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Geostatistical simulation of spatial variability of convective storms in Mexico City Valley

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Resumen

La precipitación es uno de los factores principales del ciclo hidrológico y el conocimiento de su distribución espacial es fundamental para la predicción de otras variables ambientales íntimamente relacionadas como son: el escurrimiento, las inundaciones, la recarga de los acuíferos. La mayor parte de la precipitación en la Ciudad de México es producida por tormentas convectivas, caracterizadas por una alta variabilidad espacial, lo cual implica que la modelación de su comportamiento sea muy compleja. En el presente estudio se aplicaron técnicas de simulación estocástica con enfoque geoestadístico para modelar la variabilidad espacial de la precipitación de tres tormentas convectivas. El análisis de los resultados muestra que usando la metodología propuesta se obtienen distribuciones espaciales de lluvia que reproducen las características estadísticas presentadas en la información disponible.

Palabras clave: geoestadística, variabilidad espacial de la precipitación, simulación secuencial Gaussiana, cosimulación, tormentas convectivas, radar meteorológico.

Abstract

Precipitation is one of the main components of the hydrological cycle and knowledge of its spatial distribution is fundamental for the prediction of other closely related environmental variables, for example, runoff, flooding and aquifer recharge. Most of the precipitation in Mexico City is due to convective storms characterized by a high spatial variability, implying that modeling its behavior is very complex. In this work stochastic simulation techniques with a geostatistical approach were applied to model the spatial variability of the rainfall of three convective storms. The analysis of the results shows that using the proposed methodology spatial distributions of rain are obtained that reproduce the statistical characteristics present in the available information.

Key words: geostatistics, rainfall spatial variability, sequential Gaussian simulation, cosimulation, convective storms, meteorological radar.

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Introduction

One of the most modern instruments to estimate rainfall is meteorological radar. It cover a large area (about 200 km in radius); although the estimates are not precise, due to inherent errors of the instrument itself: anomalous propagation, attenuation, etc.; to its surroundings: beam blocking due to mountains, false echoes, evaporation, etc. (Zawadzki, 1984); and to the estimation algorithms (Seo and Krajewski, 2011). The rain gauge has been the traditional instrument for rainfall estimation due to its good precision, though representativeness of its measurements is of a few meters around the instrument. Many countries of the world, to take advantage of both instruments, have systems that estimate rainfall based on a combination of meteorological radar and rain gauge estimates. However, rainfall estimation becomes very complicated when the spatial distribution is very variable, which is the case of convective or electrical storms. Various geostatistical techniques to estimate rainfall using Kalman filters (Anhert *et al*., 1986) or geostatistical estimation methods such as kriging (Krajewski, 1987) have been developed.

Since 1995 a network of 13 C-band Doppler meteorological radar equipment exists in Mexico (Valdés-Manzanilla and Aparicio, 1997). Their main objectives are to monitor the tropical cyclones in or near the Mexican national territory and to estimate rainfall with hydrological purposes. One of these radar stations is near the metropolitan area of Mexico City, at the top of Cerro de la Catedral. Also, a network of digital rain gauges with telemetry, owned by the government of Mexico City, covers much of the city area.

Because of that, Valdés-Manzanilla and Herrera (2002) designed a rainfall estimation method using both sources of meteorological information. A Kalman filter was used to calculate optimally, in real time, the mean error between rainfall estimated by radar and the one estimated by rain gauges. After applying this technique to two convective storms, the root mean square error was reduced by 1.3 and 1.9 mm during the entire storm.

Díaz-Viera, *et al*. (2009) explored different variants of kriging to estimate rainfall in the Mexico City metropolitan area using radar and rain gauge data. Their estimates obtained by cokriging with a model of linear corregionalization and collocated cokriging generated better estimates of the rainfall than obtained by ordinary kriging.

Becerra-Soriano (2009), in her master's thesis, continued these two investigations. Her objective was to evaluate the cokriging method for estimating rainfall combining radar and rain gauges measurements and using all radar images from two storms in the Mexico City area. As part of the assessment, a calculation of runoff volume was included, in order to estimate the water volume that would go into the Mexico City drainage system.

Geostatistical estimation techniques like kriging-the best unbiased linear estimators (Chilès and Delfiner, 1999)-may be optimal in the sense of minimizing the estimation error variance, but are strongly dependent on data quantity, spatial position and, the worst, they do not reproduce the spatial correlation. These techniques can generate unrealistic rainfall spatial distributions (Young, 2008; and Curtis and Clyde, 1999).

An alternative method for spatial estimation is a simulation approach, which, by definition, reproduces the statistical behavior of the phenomenon. Specifically, geostatistical simulation methods can generate multiple realizations that are statistically equivalent in terms of the first and second-order moments (Chilès and Delfiner, 1999). Here, the application of geostatistical simulation methods to model rainfall spatial variability is considered.

