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PHYSICS LETTERS B
Physics Letters B 576 (2003) 243–252
www.elsevier.com/locate/physletb

A general solution for the quark propagator in two-dimensional covariant gauge QCD

V. Gogohia, Gy. Kluge, I. Vargas de Usera

HAS, CRIP, RMKI, Department Theoretical Physics, PO Box 49, H-1525 Budapest 114, Hungary

Received 17 May 2003; received in revised form 30 August 2003; accepted 30 September 2003

Editor: L. Alvarez-Gaumé

Abstract

We have investigated a closed set of equations for the quark propagator, which has been obtained earlier within a new, nonperturbative approach to two-dimensional covariant gauge QCD. It is shown that this theory implies quark confinement (the quark propagator has no poles, indeed), as well as dynamical breakdown of chiral symmetry (a chiral symmetry preserving solution is forbidden). The above-mentioned set of equations can be exactly solved in the chiral limit. We develop an analytical formalism, the so-called chiral perturbation theory at the fundamental quark level, which allows one to find solution for the quark propagator in powers of the light quark masses. Each correction satisfies the differential equation, which can be formally solved. We develop also an analytical formalism which allows one to find solution for the quark propagator in the inverse powers of the heavy quark masses. It coincides with the free heavy quark propagator up to terms of order $1/m_Q^3$, where m_Q is the heavy quark mass. So this solution automatically possesses the heavy quark flavor symmetry up to terms of order $1/m_Q$. At the same time, we have found a general solution for the heavy quark propagator, which by no means can be reduced to the free one.

PACS: 11.15.Tk; 12.38.Aw; 12.38.Lg

1. Introduction

The investigation of two-dimensional (2D) QCD in the context of the Schwinger–Dyson (SD) dynamical equations of motion has been initiated by the pioneering paper of 't Hooft [1]. He used the free gluon propagator in the light-cone gauge, which is free from ghost complications. He used also the large N_c (the number of colors) limit technique in order to make the perturbation (PT) expansion with respect to $1/N_c$ reasonable. In this case the planar diagrams are reduced to quark self-energy and ladder diagrams, which can be summed. The bound-state problem within the Bethe–Salpeter (BS) formalism was finally obtained free from the infrared (IR) singularities. The existence of a discrete spectrum only (no continuum in the spectrum) was demonstrated. Since this pioneering paper 2D QCD continues to attract attention (see, for example, review [2] and recent papers [3–5] and references therein). Despite its simplistic vacuum structure it re-

E-mail addresses: gogohia@rmki.kfki.hu (V. Gogohia), kluge@rmki.kfki.hu (Gy. Kluge), ignacio@nextlimit.com (I. Vargas de Usera).

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doi:10.1016/j.physletb.2003.09.093

mains a rather good laboratory for the modern theory of strong interactions, which is four-dimensional (4D) QCD [6].

In our previous publications [7,8] we have investigated 2D QCD in the arbitrary covariant gauge for the first time. In these works a new, nonperturbative (NP) solution (using neither large N_c limit technique explicitly nor a weak coupling regime, i.e., ladder approximation) to 2D QCD in the covariant gauge is obtained in the context of the above-mentioned SD equations, complemented by the corresponding Slavnov–Taylor (ST) identities. It is well known, however, that covariant gauges, in general, are complicated by the ghost contributions. Nevertheless, we have shown that ghost degrees of freedom can be considerable within our approach [7]. The ghost-quark sector contains a very important piece of information on quark degrees of freedom themselves through the corresponding quark ST identity. This is just the information which should be self-consistently taken into account. In this way a close set of equations has been derived for the quark propagator [7]. The main purpose of this Letter is to exactly solve the obtained system of equations in the chiral limit and to develop analytical methods of its solution in the general case, i.e., for the nonzero current quark masses. Let us emphasize in advance that we have found a general solution for the heavy quark propagator, which by no means could be reduced to the free one. All of this will provide the necessary basis for future numerical calculations as well.

