Navigation function design for backbone connectivity in vehicle ad hoc networks

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\textbf{A R T I C L E I N F O}

Keywords:
Backbone connectivity
Power and mobility control
Interference
Navigation functions

\textbf{A B S T R A C T}

Backbone connectivity is a critical and challenging problem in vehicle ad hoc networks. Just like base stations in cellular networks, the backbones or mobile base stations in vehicle ad hoc networks play important roles in location management and engineering applications. In this paper, we discuss the one-to-one connectivity by extending the navigation functions, and present the integrated power and mobility navigation functions. Considering the interference between the backbones, we describe the interference alleviating navigation function design. We establish the control laws, describe the numerical computation, and discuss the stability of the navigation system at the end.

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1. Introduction

Backbone connectivity is a critical and challenging problem in vehicle ad hoc networks. Just like base stations in cellular networks, the backbones in vehicle ad hoc networks play important roles in location management and engineering applications [1]. The paper [1] considers the use of mobility control, such as the dynamic repositioning of backbone nodes, to provide assured coverage-connectivity in dynamic environments. We know that mobility control may only enlarge the communication delay. The paper [2] extends previous works on power control [3] and mobility control [4] for connectivity, and presents integrated power and mobility control methods for power, position and velocity, but does not consider interference between the backbone nodes. This paper presents interference alleviating connectivity control methods for one-by-one vehicle ad hoc networks. As we know, the more closely the vehicles move, the better the connectivity maintenance, but the larger the interference between the vehicles. There may be a trade-off between connectivity and interferences in a vehicle ad hoc network. We manage to deal with this problem by extending the navigation functions based on [4].

The remainder of the paper is arranged as follows. The primary integrated navigation function design for the one-to-one connectivity is discussed in Section 2. In Section 3, we consider that the smaller the distance between the backbones, the bigger the interference between them; so interference alleviating based control law design and computation for the one-to-one connectivity are considered intensively. Finally, some concluding remarks are given at the end.

2. Primary integrated navigation function design

The communication graph $G = (V, E)$ is a directed graph consisting of a set of vertices $V = \{1, 2, \ldots, N\}$ indexed by the backbones and a set of edges $E = \{(i,j)|i, j \in V\}$. $p_i(t), r_i(t) \in \mathbb{R}^2$ denote the position and directional power [1] of
Assume that the mobility of node $i$ and the power of node $i$ are described as
\begin{equation}
\dot{r}_i(t) = v_i(t), \quad \dot{p}_i(t) = u_i(t).
\end{equation}

We extend the navigation function described in [4], and present an integrated power and mobility navigation function for system (1):
\begin{equation}
G_i(t) = \frac{1}{2} \|p_{i+1}(t) - p_i(t) - r_i(t)\|^2 + \frac{1}{2} \|p_i(t) - p_{i-1}(t) - r_i(t)\|^2.
\end{equation}

**Remark 1.** We can see from the navigation function $G_i(t)$ that with the $p_i(t) - p_{i+1}(t)$ or $p_i(t) - p_{i-1}(t)$ approaching the $r_i(t)$, the $G_i(t)$ will be decreasing. In other words, we can control the power and mobility of node $i$ through $G_i(t)$. So, the integrated power and mobility navigation function $G_i(t)$ navigates $p_{i+1}(t)$ and $p_{i-1}(t)$ to connect with $p_i(t)$.

The mobility control law of each mobile backbone $i$ is defined by
\begin{equation}
v_i(t) = -\frac{\partial G_i(t)}{\partial p_i(t)}
\end{equation}
and the power control law of each mobile backbone $i$ is now defined by
\begin{equation}
u_i(t) = -\frac{\partial G_i(t)}{\partial r_i(t)}
\end{equation}
and we get a closed loop mobility control system from (1) and (3):
\begin{equation}
\dot{r}_i(t) = p_{i+1}(t) + p_{i-1}(t) - 2p_i(t)
\end{equation}
and a power control system from (1) and (4):
\begin{equation}
\dot{p}_i(t) = p_{i+1}(t) - p_{i-1}(t) - 2r_i(t).
\end{equation}