An antecedent to the present work is the master's thesis of Méndez-Venegas (2008), where he performed a simulation using only rain gauge data and a cosimulation using both rain gauge data and radar images for a single storm. The applied simulation method was sequential Gaussian (Alabert and Massonat, 1990). This paper is an extension of his work to a set of three convective storms in Mexico City.

Here, two simulations for each storm: a univariate simulation (Z^S) using only rain gauge data and a cosimulation (Z^{CS}) using rain gauge data and radar images are performed. To evaluate the results, their statistics were satisfactorily compared with those of the data.

Rain gage and radar image data

The radar data was obtained at the C-band Doppler meteorological radar station of the National Meteorological Service on Cerro de la Catedral, overlooking the metropolitan area of Mexico City (Figure 1). The radar images used are 8 bits images of 240 x 240 km with a resolution of 1 km^2. A pixel covers an area of 1 km x 1 km in a pseudo-CAPPI presentation at 4 km above sea level every fifteen minutes (Valdés-Manzanilla and Aparicio, 1997). The precipitation data is from 61 rain gauges of the Water System of Mexico City and radio reporting, every minute, of the accumulated rainfall during the storm. These rain gauges are of tipping-bucket kind with telemetry and a density of one rain gauge for every 30 km^2 (Díaz-Viera, *et al*., 2009).

The accumulated rainfall per hour for each type of measurement is calculated. The radar records an image with values of reflectivity (Z) every 15 minutes, these images to values of rain intensity (R) using a Z-R relationship are converted, subsequently four consecutive radar images are averaged for rainfall intensity to obtain effective cumulative rainfall in one hour. The relation $Z = 300R^{1.4}$ recommended by the manufacturer is used (Valdés-Manzanilla and Herrera- Zamarrón, 2000).

For rain gauge data digital files for the date and time of the storm are used. Each rain gauge has a counter that is incremented by one each time it registers a shower of 1/4 mm (Rosengaus, 2000). The cumulative rainfall per hour is calculated.

Rain gauges Z_g (Table 1) and radar images Z_r (Figures 2, 3, 4) were recorded in Mexico City during 13, 15 and 16 July 1997 (referred thereafter as storm 1, 2 and 3). Storm 2 has the largest number of gauge data (50), while storm 1 has only 23 gauge measurements. On Figures 2, 3 and 4 the gray scale images correspond to one hour accumulated precipitation given in millimeters (mm), while the cross symbols represent the locations of gauge data for this storm.

Table 1. Rain gauge and radar data basic statistics for each storm.

Statistics	Storm 1		Storm 2		Storm 3	
	Rain Gauge (Z_g) (mm)	Radar (Z_r) (mm)	Rain Gauge (Z_g) (mm)	Radar (Z_r) (mm)	Rain Gauge (Z_g) (mm)	Radar (Z_r) (mm)
Number of observations	23	2106	50	2025	40	1404
Minimum	0.25	0.00	0.25	0.00	0.25	0.00
Mean	5.10	1.95	1.47	1.81	5.77	3.39
Maximum	30.50	55.60	7.75	15.50	27.50	74.00
Standard deviation	8.38	4.67	1.60	1.95	6.55	6.43
1st Quartil	0.25	0.00	0.31	0.30	0.75	0.20
Median	2.25	0.20	1.00	1.30	4.00	1.20
3rd Quartil	4.37	1.40	1.93	2.60	7.18	3.80

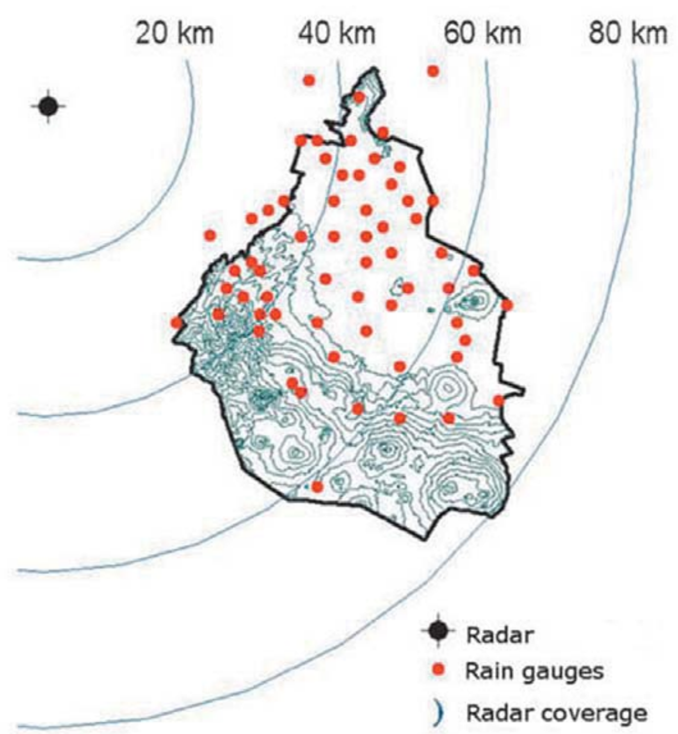

Figure 1. Location of meteorological radar on Cerro de la Catedral and the rain gauge network (red dots) of the Mexico City Water System (Rosengaus, 2000).