2. Quark SD equation

The final system of equations, obtained in Ref. [7] for the quantities in the quark sector, are presented by the quark SD equation and the quark ST identity as follows (Euclidean signature):

$$S^{-1}(p) = S_0^{-1}(p) + \bar{g}^2 \Gamma_\mu(p,0) S(p)\gamma_\mu,$$
$$\Gamma_\mu(p,0) = i d_\mu S^{-1}(p) - S(p)\Gamma_\mu(p,0)S^{-1}(p). \tag{2.1}$$

For simplicity, here we remove an overbar from the definitions of the renormalized Green's functions, retaining it only for the coupling constant $\bar{g}$ (which has the dimensions of mass) in order to distinguish it from initial ("bare") coupling constant. It contains all known finite numerical factors. $\Gamma_\mu(p,0)$ is obviously the proper quark–gluon vertex at zero momentum transfer. The Euclidean version of our parametrization of the quark propagator is as follows: $iS(p) = \hat{p}A(p^2) - B(p^2)$. It is convenient to introduce the dimensionless variables and functions as $A(p^2) = \bar{g}^{-2}A(x)$, $B(p^2) = \bar{g}^{-1}B(x)$, $x = p^2/\bar{g}^2$. Performing further some tedious algebra of the γ matrices in 2D Euclidean space, the system (2.1) can be explicitly reduced to a system of a coupled, nonlinear ordinary differential equations of the first order for the $A(x)$ and $B(x)$ quark propagator form factors, namely

$$xA' = -(1+x)A - 1 - \bar{m}_0 B,$$
$$2BB' = -A^2 + 2(\bar{m}_0 A - B)B, \tag{2.2}$$

where $A \equiv A(x)$, $B \equiv B(x)$, and the prime denotes the derivative with respect to the Euclidean dimensionless momentum variable x. For the dimensionless current quark mass, we introduce the notation $\bar{m}_0 = m_0/\bar{g}$.

The formal exact solution of the system (2.2) for the dynamically generated quark mass function is

$$B^2(c, \bar{m}_0; x) = \exp(-2x)\int_x^c \exp(2x')\tilde{\nu}(x')\,dx', \tag{2.3}$$

and c is the constant of integration. Not losing generality, it can be fixed as $c = p_c^2/\bar{g}^2$, where p_c^2 is some constant momentum squared, and

$$\tilde{\nu}(x) = A^2(x) + 2A(x)\nu(x) \tag{2.4}$$

with

$$\nu(x) = -\bar{m}_0 B(x) = xA'(x) + (1+x)A(x) + 1. \tag{2.5}$$

Then the equation determining the $A(x)$ function becomes

$$\frac{d\nu^2(x)}{dx} + 2\nu^2(x) = -A^2(x)\bar{m}_0^2 - 2A(x)\nu(x)\bar{m}_0^2. \tag{2.6}$$

2.1. Quark confinement

As was emphasized in Refs. [7,8], the important observation is that the formal exact solution (2.3) exhibits the algebraic branch point at $x = c$, which completely *excludes a pole-type singularity* at any finite point on the real axis in the x-complex plane whatever the solution for the $A(x)$ function might be. Thus the solution for the quark propagator cannot be presented as the expression having finally a pole-type singularity at any finite point $p^2 = -m^2$ (Euclidean signature), i.e.,

$$S(p) \neq \frac{\text{const}}{\hat{p} + m}, \tag{2.7}$$

certainly satisfies thereby the first necessary condition of quark confinement, formulated at the fundamental quark level as the absence of a pole-type singularity in the quark propagator [9]. It is well known that such kind of unphysical singularity (algebraic branch point at $x = c$) is due to the inevitable ghost contributions in the covariant gauge QCD. However, as was explained in Refs. [7,8], it will not cause any problems within our approach in order to calculate truly NP quantities, such as quark condensate. The absence of a pole-type singularities in the quark propagator as a criterion of confinement at the microscopic level is only first necessary condition. The second sufficient condition of this criterion, formulated at the macroscopic (hadron) level, is the existence of a discrete spectrum only (no continuum in the spectrum) in the bound-state problem within the corresponding BS formalism [1]. Its discussion is obviously beyond the scope of the present Letter.

2.2. Dynamical breakdown of chiral symmetry (DBCS)

From a coupled system of the differential equations (2.2) it is easy to see that this system *allows a chiral symmetry breaking solution only,*

$$\bar{m}_0 = 0, \qquad A(x) \neq 0, \qquad B(x) \neq 0 \tag{2.8}$$

and *forbids a chiral symmetry preserving solution,*

$$\bar{m}_0 = B(x) = 0, \qquad A(x) \neq 0. \tag{2.9}$$

Thus any nontrivial solution automatically breaks the γ_5 invariance of the quark propagator, and therefore *certainly* leads to the spontaneous chiral symmetry breakdown at the fundamental quark level ($m_0 = 0$, $\overline{B}(x) \neq 0$, dynamical quark mass generation). In all previous investigations a chiral symmetry preserving solution always exists. For simplicity, we do not distinguish between $B(x)$ and $\overline{B}(x)$, calling both dynamically generated quark mass functions.

