are the radiation heat fluxes emitting from the two sides of layer \( i \); \( e_i G_e^i, e_r G_r^i \) are the radiation heat transfer absorbed from \( i - 1 \) direction and \( i + 1 \) direction, respectively.

The above heat fluxes can be described based on Fourier’s law and radiation law:

\[
\begin{align*}
q_{le}^i &= \lambda_{i+1} \frac{T_i - T_{i+1}}{\Delta x} \\
q_{re}^i &= \lambda_{i-1} \frac{T_{i-1} - T_i}{\Delta x} \\
q_{fe}^i &= q_{fe}^i = e_i \sigma(T_i^4)
\end{align*}
\]

where \( \lambda_i \) is the mean thermal conductivity of the solid and gas between layer \( i \) and \( i + 1 \); \( \lambda_{i+1} \) is the mean thermal conductivity of the solid and gas between layer \( i + 1 \) and \( i + 2 \); \( \lambda_{i-1} \) is the mean thermal conductivity of the solid and gas between layer \( i \) and \( i - 1 \); \( T_{i+1}, T_i, T_{i-1} \) are the temperature values of the corresponding layers, respectively; \( \Delta x \) is the distance between the corresponding layers; \( e_i \) is the emissivity/absorptivity of layer \( i \); \( \sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4) \) is Stefan–Boltzmann’s constant.

Here, the mean thermal conductivity of the solid and gas between layers can be given as:

\[
\begin{align*}
\lambda_{i+1} &= \frac{2\lambda_i \lambda_{i+1}}{\lambda_i + \lambda_{i+1}} \\
\lambda_{i-1} &= \frac{2\lambda_i \lambda_{i-1}}{\lambda_i + \lambda_{i-1}}
\end{align*}
\]

where \( \lambda_i \) is the thermal conductivity of layer \( i \); \( \lambda_{i+1} \) is the thermal conductivity of layer \( i + 1 \); \( \lambda_{i-1} \) is the thermal conductivity of layer \( i - 1 \).

For a steady stability balance:

\[
q_{le}^i - q_{re}^i + e_i (G_e^i + G_r^i) - q_{fe}^i - q_{ge}^i = 0
\]

For a transient heat transfer process, the energy variation during \( \Delta \tau \) s can be described as:

\[
q_{le}^i - q_{re}^i + e_i (G_e^i + G_r^i) - q_{fe}^i - q_{ge}^i = \Delta \tau \rho_i c_i (T_{i+1}^k - T_i^k)
\]

where superscript \( k \) refers to time \( k \) s; \( T_{i+1}^k, T_i^k \) are the temperature values of layer \( i \) at time \( k \) s; \( \rho_i \) is the density of layer \( i \); \( c_i \) is the specific heat capacity of layer \( i \). The transient temperature field of thermal insulation can be calculated based on the energy equation of transient heat transfer above.

The densities of incident forward radiant \( (G_e^i) \) and backward radiant \( (G_r^i) \) fluxes are governed by the following relationships:

\[
\begin{align*}
G_e^{i+} &= \theta_i e_i \sigma(T_{i+1}^k)^4 + \gamma_i G_r^{i-} \\
G_e^{i-} &= \theta_i e_i \sigma(T_{i-1}^k)^4 + \gamma_i G_r^{i+}
\end{align*}
\]

where \( \theta, e, \gamma \) stand for radiation transmittance, emissivity/absorptivity and reflectivity, respectively.

The forward radiation flux \( (G_e^i) \) is composed of three parts: the forward radiation flux of layer \( i - 1 \) passing through layer \( i \) and \( i + 1 \), the radiation flux emitted from layer \( i - 1 \), the backward radiation flux of layer \( i - 1 \) reflected by layer \( i - 1 \). Similarly, the backward radiation flux \( (G_r^i) \) is composed of three parts: the backward radiation flux of layer \( i + 1 \) passing through layer \( i + 1 \), the radiation emitted from layer \( i + 1 \), the forward radiation flux of layer \( i + 1 \) reflected by layer \( i + 1 \).