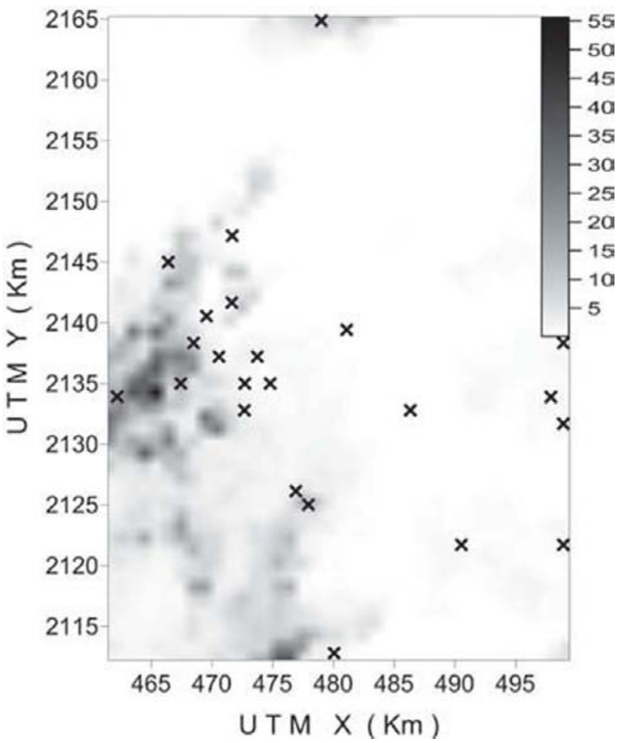

Figure 2. The gray scale image is one hour accumulated rainfall in millimeters (mm) calculated from radar images corresponding to storm 1. The cross symbols represent the locations where gauge data are available for this storm.

Sequential Gaussian Simulation

Since the early 1990's, sequential Gaussian simulation has gained in popularity (Deutsch, 2002). A new simulated value is obtained from the estimated conditional probability distribution function using observational and previously simulated values in a neighborhood of a given location applying a kriging method (Chilès and Delfiner, 1999).

The theory behind sequential Gaussian simulation is based on using previously simulated value and input data throughout the simulation process. In practice, only the closest conditioning data are used.

Geostatical simulation of three storms

The sequential Gaussian method was applied using the rain gauge data and radar data. During the exploratory data analysis, several statistical parameters were computed (Table 1) and histogram graphics were generated. It was found that the data do not have normality; consequently, an anamorphosis transformation was applied to them, which ensured normality in the transformed data (Chilès and Delfiner, 1999).

Variograms were calculated and a model was adjusted to each using weighted least squares. The model with the lowest sum of squares errors was chosen and validated using cross validation. The

Geostatistical simulation methodology

Geostatistical methodology basically consists of three phases: exploratory data analysis, variographic analysis and estimation and/or simulation. The geostatistical simulation is applied in this work. Standard procedures are followed (Díaz-Viera, *et al*., 2009; and Méndez-Venegas, 2008).

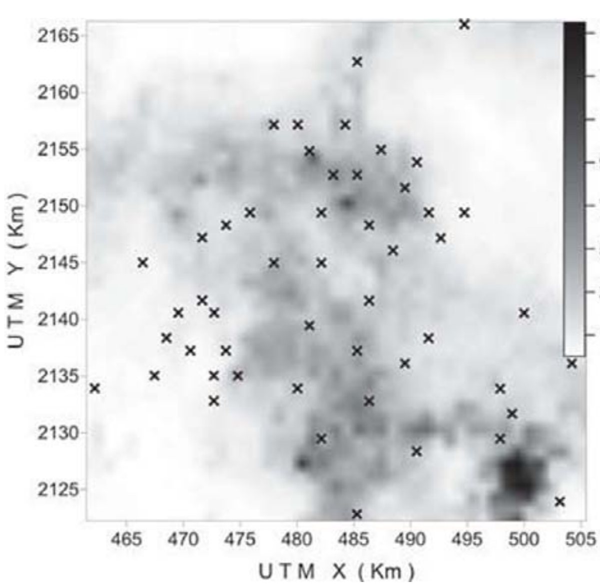

Figure 3. As in Figure 2, for storm 2.