A few remarks are in order. A nonzero, dynamically generated quark mass function defined by condition (2.8) is the order parameter of DBCS at the fundamental quark level. At the phenomenological level the order parameter of DBCS is the nonzero chiral quark condensate determined as $\langle \bar{q}q \rangle_0 \sim -\bar{g} \int_0^{c_0} dx \times B_0(c_0, x)$ within our approach (see Ref. [8] and $B_0(c_0, x)$ is explicitly given below in Eq. (2.11)). In general, it can be formally zero, even when the mass function is definitely nonzero. Thus the nonzero, dynamically generated quark mass is a much more appropriate condition of DBCS than the quark condensate. One may say that this is the first necessary condition of DBCS, while the nonzero chiral quark condensate is the second sufficient one.

2.3. The chiral limit

In the chiral limit ($\bar{m}_0 = 0$) the system (2.2) can be solved exactly. The solution for the $A(x)$ function is

$$A_0(x) = -x^{-1}\{1 - \exp(-x)\}. \tag{2.10}$$

It has thus the correct asymptotic properties (see Fig. 1). It is regular at small x and asymptotically approaches the free propagator at infinity ($x \to \infty$), which can be formally achieved by the two ways: $p^2 \to \infty$ at fixed $\bar{g}^2$ and/or by $\bar{g}^2 \to 0$ as well. Let us note that the last limit is known as the PT one. For the dynamically generated quark mass function $B(x)$ the exact solution is

$$B_0^2(c_0, x) = \exp(-2x) \int_x^{c_0} \exp(2x') A_0^2(x')\, dx', \tag{2.11}$$

where $c_0 = p_0^2/\bar{g}^2$ is an arbitrary constant of integration and p_0^2 is some constant momentum squared for the chiral limit case. It is regular at zero. In addition, it also has algebraic branch points at $x = c_0$ and at infinity (at fixed c_0, i.e., when $\bar{g}^2$ is fixed). As in the general (nonchiral) case, these unphysical singularities are caused by the inevitable ghost contributions in the covariant gauges (for general behavior of this solution see Fig. 2).

As was mentioned above, $A_0(x)$ automatically has a correct behavior at infinity (it does not depend on the constant of integration, since it was specified in order to get regular at zero solution). In the PT limit ($\bar{g}^2 \to 0$), the constant of the integration c_0 and the

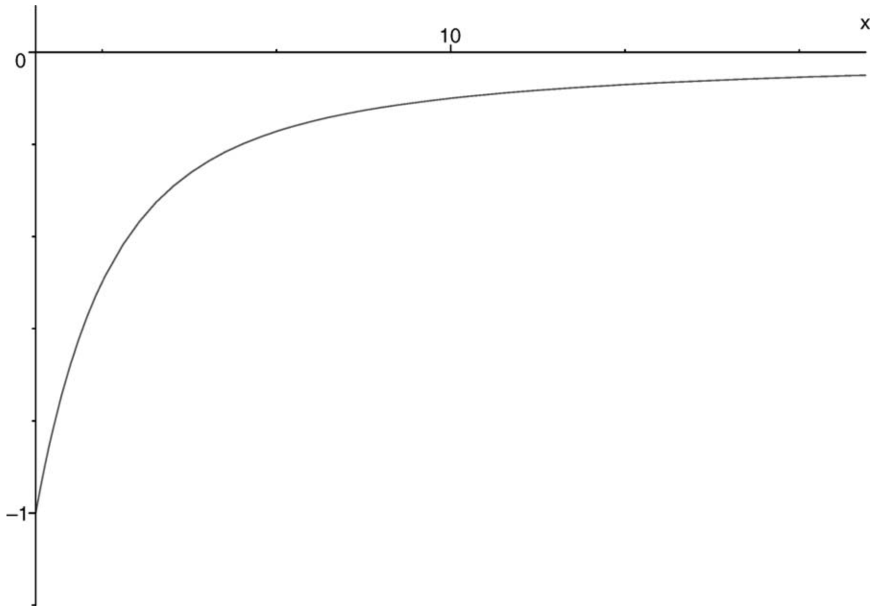

Fig. 1. $A_0(x)$ as given by Eq. (2.6).

