Conical diffraction in one dimensional media containing the negative index defect

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Abstract

Conical diffraction in one dimensional lattice formed by waveguide array results in the two quasi beams. Scattering of the electromagnetic wave of either of two beams on defect is investigated. The defect is represented by the waveguide fabricated from negative refractive index material. It is embedded into waveguide array that consists of waveguides with positive refractive index. The numerical simulation demonstrates the strong reflection of the electromagnetic wave from defect of this kind. © 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

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1. Introduction

Arrays of waveguides coupled due to the tunneling of light from one waveguide to the other are important for various applications in integrated optics. Recently waveguide arrays (WA) attract attention inasmuch as they possess the extraordinary optical properties such as anomalous refraction and diffraction [Pertsch et al. (2002), Moison et al. (2009), Longi (2009)]. This opens the way to light control in a functionalities discrete space, i.e., discrete photonics. In recent years, the development of technology has led to the creation of new materials with very unusual optical properties [Smith et al. (2000), Pendry (2000), Eleftheriades et al. (2002), Maimistov et al. (2007)]. For example,
these materials can behave as a medium with negative index of refraction. The conventional media are characterized by the positive refractive index.

Due to conical diffraction in WA two quasi beams are formed from incident wave, which has been directed in one waveguide of WA. Diffraction angle in this case is defined by the coupling between neighboring waveguides. As the coupling constant is due to tunnel penetration of light from one waveguide into another waveguide the substitution the one waveguide for another with alternative parameters can be considered as the sitting of defect into array.

In this paper we will consider an optical waveguide array that is prepared from the waveguides, which are characterized by positive refractive index (PRI) material. Defect in this array is presented by waveguide manufactured from the negative refractive index (NRI) material. The numerical simulation of the reflection of the electromagnetic wave from the NRI defect was produced. The numerical results demonstrated a strong scattering of the electromagnetic wave in WA. Effect of this scattering is more than the same in the case of the conventional defect waveguide.

2. Model description

Let us consider an array of waveguides where all waveguides marked by index \( j \) \((-\infty < j < j_0 \) and \( j_0 < j < \infty \) ) are characterized by positive refractive index. Waveguide marked by index \( j_0 \) is characterized by negative refractive index. This waveguide can be viewed as a defect in a one dimensional lattice. Propagation of an electromagnetic wave in such an array is governed by the system of equation:

\[
\begin{align*}
    iq_{j,z} + iq_{j,t} + \kappa(q_{j-1} + q_{j+1}) &= 0, & j &< j_0, \\
    iq_{j_0,z} - iv \cdot q_{j_0,t} + \kappa_0(q_{j_0-1} + q_{j_0+1}) &= 0, & j &= j_0, \\
    iq_{j,z} + iq_{j,t} + \kappa(q_{j-1} + q_{j+1}) &= 0, & j &> j_0.
\end{align*}
\]  

(1)

where \( q_j \) is the normalized envelope of the wave localized in \( j \)-th waveguide, and \( v \) is the relative group velocity of the wave in \( j \)-th waveguide. Coupling between neighboring waveguides is defined by parameters \( \kappa \) and \( \kappa_0 \). For the sake of simplicity we assume that \( v = 1 \). Thus the defect properties of \( j_0 \)-th waveguide is defined only by the sign of index refraction.

Here we will consider the continuous wave approximation. The time derivatives must be dropped in the Eqs. (1). In the case of ideal WA (i.e., there is no defect) Eqs. (1) can be solved exactly [Somekh at al. (1973)]:

\[
q_j(z) = (-i)^j J_j(2\kappa z)
\]

where \( J_j(z) \) is the Bessel functions. This expression was found provided the incident wave was introduced only to waveguide marked by index \( j = 0 \). To estimate the influence of the defect the total transferred energy function

\[
P(z) = \sum_{j=j_0+1}^{j_0-1} |q_j(z)|^2.
\]

(2)

is introduced. In the case of ideal WA it is \( P_0(z) = \sum_{j_0+1}^{j_0-1} |J_j(2\kappa z)|^2 \). In the case of WA with defect the total transferred energy function (2) was calculated from the numerical solution of the Eqs. (1).

3. Results of numerical simulation

Study of the scattering diffracted wave was conducted for WA containing 50 waveguides. Incident wave was introduced only to 25-th waveguide, and 32-th waveguide is taking as defected. Fig. 1 shows the distribution of the intensities \( a_d(z) = |q_d(z)|^2 \). In this case we assume that \( \kappa = \kappa_0 \).

Let us consider the coupling constant \( \kappa \) and \( \kappa_0 \). Parameter \( K = \kappa_0 / \kappa \) is introduced. Function \( P(z) \) was calculated for different values of parameter \( K \). For the sake of simplicity we assume \( \kappa = 1 \). The curves in Fig. 2 correspond the different parameter \( K \). Fig. 2 shows that increasing of the \( \kappa_0 \) ( \( K = \kappa_0 / \kappa \) at \( \kappa = 1 \)) results in
saturation of the $P(z)$.

![Graph](image)

Fig. 1. Intensity distribution on channels of WA vs propagating distance $z$.

![Graph](image)

Fig. 2. Total transferred energy $P(z)$ at different coupling constant. Dashed line represents the $P(z)$.

Now, let us consider the defected waveguide fabricated from PRI material. This waveguide can be act as defect only if $\kappa_0 \neq \kappa$. Numerical simulations demonstrate that total transferred energy function $P(z)$ can be both increasing and decreasing function on $K$. If $\kappa_0 < \kappa$, then decreasing of $\kappa_0$ results in decreasing of the asymptotic at $z \to \infty$ value of the total transferred energy, Fig.3. If $\kappa_0 > \kappa$, then increasing of $\kappa_0$ (id $K$) results in decreasing of the asymptotic value of the total transferred energy too. However, in this case the total transferred energy approaches some constant value, Fig.4. It means that strong coupling between regular and defect waveguides is favorable to transferring electromagnetic wave through defect.
Fig. 3. Total transferred energy function $P(z)$ at different coupling constant. The defected waveguide fabricated from PRI material. Dashed line represents the $P_0(z)$.

Fig. 4. Total transferred energy function $P(z)$ in the case of PRI and NRI defect. Dashed line represents the $P_0(z)$.

4. Conclusion

We analyzed the discrete diffraction in the waveguide array containing the defect. We considered the defected waveguide fabricated from both NRI and PRI material. Using the numerical simulation we are demonstrated that the power of scattering wave is more than the power of scattering wave in the case of the conventional defect waveguide. It has been known that the two closely located waveguides can be coupled due to the tunneling of light.
from one waveguide to the other. If waveguide is fabricated from material with a PRI then wave vector of the electromagnetic wave is parallel to the Poynting vector. This wave is named forward wave. Oppositely, if waveguide is fabricated from materials with a PRI then wave vector of the electromagnetic wave is antiparallel to the Poynting vector. In this case this wave is referred to as backward wave. The scattering on the NRI defect gives rise to interaction between forward and backward waves. That results in to increasing of the scattering power.

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References