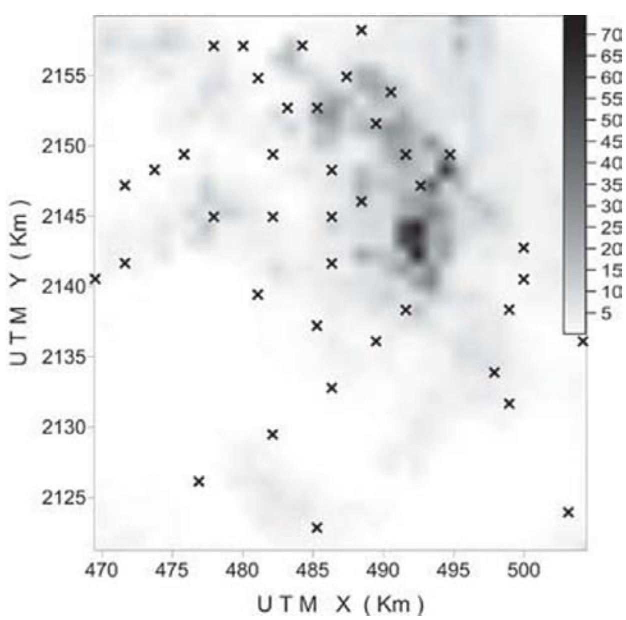

Figure 4. As in Figure 2, for storm 3.

leave-one-out method (Journel and Huijbregts, 1978) was used for cross-validation which involves removing each sample and estimating the value at that point using the kriging equations and the variogram model obtained. As a result, a map of the differences between actual and estimated values is obtained. The adjusted models were spherical.

Results and discussion

For the simulation of storm 1 (Figure 5) rain gauge data and the model shown in the first line of Table 4 were used. For the cosimulation of this storm (Figure 6) rain gauge and radar data of the corresponding storm and the model showed in Table 2 were used. The simulation of storm 2 (Figure 8) was done using rain gauge data and the model in the third line of Table 4; for the cosimulation (Figure 9) the rain gauges and radar data of storm 2 and the model shown in Table 7 were used. For storm 3, as in the previous two cases, the univariate simulation (Figure 11) only uses rain gauge data and a model (fifth line, Table 4) and the cosimulation (Figure 12) was done with all the information available and a model (Table 8).

Table 2. Variogram models in the linear corregionalization model for storm 1.

Variable	*Model*	*Nugget*	*Sill*	*Range (km)*
Rain gauge (Z_g^A)	*Spherical*	*0.3*	*1.05*	*20*
Radar (Z_r^A)	*Spherical*	*0.2*	*1.05*	*20*
Radar (Z_g^A) *– Rain gauge* (Z_r^A)	*Spherical*	*0.15*	*0.95*	*20*

Table 3. Rain gauge data, simulation and cosimulation basic statistics for storm 1.

Statistics	**Rain gauge (Z_g)(mm)**	**Simulation (Z_g^S)(mm)**	**Cosimulation (Z_g^{CS})(mm)**
Number of observations	23	2106	2106
Minimum	0.25	0.25	0.25
Mean	5.10	4.87	4.71
Maximum	30.50	30.50	30.50
Standard deviation	8.38	8.13	9.00
1st Quartil	0.25	0.25	0.25
Median	2.25	0.50	0.25
3rd Quartil	4.37	4.32	2.88

Table 4. Comparison the fitted variograms models for the rain gauge data versus the outcomes of the simulation.

Variable	**Model**	**Nugget**	**Sill**	**Range (km)**
Rain gauge data (storm 1)	Spherical	0.15	1.05	16
Simulation (storm 1)	Spherical	0.15	0.95	17
Rain gauge data (storm 2)	Spherical	0	1.11	13.81
Simulation (storm 2)	Spherical	0.05	1.2	14
Rain gauge data (storm 3)	Spherical	0.1	1.3	20
Simulation (storm 3)	Spherical	0.15	0.85	19

Table 5: Comparison the fitted variograms models for the rain gauge data with the linear corregionalization model versus the outcomes of the cosimulation.