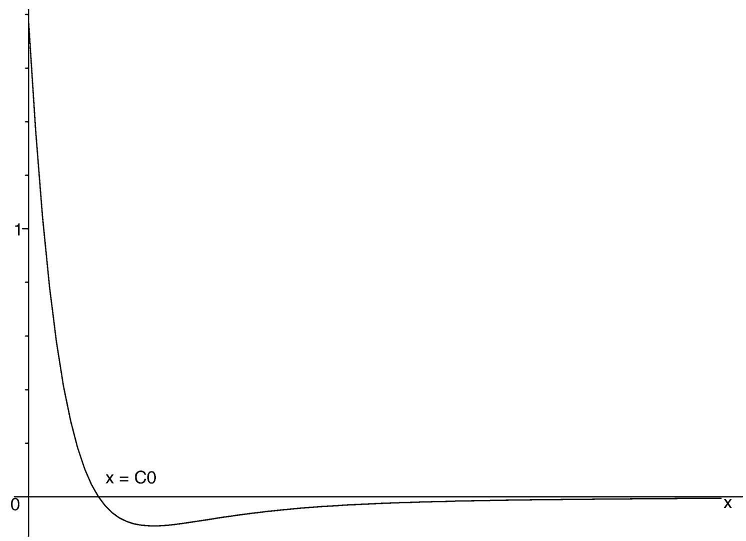

Fig. 2. The dynamically generated quark mass function as given by Eq. (2.7).

variable x go to infinity uniformly ($c_0, x \to \infty$), so the dynamically generated quark mass function (2.11) identically vanishes in this limit, in accordance with the vanishing current light quark mass in the chiral limit. Obviously, we have to keep the constant of integration c_0 in Eq. (2.11) arbitrary but finite in order to obtain a regular at zero point solution. The problem is that if $c_0 = \infty$, then the solution (2.11) does not exist at all at any finite x, in particular at $x = 0$. This is valid, of course, for the general solution (2.3) as well.

3. Nonzero current quark masses

Let us formulate and develop now the calculation scheme, which gives the solution of the system (2.2) step by step in powers of the light nonzero current quark masses, as well as in the inverse powers of the heavy quark masses. For this purpose, it is much more convenient to start from the ground system itself, Eqs. (2.2), rather than to investigate the general solution (2.3). For this purpose, let us rewrite the ground system (2.2) as follows:

$$xA' + (1+x)A + 1 = -\bar{m}_0 B,$$
$$2BB' + A^2 + 2B^2 = 2\bar{m}_0 AB. \qquad (3.1)$$

As was mentioned above, we are interested in the solutions which are *regular at zero* and asymptotically approach the free quark case at infinity. Because of our parametrization of the quark propagator, its asymptotics have to be determined as follows (Euclidean signature): $A(x) \sim_{x\to\infty} -1/(x+\bar{m}_0^2)$, $B(x) \sim_{x\to\infty} -\bar{m}_0/(x+\bar{m}_0^2)$, and neglecting $\bar{m}_0^2$ in the denominators for light quarks. The ground system (3.1) is very suitable for numerical calculations.

3.1. Light quarks

Let us now develop the above-mentioned analytical formalism, which makes it possible to find solution of the ground system (3.1) step by step in powers of the light (u, d, s) nonzero current quark masses, the so-called chiral perturbation theory at the fundamental quark level. For this purpose it is convenient to present the quark propagator form factors A and B as follows:

$$A(x) = \sum_{n=0}^{\infty} \bar{m}_0^n A_n(x),$$
$$B(x) = \sum_{n=0}^{\infty} \bar{m}_0^n B_n(x), \qquad (3.2)$$

where it is formally assumed that $\bar{m}_0^{(u,d,s)} \ll 1$. Substituting these expansions into the ground system (3.1) and omitting some tedious algebra, one obtains

$$xA_0'(x) + (1+x)A_0(x) + 1 = 0,$$
$$2B_0(x)B_0'(x) + A_0^2(x) + 2B_0^2(x) = 0, \qquad (3.3)$$

and for $n = 1, 2, 3, \ldots$, one obtains

$$xA_n'(x) + (1+x)A_n(x) = -B_{n-1}(x),$$
$$2P_n(x) + M_n(x) + 2Q_n(x) = 2N_{n-1}(x), \qquad (3.4)$$

where

$$P_n(x) = \sum_{m=0}^{n} B_{n-m}(x)B_m'(x),$$
$$M_n(x) = \sum_{m=0}^{n} A_{n-m}(x)A_m(x),$$
$$Q_n(x) = \sum_{m=0}^{n} B_{n-m}(x)B_m(x),$$
$$N_n(x) = \sum_{m=0}^{n} A_{n-m}(x)B_m(x). \qquad (3.5)$$