Let $r(t) = (r_1(t), r_2(t), \ldots, r_N(t))^\top$, $p(t) = (p_1^T(t), p_2^T(t), \ldots, p_N^T(t))^\top$, $q(t) = (p^T(t), r^T(t))^\top$, and initial condition $g(0) = (p^T(0), r^T(0))^\top = C$. $\otimes$ is the Kronecker product, and $I_2$ is the $2 \times 2$ identity matrix. From (4) and (5), we get the stack vector form as
\begin{equation}
dq(t) = -(A \otimes I_2)q(t)
\end{equation}
where
\begin{equation}
A = \begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix},
\quad
A_{11} = \begin{pmatrix}
2 & -1 & 0 & \cdots & 0 & 0 \\
-1 & 2 & -1 & \cdots & 0 & 0 \\
0 & -1 & 2 & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & -1 & 2
\end{pmatrix}_{N \times N},
\quad
A_{12} = 0_{N \times N}
\end{equation}
\begin{equation}
A_{21} = \begin{pmatrix}
0 & -1 & 0 & \cdots & 0 & 0 \\
1 & 0 & -1 & \cdots & 0 & 0 \\
0 & 1 & 0 & -1 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & 1 & 0
\end{pmatrix}_{N \times N},
\quad
A_{22} = \begin{pmatrix}
2 & 0 & 0 & \cdots & 0 & 0 \\
0 & 2 & 0 & \cdots & 0 & 0 \\
0 & 0 & 2 & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & 0 & 2
\end{pmatrix}_{N \times N}.
\end{equation}

**Lemma 1 ([5]).** If $A_{11}$ and $A_{22}$ are square matrices,
\begin{equation}
\det A = \det \begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix} = (\det A_{11})(\det A_{22}).
\end{equation}

**Lemma 2.** $\det A_{11} \neq 0$.

**Proof.** Consider $d_N = \det A_{11} = 2d_{N-1} - d_{N-2}$; we get $d_N = d_{N-1} = d_{N-1} - d_{N-2}$, $d_N$ as equal difference sequences. We easily get $d_1 = 2$ and $d_2 = 3$, so $d_N = d_{N-1} = 1$ and $d_N = d_{N-1} + 1$, and we get $\det A_{11} = d_N \neq 0$. The lemma is proved. \qed

The conclusion is summarized in the following theorem:

**Theorem 1.** Assume that the mobility of (1) is driven by (5) and the power of (1) is driven by (6), and that the relative position $p_i(t) - p_{i+1}(t)$ or $p_i(t) - p_{i-1}(t)$ and power range $r_i(t)$ approach zero.
3. Interference alleviating navigation function design

Only for connectivity by the control law discussed in Section 2 do we see from Theorem 1 that the backbone nodes may close together. We know that the smaller the distance between the nodes, the bigger the interference between them. For a packet reception to be successful, [6] suggests that \( r_i(t) = (1 + c_1) r_i(t) \), where \( r_i(t) \) stands for the interference range, \( r_i(t) \) stands for the communication range, and \( c_1 \) is a positive parameter. Consider the limited physical length between the backbone nodes; we propose the integrated control law design with one-by-one network and interference based backbone connectivity in this section, for the system (1). We construct an interference alleviating navigation function

\[
G_i(t) = \frac{1}{2} \| p_{i+1}(t) - p_i(t) - r_i(t) \|^2 + \frac{1}{2} \| p_i(t) - p_{i-1}(t) - r_i(t) \|^2 + \frac{1}{2} \| p_{i-1}(t) - p_i(t) - r_{i-1}(t) \|^2 \\
+ \frac{1}{2} \| p_{i-1}(t) - p_{i+1}(t) - r_{i-1}(t)(1 + c_1) \|^2 + \frac{1}{2} \| r_i(t) - c_2 \|^2.
\]

(8)

Remark 2. We can see that the first two terms in (8) are same as two terms in (2) in view of the node i for connectivity. The second two terms in (8) are to maintain that a packet reception is successful in view of the node \( i - 1 \). The last term in (8) is to control that \( r_i(t) \) is not approaching zero, but some positive parameter \( c_2 \) for adjacency links.