Variable	**Model**	**Nugget**	**Sill**	**Range (km)**
Rain gauges data (storm 1)	Spherical	0.3	1.05	20
Cosimulation (storm 1)	Spherical	0.17	0.8	20
Rain gauges data (storm 2)	Spherical	0.22	1.06	21
Cosimulation (storm 2)	Spherical	0.17	0.95	21
Rain gauge data (storm 3)	Spherical	0.25	1.3	25
Cosimulation (storm 3)	Spherical	0.2	0.9	25

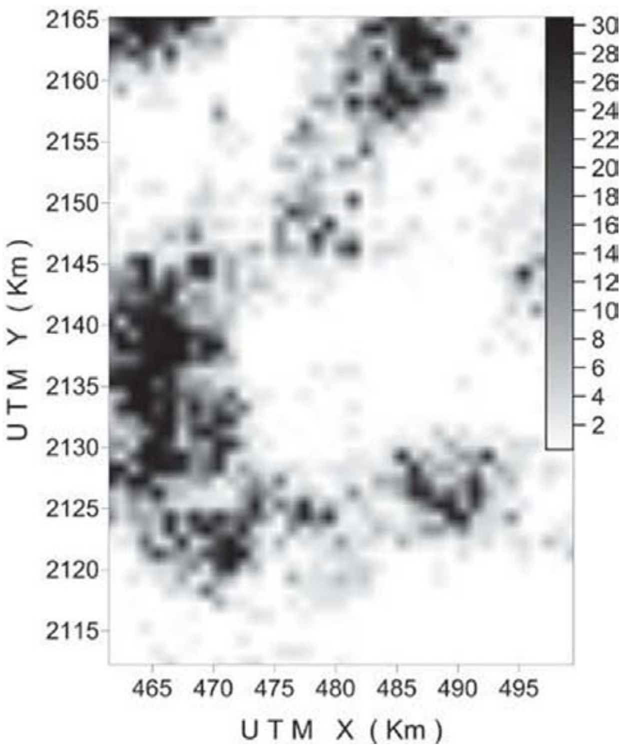

Figure 5. Simulation of one hour accumulated rainfall in millimeters for storm 1.

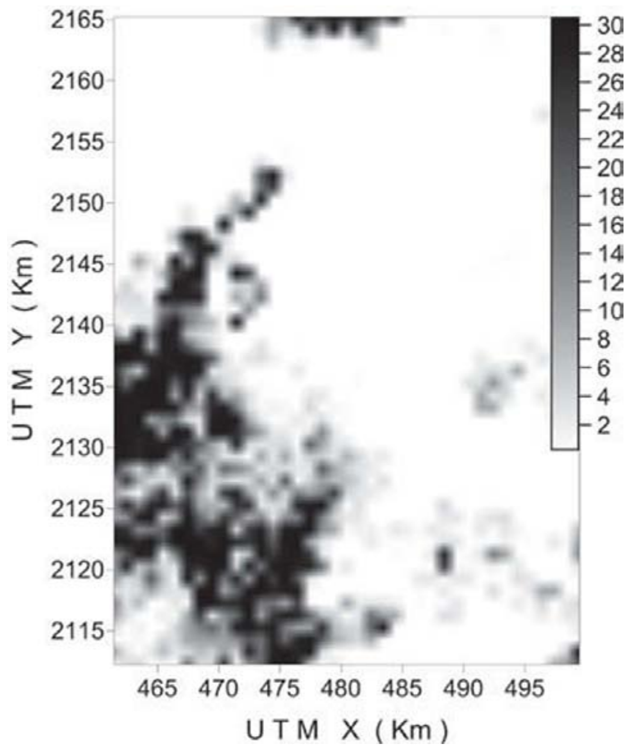

Figure 6. Cosimulation of one hour accumulated rainfall in millimeters for storm 1.

The superscript *A* indicates that the variable was applied the anamorphosis transformation.

Results for each storm were compared with the corresponding sample information. The simulations reproduce adequately the statistical values (Tables 3, 6 and 9), the histograms and box plots (Figures 7, 10 and 13), as the variogram model of the data (Tables 4 and 5).