Is is obvious that the system (3.3) describes the ground system (3.1) in the chiral limit ($\bar{m}_0 = 0$). As we already know, it can be solved exactly (see below as well). The first nontrivial correction in powers of small $\bar{m}_0$ is determined by the following system, which follows from Eqs. (3.4) and (3.5), and it is

$$xA_1' + (1+x)A_1 = -B_0,$$
$$(B_1B_0' + B_0B_1') + A_0A_1 + 2B_0B_1 = A_0B_0, \qquad (3.6)$$

where we omit the dependence on the argument x, for simplicity. In the similar way can be found the system of equations to determine terms of order m_0^2 in the solution for the quark propagator and so on.

Let us present a general solution to the first of Eqs. (3.4), which is

$$A_n(x) = -x^{-1}e^{-x}\int_0^x dx'\, e^{x'} B_{n-1}(x'). \qquad (3.7)$$

It is always regular at zero, since all $B_n(x)$ are regular as well. The advantage of the developed chiral perturbation theory at the fundamental quark level is that each correction in the powers of small current quark masses is determined by the corresponding system of equations which can be formally solved exactly.

Let us write down the system of solutions approximating the light quark propagator up to first correction, i.e.,

$$A(x) = A_0(x) + \bar{m}_0 A_1(x) + \cdots,$$
$$B(x) = B_0(x) + \bar{m}_0 B_1(x) + \cdots. \quad (3.8)$$

This system is

$$A_0(x) = -x^{-1}\left(1 - e^{-x}\right), \qquad A_0(0) = -1, \quad (3.9)$$

$$B_0^2(x) = e^{-2x} \int_x^{c_0} dx'\, e^{2x'} A_0^2(x'), \quad (3.10)$$

and

$$A_1(x) = -x^{-1} e^{-x} \int_0^x dx'\, e^{x'} B_0(x'), \quad (3.11)$$

$$B_1(x) = e^{-2x} B_0^{-1}(x) \int_{c_1}^x dz\, e^{2z} A_0(z)\left[B_0(z) - A_1(z)\right]. \quad (3.12)$$

In physical applications we also need $B^2(x)$, so we have

$$B^2(x) = B_0^2(x) + 2\bar{m}_0 B_0(x) B_1(x) + \cdots$$
$$= B_0^2(x) + 2\bar{m}_0 e^{-2x} \int_{c_1}^x dz\, e^{2z} A_0(z)\left[B_0(z) - A_1(z)\right] + \cdots, \quad (3.13)$$

and the relation between constants of integration c_0 and c_1 remains, in general, arbitrary. However, there exists a general restriction, namely $B^2(x) \geqslant 0$ and it should be real, which may lead to some bounds for the constants of integration, while $x \leqslant c_0$ always remains valid.

3.2. Heavy quarks

For heavy quarks (c, b, t) it makes sense to replace $\bar{m}_0 \to \bar{m}_Q$. In this case it is convenient to find solution for heavy quark form factors A and B as follows:

$$\bar{m}_Q^2 A(x) = \sum_{n=0}^{\infty} \bar{m}_Q^{-n} A_n(x),$$
$$\bar{m}_Q B(x) = \sum_{n=0}^{\infty} \bar{m}_Q^{-n} B_n(x), \quad (3.14)$$

and for heavy quark masses it is formally assumed that $\bar{m}_Q^{(c,b,t)} \gg 1$, i.e., the inverse powers are small. Substituting these expansions into the first equation of the ground system (3.1) and omitting some tedious algebra, one finally obtains

$$B_0(x) = -1, \qquad B_1(x) = 0, \quad (3.15)$$

and

$$x A_n'(x) + (1 + x) A_n(x) = -B_{n+2}(x),$$
$$n = 0, 1, 2, 3, \ldots. \quad (3.16)$$

