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Estimation of work-hardening curve for large strain using friction-free compression test

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Abstract

Forming simulation has been carried out for prediction of forming limits. Work-hardening curve is deeply involved in the reliability of the forming simulation. Measurement method of large strain region has not been established. In this paper, a new method of determining work-hardening curve of large strain region is proposed. The proposed method consists of compression test and optimization scheme. In the optimization, using the least-square method, the work-hardening curve is determined from the relationship of load and displacement obtained by the compression test and finite element analysis. Jigs are designed to eliminate the effects of friction by constraining the slip between a specimen and jigs. Dumbbell-shaped specimens are designed to prevent re-contact with the jigs. In the experiment, low carbon steel was used as specimens. The proposed method was carried out and the work-hardening curve of large strain region was obtained.

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Keywords: Inverse method; Finite element method; Work-hardening

1. Introduction

In product design, numerical simulations are performed using the finite element method to reduce the number of trial. The precision required in numerical simulations has increased as processing accuracy has improved. The accuracy of material data is an important factor determining the accuracy of simulations. The work-hardening curve mainly determines the deformation behaviour of a material. A large strain occurs in the processing involving

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large deformations, such as forging. When simulating such process, data for the material under a large strain is required.

Work-hardening curves are generally obtained by a tensile test. Before necking, stress and strain are calculated from the load and elongation. After the onset of necking, they cannot be calculated directly because non-uniform deformation occurs. Data after necking is generally obtained by simple extrapolation. In this case, a difference may occur between the actual value and the extrapolated data. Several methods of estimating the work-hardening curve after necking have been proposed. Murayama et al. (2012) and Tsuchida et al. (2012) proposed a method of calculating the true stress by using the Bridgeman formula with the radius of curvature and cross-section radius in the region of necking. Joun et al. (2008) and Kamaya et al. (2011) proposed an estimation method using a tensile test and finite element method. Since breakage may occurs after necking, it is difficult to measure a large strain in the tensile test. In the compressing test, the effect of friction is considerable under large strain, which causes unignorable error. Correction method of the effect of friction by slab method (Siebel, 1923; Avitzur, 1968; Ettouney, 1983) and upper-bound method (Chen et al., 2000) was proposed. Osakada et al. (1981) proposed a method of removing the effect of friction by restraining the end surface using a tool with concentric grooves. To investigate the properties of material subjected to the double-compression test, Yanagida and Yanagimoto (2008) used inverse approach for obtaining a flow curve equation. As an inverse approach, estimation using a least-square method is also performed (Lin et al., 2009). The parameter of the work-hardening curve is updated on the basis of changes in the objective function calculated from the analytical value and the experimental value obtained from the compression test. However in this method, the effect of friction is also a problem.

In this study, to increases the accuracy of large deformation analysis, an estimation method for the work-hardening curve for a large strain by inverse analysis based on a compression test is proposed. The curve is estimated using a least-squares method from the results of a finite element analysis and a compression test. To avoid the effect of friction, we designed special compression jig and specimen.

<table>
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<tr>
<th>Nomenclature</th>
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<tr>
<td>( K )</td>
</tr>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>( d )</td>
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<td>( x )</td>
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2. Estimation process

In the proposed method, the work-hardening curve is optimized by the Gauss-Newton method, which is a least-squares method. If the initial convergence value is close to a solution, the convergence is faster in this method. The work-hardening curve is expressed by \( n \)-power law in the proposed method. As an initial value for the optimization, the value obtained from tensile tests was chosen to make the convergence faster. The objective function is calculated as the distance of the analytical and experimental values of the load as

\[
E(K,n) = \frac{1}{2} \sum_{i=1}^{m} (d_i - x_i)^2.
\]

(1)

where \( m \) is the number of reference values of load of different reduction ratios. To update the current state, incremental parameters are calculated as

\[
J^T \frac{\Delta K}{\Delta n} = J^T (d - x),
\]

(2)
where

\[
J = \begin{bmatrix}
\frac{\partial x_1}{\partial K} & \frac{\partial x_1}{\partial n} \\
\vdots & \vdots \\
\frac{\partial x_m}{\partial K} & \frac{\partial x_m}{\partial n}
\end{bmatrix},
\]

(3)

\[
d = [d_1 \ldots d_m]^T,
\]

(4)

\[
x = [x_1 \ldots x_m]^T.
\]

(5)

Partial differentiation of Jacobian \(J\) is determined by the difference calculation.

\[
\frac{\partial x}{\partial K} = \frac{x(K + \Delta K) - x(K)}{\Delta K},
\]

(6)

\[
\frac{\partial x}{\partial n} = \frac{x(n + \Delta n) - x(n)}{\Delta n}.
\]

(7)

New parameters are calculated as

\[
K^{k+1} = K^k + \Delta K,
\]

(8)

\[
n^{k+1} = n^k + \Delta n.
\]

(9)

The objective function is calculated using the updated parameters. This process is repeated until the update of object function becomes sufficiently small.