Assuming that the bounding solid surfaces are emitting/reflecting surfaces, the radiant boundary conditions are:

\[
\begin{align*}
G_e^{k+} &= e_i \sigma(T_1^k)^4 + (1 - e_i) G_r^{k-} \\
G_e^{k-} &= e_i \sigma(T_L^k)^4 + (1 - e_i) G_r^{k+}
\end{align*}
\]

Matrix equations made by the equations for the densities of incident radiant fluxes Eq. (7) and the boundaries conditions Eq. (8) can be given below:

\[
\begin{bmatrix}
\theta_1 e_1 \sigma(T_1^k)^4 + \gamma_1 G_r^{k-} & \theta_1 e_1 \sigma(T_1^k)^4 + \gamma_1 G_r^{k-} & \ldots & \theta_1 e_1 \sigma(T_1^k)^4 + \gamma_1 G_r^{k+} \\
\theta_2 e_2 \sigma(T_2^k)^4 + \gamma_2 G_r^{k-} & \theta_2 e_2 \sigma(T_2^k)^4 + \gamma_2 G_r^{k-} & \ldots & \theta_2 e_2 \sigma(T_2^k)^4 + \gamma_2 G_r^{k+} \\
& \ldots & \ldots & \ldots \\
\theta_n e_n \sigma(T_n^k)^4 + \gamma_n G_r^{k-} & \theta_n e_n \sigma(T_n^k)^4 + \gamma_n G_r^{k-} & \ldots & \theta_n e_n \sigma(T_n^k)^4 + \gamma_n G_r^{k+}
\end{bmatrix}
\]

Eqs. (5) and (9) are used to calculate the steady stability of high-temperature insulation, while Eqs. (6) and (9) are used to calculate the transient temperature distribution of the high-temperature insulation. To solve the steady heat transfer process, an initial temperature distribution is given which can be used to calculate incident radiation fluxes based on Eq. (9). Eq. (5) is solved only after the temperature at each volume element remains unchanged with successive iterations and the steady-state conditions are achieved. Eq. (6) is developed to solve the transient heat transfer conditions. Similarly, an initial temperature distribution at time \( 0 \) s needs to be given which can be used to calculate incident radiation fluxes based on Eq. (9), and then temperature distributions at time \( 1 \) s, \( 2 \) s, \ldots, \( k \) s can be calculated. Particularly, when the analytical model is used for multilayer insulations, the reflectivity screen between space materials must be divided into a separate layer.

### 4. Parameter estimation

The effective conductivity of the solid and gas, radiation transmittance, radiation absorptivity and radiation reflectivity are
not known and are estimated using parameter estimation techniques to obtain equivalent parameters. The strategy function is the effective thermal conductivities based on the least-squares minimization of the difference between the measured and predicted values for fibrous samples.

\[ S = \sum_{i=1}^{n} \left[ k_{em}(i) - k_{ep}(\epsilon_i, \gamma_i, \alpha_i) \right]^2 \]  

(10)

where \( k_{em} \) is the measured effective conductivity; \( k_{ep} \) is the predicted effective conductivity.

Eq. (10) is subject to the following physical constraints:

\[
\begin{align*}
0 < \epsilon &< 1 \\
0 < \gamma &< 1 \\
1 - \epsilon - \gamma &> 0
\end{align*}
\]

(11)

Effective thermal conductivities for the fibrous thermal insulation samples (see Fig. 2) are: Sample 1 aluminum silicate fibers (AS) with the density of 190 kg/m\(^3\) and the thickness of 12.10 mm, Sample 2 CA with the density of 310 kg/m\(^3\) and the thickness of 10.47 mm, Sample 3 NP2 with the density of 380 kg/m\(^3\) and the thickness of 8.53 mm were measured based on a water-cooled plate thermal conductivity apparatus as mentioned in Section 2. Fig. 2 shows the relationship between the effective thermal conductivity and the average temperature for the three samples.

It is assumed that the equivalent radiation transmittance, absorptivity and reflectivity are independent of temperature, while the conductivity of the gas and solid is a linear function of temperature:

\[ k_c = k_{c1} + k_{c2} T \]  

(12)

The radiation transmittance of fibrous insulation is \( \theta = 1 - \epsilon - \gamma \). Therefore a total of four parameters (\( \epsilon, k_{c1}, k_{c2}, \gamma \)) need to be estimated.

The MATLAB optimization toolbox is used to solve the least-squares minimization problem and the results of the least-squares minimization parameter estimation are discussed below. For Sample 1, the estimated equivalent radiation absorptivity/emissivity is 0.64, the equivalent radiation transmittance is 0.27, the equivalent radiation reflectivity is 0.09 and the equivalent conductivity of the gas and solid is \( k_c = 1.12 \times 10^{-2} + 4.27 \times 10^{-5} T \). For Sample 2, the estimated equivalent radiation absorptivity/emissivity is 0.75, the equivalent radiation transmittance is 0.23, the equivalent radiation reflectivity is 0.01 and the equivalent conductivity of the gas and solid is \( k_c = 2.75 \times 10^{-2} + 1.89 \times 10^{-5} T \). For Sample 3, the estimated equivalent radiation absorptivity/emissivity is 0.81, the radiation equivalent transmittance is 0.11, the equivalent radiation reflectivity is 0.08 and the equivalent conductivity of the gas and solid is \( k_c = 1.57 \times 10^{-2} + 5.02 \times 10^{-6} T \).