In the same way, by equating terms at equal powers in the inverse of heavy quark masses, from second of the equations of the ground system (3.1), one finally obtains

$$P_0(x) + Q_0(x) - N_0(x) = 0,$$
$$P_1(x) + Q_1(x) - N_1(x) = 0, \quad (3.17)$$

and

$$P_{n+2}(x) + Q_{n+2}(x) - N_{n+2}(x) = -\frac{1}{2} M_n(x),$$
$$n = 0, 1, 2, 3, \ldots, \quad (3.18)$$

where $P_n(z), M_n(z), Q_n(z), N_n(z)$ are again given by Eqs. (3.5). Solving Eqs. (3.17) and taking into account Eq. (3.15), one obtains

$$A_0(x) = B_0(x) = -1, \qquad A_1(x) = B_1(x) = 0, \quad (3.19)$$

so the final system to be solved further becomes

$$x A_n'(x) + (1 + x) A_n(x) = -B_{n+2}(x),$$
$$P_{n+2}(x) + Q_{n+2}(x) - N_{n+2}(x) = -\frac{1}{2} M_n(x),$$
$$n = 0, 1, 2, 3, \ldots. \quad (3.20)$$

It is possible to show that all odd terms are simply zero, i.e., $A_{2n+1}(x) = B_{2n+1}(x) = 0$, $n = 0, 1, 2, 3, \ldots$.

The explicit solutions for a few first nonzero terms are

$$A_0(x) = B_0(x) = -1, \quad (3.21)$$

$$A_2(x) = x + \frac{3}{2}, \qquad B_2(x) = x + 1. \quad (3.22)$$

$$A_4(x) = -x^2 - \frac{3}{2}x - \frac{15}{2},$$
$$B_4(x) = -x^2 - \frac{7}{2}x - \frac{3}{2}. \quad (3.23)$$

Thus our solutions for the heavy quark form factors look like

$$A(x) = \frac{1}{\bar{m}_Q^2} \sum_{n=0}^{\infty} \bar{m}_Q^{-n} A_n(x) = -\frac{1}{\bar{m}_Q^2} + \frac{x}{\bar{m}_Q^4} - \frac{x^2}{\bar{m}_Q^6} + \cdots + D_A(x), \quad (3.24)$$

where

$$D_A(x) = \frac{3}{2\bar{m}_Q^4} - \frac{3x+15}{2\bar{m}_Q^6} + \cdots, \quad (3.25)$$

and

$$B(x) = \frac{1}{\bar{m}_Q} \sum_{n=0}^{\infty} \bar{m}_Q^{-n} B_n(x) = -\frac{1}{\bar{m}_Q} + \frac{x}{\bar{m}_Q^3} - \frac{x^2}{\bar{m}_Q^5} + \cdots + D_B(x), \quad (3.26)$$

where

$$D_B(x) = \frac{1}{\bar{m}_Q^3} - \frac{7x+3}{2\bar{m}_Q^5} + \cdots. \quad (3.27)$$

Summing up, one obtains

$$A(x) = -\frac{1}{x + \bar{m}_Q^2} + D_A(x),$$
$$B(x) = -\frac{m_Q}{x + \bar{m}_Q^2} + D_B(x). \quad (3.28)$$

In terms of the Euclidean dimensionless variables, the quark propagator is

$$iS(x) = \hat{x}A(x) - B(x). \quad (3.29)$$

Using our solutions, obtained above, it can be written down as follows:

$$iS_h(x) = iS_0(x) + \hat{x}D_A(x) - D_B(x), \quad (3.30)$$

where $iS_0(x)$ is nothing else but the free quark propagator with the substitution $\bar{m}_0 \to \bar{m}_Q$, i.e.,

$$iS_0(x) = -\frac{\hat{x} - \bar{m}_Q}{x + \bar{m}_Q^2}. \quad (3.31)$$

Since $\hat{x}D_A(x) - D_B(x)$ is of order $\bar{m}_Q^{-3}$, then Eq. (3.30), becomes

$$iS_h(x) = iS_0(x) + O\left(\bar{m}_Q^{-3}\right), \quad (3.32)$$

i.e., it is reduced to the free quark propagator up to terms of order $1/\bar{m}_Q^3$.