Verification of the uniqueness of the solution is required for the inverse method. By performing estimations using specimens of same material with different shapes, we can confirm that the obtained result is reliable if the same work-hardening curves are obtained with different specimens.

3. Experimental conditions

3.1. Constrained surface of the specimen

Friction is generated on the specimen surface in the compression test, causing its deformation into a barrel shape. Since the deformed shape is changed depending on an amount of the friction, the load also varies. It is necessary to match the conditions of the experiment and analysis and to consider the friction in the analysis. However, it is difficult to correctly model the behavior of friction at a high surface pressure.

The specimen’s edges are restrained to suppress the slip of the end surface in the proposed method as shown in Fig. 1. Because the deformed shape does not depend on the magnitude of friction, it is possible to eliminate the effect of friction in the numerical analysis. Therefore, the proposed method makes it possible to facilitate the estimation process because only the material properties are the variables in optimization.

![Fig. 1. Schematic illustration of proposed method.](image-url)
3. 2. Dumbbell-shaped specimens

Specimens are deformed into a barrel owing to the effect of friction in the case of compression tests with a restrained specimen edge, as in the present method, this phenomenon is particularly remarkable, and the center of the specimen greatly swells. As the expansion of the specimen increases, the contact area between the specimen and jig also increases, which causes energy consumption according to the friction. It is necessary to increase the compression ratio when estimating the work-hardening curve for a large strain. In this case, accurate estimation is difficult because the area of contact between the jig and the expanding specimen increases.

Therefore, a compression test is performed with a dumbbell-shaped specimen, as shown in Fig. 2, to allow the large expansion of the central part of the specimen. A dumbbell-shaped specimen has a smaller contact area between the jig and specimen after compression than a cylindrical specimen as shown in Fig. 3. The red mesh is a deformable specimen, and the blue mesh represents a rigid jig. Thus, it is possible to estimate large strains by suppressing the effect of friction.

![Fig. 2. Specimen shape.](image)

![Fig. 3. Specimen shape after compression.](image)

4. Test conditions

The work-hardening curve of carbon steel named S15C was estimated using the proposed method. In this study, it is assumed that an \( n \)-power law can express this curve. The initial value used in the inverse calculation was obtained by a tensile test. For the initial parameters, a plastic coefficient of 804.8 MPa and a work-hardening exponent of 0.2 were obtained by curve fitting.

The load history for various compression rates was obtained by finite element analysis and measured in compression tests, and the results were used to update the initial parameter. The load was estimated for specimens shown in Fig. 4 which have different shapes, to verify the uniqueness of the solution. Height and center diameter of each specimen is shown in Table. 1. Since buckling occurred in specimen A in the experiment, the remaining specimens were used in the later process.
5. Experimental results

Experimental results are shown in Fig. 5. The work-hardening curves were roughly the same for all specimens and could be estimated regardless of the specimen shape. The equivalent stress was overestimated when the results of the tensile test were simply extrapolated, and an error of about 10% occurred at an equivalent plastic strain of 1.6. Therefore, simple extrapolation of the result of tensile test is inappropriate when large deformation analysis is carried out. Especially, in forging analyses, such a large error is crucial to the accurate prediction of cleavage. This result suggests the necessity of accurate estimation method of work-hardening curve.

The stress obtained from the tensile test was smaller than the estimation result at a lower strain region. Thus, it is considered the parameters were determined to match the work-hardening curve at larger strain region because the effect of a large strain on the load is larger than that of small strain. It is considered that if the curve had been represented by an expression with more degrees of freedom, the estimated results and those from the tensile test

<table>
<thead>
<tr>
<th>Specimen</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<tbody>
<tr>
<td>L [mm]</td>
<td>39</td>
<td>32.5</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>R [mm]</td>
<td>7.8</td>
<td>7.8</td>
<td>7.8</td>
<td>6.5</td>
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would have been consistent in low strains. An expression by curve with more parameters (Hasegawa et al., 2009) and piecewise linear function (Koede et al., 2007) is proposed, but improvement of convergence is required when the number of parameter is increased.

6. Conclusion

A new method of estimating the work-hardening curve for large deformation analysis by inverse method based on a compression test was proposed. To achieve larger deformation, we designed special jig and specimen. Namely, restraining the end of the specimen eliminated the effect of friction. Contact between the jig and specimen was avoided by using a dumbbell-shaped specimen. We conducted compression experiments, and the proposed method was carried out based on the experimental data. It was possible to estimate the work-hardening curve accurately without considering friction using this method. The proposed approach was proved to be effective when high accuracy in large deformation forming simulation is required.

References