From (8), the power control law of each mobile backbone \( i \) is defined by \( u_i(t) = -\frac{\partial G_i(t)}{\partial r_i(t)} \), and the mobility control law of each mobile backbone \( i \) is now defined by \( u_i(x) = -\frac{\partial G_i(t)}{\partial p_i(x)} \); we get closed loop power control and the mobility control system

\[
\begin{align*}
\dot{p}_i(t) &= p_{i+1}(t) - 3p_i(t) + 2p_{i-1}(t) - r_i(t) \\
\dot{r}_i(t) &= 2p_{i+1}(t) - p_i(t) - p_{i-1}(t) - 3r_i(t) + c_2.
\end{align*}
\]

(9)  (10)

From (9) and (10), we get the stack vector form as

\[
\frac{dq(t)}{dt} = -[A \otimes I_2]q(t) + B \otimes I_2
\]

(11)

where

\[
A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad A_{11} = \begin{pmatrix} 3 & -1 & 0 & \ldots & 0 & 0 \\ -2 & 3 & -1 & \ldots & 0 & 0 \\ 0 & -2 & 3 & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & -2 & 3 \end{pmatrix}_{N \times N} \\
A_{12} = \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 1 & \ldots & 0 \end{pmatrix}_{N \times N}, \quad A_{21} = \begin{pmatrix} 1 & -2 & 0 & \ldots & 0 & 0 \\ 1 & 1 & -2 & \ldots & 0 & 0 \\ 0 & 1 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & \ldots & 1 \end{pmatrix}_{N \times N} \\
A_{22} = \begin{pmatrix} 3 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 3 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 3 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 3 \end{pmatrix}_{N \times N}, \quad B = \begin{pmatrix} 0 & 0 & \ldots & 0 & 0 & c_2 & c_2 & \ldots & c_2 \end{pmatrix}^T_{1 \times 2N}.
\]

We apply a similar transform to \( A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \), and get \( A_{11}' = 0 \). From Lemma 1, we have

\[
\det(A - \lambda I) = \det \begin{pmatrix} A_{11}' - \lambda I_{11} & 0 \\ A_{21}' & A_{22}' - \lambda I_{22} \end{pmatrix} = \det(A_{11}' - \lambda I_{11})\det(A_{22}' - \lambda I_{22})
\]

where

\[
A_{11}' = \begin{pmatrix} 3 & -1 & 0 & \ldots & 0 & 0 \\ -7 & 11 & -1 & \ldots & 0 & 0 \\ -3 & 3 & -1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & -1 & \frac{7}{3} & \frac{11}{3} \end{pmatrix}_{N \times N}, \quad A_{22}' = A_{22} = \begin{pmatrix} 3 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 3 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 3 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 3 \end{pmatrix}_{N \times N}.
\]
Applying a similar transform again to $A'_1$, we have

$$
A''_{11} = \begin{pmatrix}
3 + \frac{b_2}{a_2} & 0 & 0 & \cdots & 0 & 0 \\
b_2 & a_2 & 0 & \cdots & 0 & 0 \\
c_3 & b_3 & a_3 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & c_{N-1} & b_{N-1} & a_{N-1} & 0 & 0 \\
0 & 0 & \cdots & c_N & b_N & a_N
\end{pmatrix}_{N \times N}
$$

where $a_{N-1} = a_N + \frac{b_N}{a_N}$, $b_{N-1} = b_N + \frac{c_N}{a_N}$, $c_{N-1} = c_N = -\frac{4}{3}$.

The initial conditions are $a_N = \frac{11}{3}$, $b_N = -\frac{7}{3}$, $c_N = -\frac{1}{3}$.

We have $a_{N-1} = \frac{100}{33}$, $a_{N-2} = \frac{7560}{3300}$, $a_{N-3} \approx 1.18$, $a_{N-4} \approx -1.2 < 0$. The conclusion is summarized in the following theorem.

**Theorem 2.** Assume the mobility of (1) is driven by (9), and the power of (1) is driven by (10); the relative position $p_i(t) - p_{i+1}(t)$ or $p_i(t) - p_{i-1}(t)$ does not come close any longer in system (11).

4. **Conclusion**

We intensively discuss the navigation function design methods for connectivity in vehicle ad hoc networks. Only considering the mobility and power, we construct an integrated navigation function and get the result that with time increase, the communication radius approaches zero; all the backbones go together and result in interference. Considering the mobility, power and interference, we design an interference alleviating navigation function to deal with this problem. In this paper, we just consider a one-to-one backbone connectivity situation. The general connectivity in vehicle ad hoc networks may have many challenges. We are managing to deal with those challenges.

**References**