3.3. Heavy quarks flavor symmetry

It is instructive to show explicitly that our solution (3.32) possesses the heavy quark flavor symmetry [10, 11]. We will show that the quark propagator to leading order (up to terms of order $1/m_Q$) in the inverse powers of the heavy quark mass will not depend on it, i.e., it is a manifestly flavor independent to the leading order of this expansion. For this purpose, we must take into account that the argument x, which is the dimensionless momentum of the heavy quark, contains itself the heavy quark mass m_Q. In other words, a standard heavy quark momentum decomposition should be used, namely

$$p_\mu = m_Q \upsilon_\mu + k_\mu, \quad (3.33)$$

as well as

$$\hat{x} = \gamma_\mu x_\mu = \gamma_\mu (m_Q \upsilon_\mu + y_\mu), \quad (3.34)$$

where υ is the four-velocity with $\upsilon^2 = -1$ (Euclidean signature). It should be identified with the four-velocity of the hadron. The "residual" momentum k is of dynamical origin. In these terms the Euclidean dimensionless dynamical momentum variable $x = p^2/\bar{g}^2$ then becomes

$$x = -\bar{m}_Q^2 - 2\bar{m}_Q t - z, \quad (3.35)$$

where we denote $t = (\upsilon \cdot y)$ with $y_\mu = k_\mu/\bar{g}$ and $z = k^2/\bar{g}^2$.

Substituting these expressions into the Eq. (3.31) and taking into account only the leading order term in the inverse powers of m_Q, one finally obtains

$$iS_h(\upsilon, y) = iS_0(\upsilon, y) + O\left(\frac{1}{m_Q}\right), \quad (3.36)$$

where

$$iS_0(\upsilon, y) = \frac{1}{\upsilon \cdot y} \frac{\hat{\upsilon} - 1}{2}, \quad (3.37)$$

which is exactly the heavy quark propagator [11]. Thus our propagator does not depend on m_Q to leading order in the heavy quark mass limit, $m_Q \to \infty$, i.e., in this limit it possesses the heavy quark flavor symmetry, indeed.

4. The general solution for heavy quarks

It is easy to understand that the chiral perturbation theory at the fundamental quark level developed for light quarks in Subsection 3.1 completely coincides with the general solution (2.3), complemented by Eqs. (2.4), (2.5) and (2.6). We use these equations in the chiral limit as input information in the expansions (3.2). However, things are not so straightforward in the case of heavy quarks. Developing the chiral perturbation theory in the inverse powers of the heavy quark masses in Subsection 3.2, we do not use the general solution (2.3), only the system (2.2) itself. In this subsection we will show that the general solution (2.3), complemented by Eqs. (2.4), (2.5) and (2.6), for the heavy quark mass function possesses much more information than the direct solution of the system (2.2) on account of the expansions (3.14) provides at all.

Starting from the expansion (3.14) for the $A(x)$ function, which contributes into the quark wave function renormalization only, and using exact Eqs. (2.5) and (2.6), it is possible to show explicitly that it is determined by the solution (3.28), i.e., it is

$$A(x) = -\frac{1}{x+\bar{m}_Q^2} + D_A(x), \tag{4.1}$$

where the $D_A(x)$ function is given in Eq. (3.25). Thus in the case of the $A(x)$ function the straightforward solution of the initial system (2.2) completely coincides with exact solution, indeed.

Unfortunately, things are not so simple for the heavy quark mass function $B(x)$, which should be determined with the help of the exact solution (2.3), on account of the solution (4.1). Substituting it first into the relation (2.5) and then using the relation (2.4) and doing some tedious algebra, one finally obtains

$$\tilde{\nu}(x) = \frac{1-2\bar{m}_Q^2}{(x+\bar{m}_Q^2)^2} + \frac{2\bar{m}_Q^2}{(x+\bar{m}_Q^2)^3} + \overline{D}_A(x), \tag{4.2}$$

where

$$\begin{aligned}\overline{D}_A(x) &= D_A^2(x) + 2D_A(x) - \frac{2x}{(x+\bar{m}_Q^2)}D'_A(x) \\ &\quad + \frac{2x}{(x+\bar{m}_Q^2)^2}D_A(x) + 2xD_A(x)D'_A(x) \\ &\quad - \frac{2(3+2x)}{(x+\bar{m}_Q^2)}D_A(x) + 2(1+x)D_A^2(x).\end{aligned} \tag{4.3}$$

So the general solution (2.3) becomes

$$\begin{aligned}&B^2(c,\bar{m}_Q;x) \\ &\quad = \big(1-2\bar{m}_Q^2\big)\exp(-2x)\int_x^c \frac{\exp(2x')}{(x'+\bar{m}_Q^2)^2}\,dx' \\ &\qquad + 2\bar{m}_Q^2\exp(-2x)\int_x^c \frac{\exp(2x')}{(x'+\bar{m}_Q^2)^3}\,dx' \\ &\qquad + \overline{D}_B(c,\bar{m}_Q;x),\end{aligned} \tag{4.4}$$

where

$$\overline{D}_B(c,\bar{m}_Q;x) = \exp(-2x)\int_x^c \exp(2x')\overline{D}_A(x')\,dx'. \tag{4.5}$$