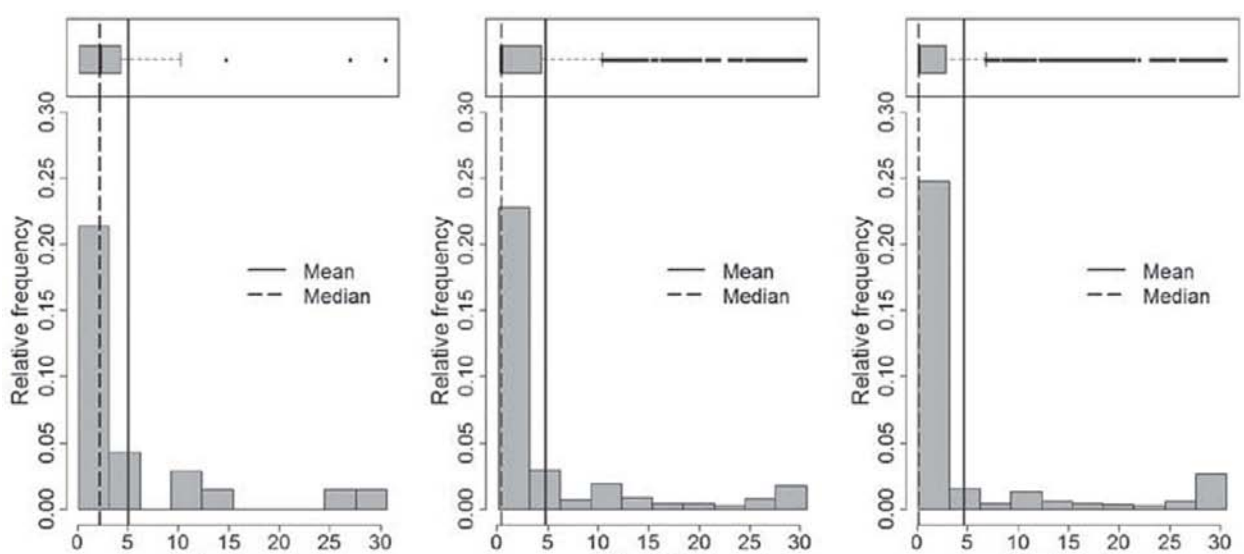

Figure 7. Histograms and box plots of rain gauge data Z_g, univariate Z_g^s and cosimulation Z_g^{cs} for storm 1 (mean value, solid line and median, dashed line).

Table 6. Rain gauge data, simulation and cosimulation basic statistics for storm 2.

Statistics	Rain gauge (Z_g)(mm)	Simulation (Z_g^s)(mm)	Cosimulation (Z_g^{cs})(mm)
Number of observations	50	2025	2025
Minimum	0.25	0.25	0.25
Mean	1.47	1.39	1.38
Maximum	7.75	7.75	7.75
Standard deviation	1.60	1.54	1.53
1st Quartil	0.31	0.25	0.25
Median	1.00	0.99	1.00
3rd Quartil	1.93	1.95	1.82

Table 7. Variogram models in the linear corregionalization model for storm 2.

Variable	Model	Nugget	Sill	Range (km)
Rain gauge (z_g^A)	Spherical	0.22	1.06	21
Radar (z_r^A)	Spherical	0.22	1.08	21
Radar (z_g^A) – Rain gauge (z_r^A)	Spherical	0.16	0.97	21

Table 8. Variogram models in the linear corregionalization model for storm 3.

Variable	Model	Nugget	Sill	Range (km)
Rain gauge (z_g^A)	Spherical	0.25	1.3	25
Radar (z_r^A)	Spherical	0.25	1.4	25
Radar (z_r^A) – Rain gauge (z_g^A)	Spherical	0.17	1.25	25

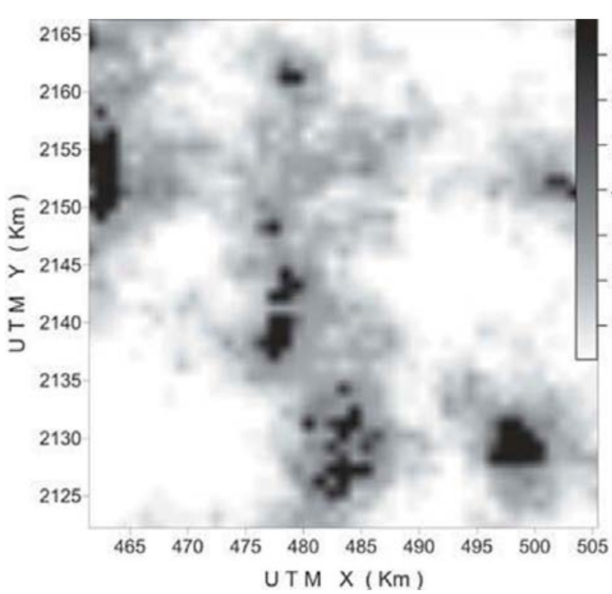

Figure 8. As in Figure 5, for storm 2.

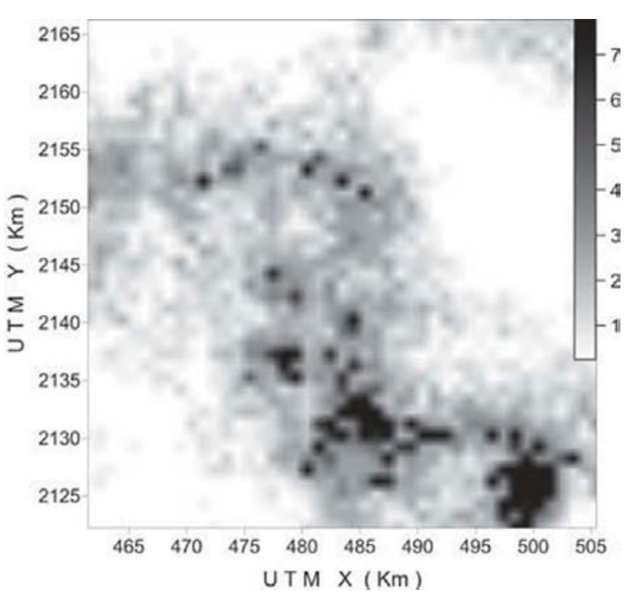

Figure 9. As in Figure 6, for storm 2.