Therefore, the heat transfer process of thermal insulation can be determined by five equivalent parameters which can be used to calculate the conductivity of the solid and gas at a certain temperature and to investigate the transient process of high-temperature thermal insulation. Here, the linear function between conductivity and temperature is useful in evaluating the thermal behavior of thermal insulation. The evaluation of estimated parameters and the transient heat transfer through multilayer thermal insulation are discussed below.

5. Evaluation of the parameter estimation

For evaluating the numerical model, numerical predictions of effective thermal conductivities based on the estimated parameters are compared with experimentally measured effective conductivities. The comparisons between the measured and predicted values are discussed below (see Fig. 3).

Figs. 3(a)–(c) show the effective thermal conductivity differences between the three predicted and measured samples. Clearly, the predicted effective thermal conductivity shows a good agreement with the measured result with the maximum deviations of less than 4% as shown in Fig. 3(d). Siegel and Howell reported that for optically dense \( \tau_s \geq 1 \) porous materials the effective thermal conductance can be expressed in terms of the local radiation temperature \( T_R \) as

\[ k_{eff} = k_c + \frac{16}{3} \frac{\bar{n}^2}{\beta} T_R^3 \]  

(13)

where \( \bar{n} \) is effective index of refraction; \( \beta \) is extinction coefficient. Therefore, the effective thermal conductivity can be analogously described as a linear function of \( T_R^3 \) and the same relationships are observed for both measured and predicted samples. Due to the fact that the predicted effective thermal conductivity can be expressed as a linear function of \( T_R^3 \) and agrees well with the measured effective thermal conductivity, the theoretical analysis mode can be used to solve the heat transfer inside thermal insulations. Therefore the assumption of equivalent conductivity of the gas and solid as a linear function of temperature and the equivalent radiation transmittance, absorptivity, reflectivity is reasonable.

6. Transient analysis for multilayer thermal insulation

Multilayer thermal insulation is composed of highly reflective radiation shields (screens), which can significantly reduce the radiation heat transfer at high temperatures, and high-temperature insulation materials as spacers. In the present study, a theoretical numerical model and estimated parameters are used to investigate the transient temperature variation in multilayer thermal insulation. Particularly, when the theoretical numerical model is used to study multilayer thermal insulation, the reflectivity screen between space materials must be divided into a separate layer. Here, the transient heat behavior of Sample 4 is discussed. Sample 4, which is composed of 5 mm CA and 10 mm NP2 with five carbon screens per 2 mm, is
tested during 0–220 s with variation temperatures at the top of CA. The temperature variation at the depth of 5 mm is measured. In order to solve the transient heat transfer conditions, Eqs. (6) and (9) are used to calculate the temperature variation at the depth of 5 mm (see Fig. 4).

Fig. 4 shows the temperature differences between the predicted temperature and the measured ones at the depth of 5 mm. Fig. 5 shows the deviation of the predicted temperature from the measured temperature against time. The deviation is from $\%6.5$ to $\%1.0$. After 100 s, the deviation is stable at near $\%5.5$. As seen from the figures above, the calculation model underestimates the temperature variation of multilayer thermal insulation within $\%6.5$. Clearly the measured temperature variation and the predicted values correspond well inside multilayer insulations. Therefore, the validity of the estimated equivalent parameters for high-temperature insulations can be confirmed and the calculation model can be used to investigate the thermal behaviors of multilayer thermal insulation.

7. Conclusion

This paper investigates a theoretical model combining radiation and conduction heat transfer to study the heat transfer behavior of high-temperature insulation based on the finite volume method. The theoretical model can be used for single fibrous thermal insulation and it may be used to investigate
the thermal behavior of multilayer thermal insulation. In this paper, we use the equivalent radiation transmittance, absorptivity, reflectivity to investigate the radiation heat transfer behavior and a linear function to describe the relationship between temperature and conductivity of the solid and gas. These parameters can be estimated by inverse problems method as mentioned above.

Comparisons, with deviations less than 4%, have been carried out between the predicted effective thermal conductivities and the measured ones in high-temperature insulation and the deviations of comparisons between the predicted transient temperature variations and the measured ones are less than 6.5% at the depth of 5 mm in multilayer thermal insulation. The results confirm the validity of the results and the good behavior of the theoretical model.

In future work, we decide to use the equivalent parameters to simulate the thermal behaviors of fibrous insulation and analyze the influence on the thermal behaviors by changing parameters. Also, our theoretical model provides a way to make optimization of multilayer thermal insulation. We decide to find out the best situation for the screens in multilayer thermal insulation to improve the thermal behaviors of the insulations and which kind of thermal insulation materials is suitable for being the space layers in multilayer thermal insulation.

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