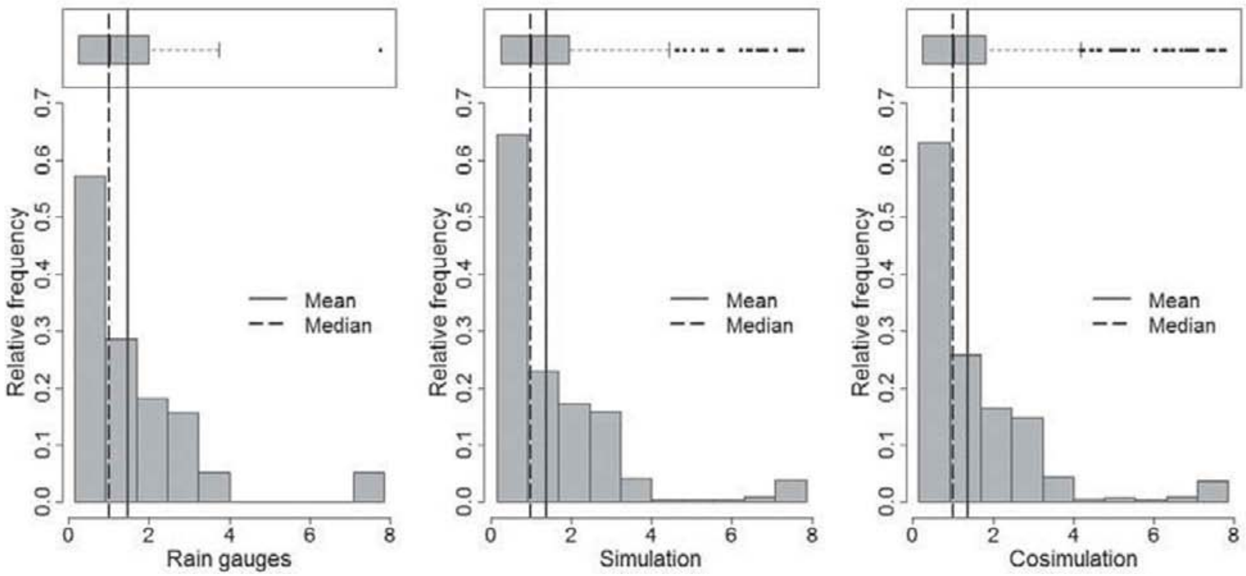

Figure 10. As in Figure 7, for storm 2.

Table 9. Rain gauge data, simulation and cosimulation basic statistics for storm 3.

Statistics	**Rain gauge (Z_g)(mm)**	**Simulation (Z_g^s)(mm)**	**Cosimulation (Z_g^{cs})(mm)**
Number of observations	40	1404	1404
Minimum	0.25	0.25	0.25
Mean	5.77	5.45	5.62
Maximum	27.50	27.50	27.50
Standard deviation	6.55	6.58	6.53
1st Quartil	0.75	0.55	0.25
Median	4.00	3.31	3.85
3rd Quartil	7.18	6.47	8.12

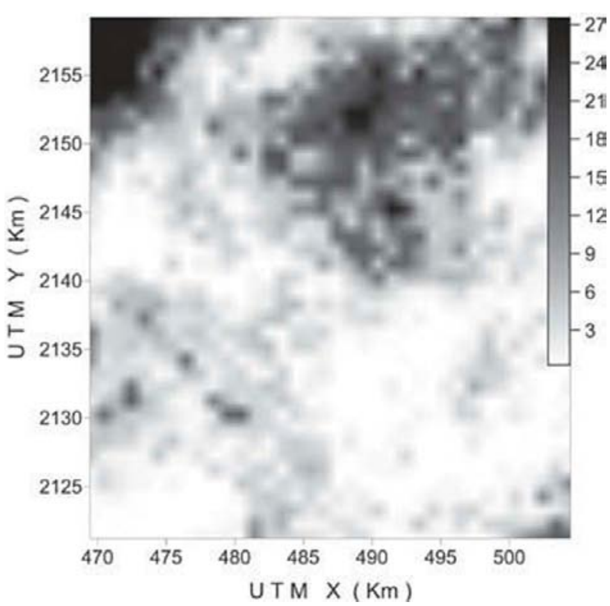

Figure 11. As in Figure 5, for storm 3.

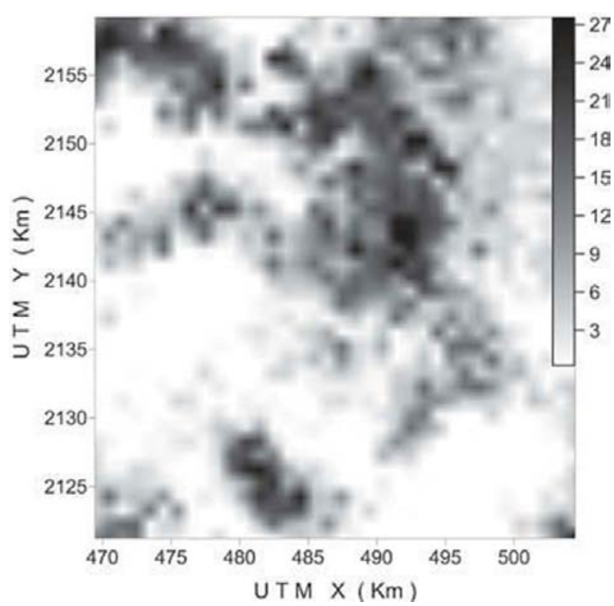

Figure 12. As in Figure 6, for storm 3.

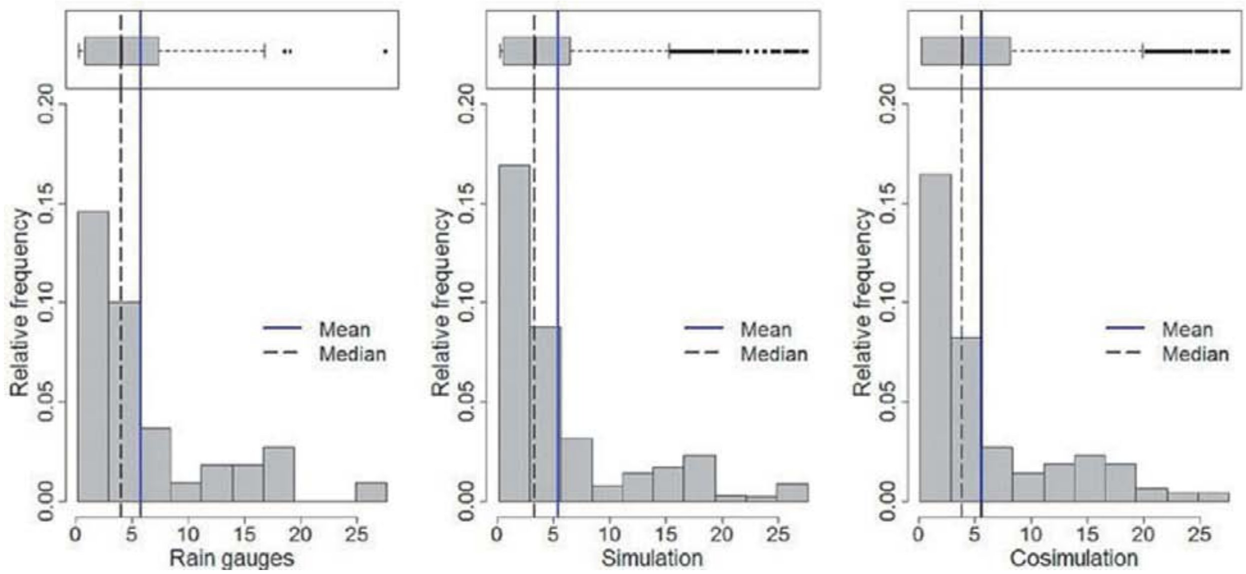

Figure 13. As in Figure 7, for storm 3.

For all storms, simulations estimate adequately precipitation. Comparison of radar images with simulations is complex because radar images are not taken at ground level. In radar images the precipitation falls in a small region and is greater than those recorded by rain gauges and from simulations, this may be due to evaporation of rainfall before it reaches the ground, because of high elevation of the radar beam over the valley of Mexico (Zawadzki,1984).

Conclusions

Spatial stochastic simulations using the sequential Gaussian method reproduce adequately data statistics (minimum, maximum, mean value, median, variance, histogram, variogram model, etc.) in both univariate and bivariate cases. Simulation can be an ideal tool to model the spatial distribution of rainfall.

When there is enough information to accurately estimate the variogram (Storm 2), simulations using only rain gauge data generated consistent estimations with the variability and the spatial distribution of the rainfall, but cosimulations with rain gauge data and radar images generated more precise and detailed estimations of the spatial distribution.

Using the simulation approach, rainfall distributions in storms could be generated from their statistical properties. Simulation is a powerful tool for studying the phenomena involved in precipitation.

For optimal performance of the simulation procedures, it is necessary to follow a methodology consistent with hypothesis, as those described here.

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