The first two integrals can be explicitly expressed in terms of the corresponding integral exponential function $Ei(x)$, so one has

$$\begin{aligned}&B^2(c,\bar{m}_Q;x) \\ &\quad = \big(1-2\bar{m}_Q^2\big)e^{-2x} \\ &\qquad \times\bigg[\frac{e^{2x}}{(x+\bar{m}_Q^2)} - \frac{e^{2c}}{(c+\bar{m}_Q^2)} \\ &\qquad\quad - 2e^{-2\bar{m}_Q^2}\big[Ei\big(2(x+\bar{m}_Q^2)\big) \\ &\qquad\qquad - Ei\big(2(c+\bar{m}_Q^2)\big)\big]\bigg] \\ &\qquad + 2\bar{m}_Q^2e^{-2x}\bigg[-\frac{e^{2c}}{2(c+\bar{m}_Q^2)^2} - \frac{e^{2c}}{(c+\bar{m}_Q^2)} \\ &\qquad\quad + 2e^{-2\bar{m}_Q^2}Ei\big(2(c+\bar{m}_Q^2)\big) \\ &\qquad\quad + \frac{e^{2x}}{2(x+\bar{m}_Q^2)^2} + \frac{e^{2x}}{(x+\bar{m}_Q^2)} \\ &\qquad\quad - 2e^{-2\bar{m}_Q^2}Ei\big(2(x+\bar{m}_Q^2)\big)\bigg] \\ &\qquad + \overline{D}_B(c,\bar{m}_Q;x).\end{aligned} \tag{4.6}$$

It is convenient to separate the dependence on the constant of integration c, so after some algebra one obtains

$$\begin{aligned}&B^2(c,\bar{m}_Q;x) \\ &\quad = \frac{1}{(x+\bar{m}_Q^2)} + \frac{\bar{m}_Q^2}{(x+\bar{m}_Q^2)^2}\end{aligned}$$

$$
\begin{aligned}
&- 2e^{-2(x+\bar{m}_Q^2)} Ei\big(2\big(x+\bar{m}_Q^2\big)\big)\\
&+ \overline{D}_B(c,\bar{m}_Q;x) + \widetilde{D}_B(c,\bar{m}_Q;x),
\end{aligned}
\tag{4.7}
$$

where

$$
\begin{aligned}
&\widetilde{D}_B(c,\bar{m}_Q;x)\\
&\quad = -\frac{1-2\bar{m}_Q^2}{(c+\bar{m}_Q^2)} e^{2(c-x)}\\
&\qquad + 2e^{-2(x+\bar{m}_Q^2)} Ei\big(2\big(c+\bar{m}_Q^2\big)\big)\\
&\qquad - 2\bar{m}_Q^2\left[\frac{1}{2(c+\bar{m}_Q^2)^2} + \frac{1}{(c+\bar{m}_Q^2)}\right] e^{2(c-x)}.
\end{aligned}
\tag{4.8}
$$

Evidently, by no means the exact solution (4.7) can be reduced to the free quark propagator solution (3.28) for the $B(x)$ function, except, maybe, of the asymptotical regime (see below). It is regular at zero.

It identically vanishes in the PT limit $\bar{g}^2 \to 0$, as it should be, since in this case $c = x \to \infty$ uniformly (see general solution (2.3)). In the heavy quark mass infinite limit ($m_Q \to \infty$ and $\bar{g}^2$ fixed), the quark momentum also goes to infinity (i.e., $x \to \infty$ see, for example Eq. (3.33)). In this case the constant of integration c remains finite, and therefore the composition $\widetilde{D}_B(c,\bar{m}_Q;x)$ also vanishes. Using further the asymptotics of the integral exponential function $Ei(z)$ as follows:

$$
Ei(z) = e^z\left[\frac{1}{z} + \frac{1}{z^2} + O\left(\frac{1}{z^3}\right)\right], \quad z \to \infty, \tag{4.9}
$$

from Eq. (4.7) one obtains

$$
B^2(c,\bar{m}_Q;x) \underset{x,\bar{m}_Q^2\to\infty}{\sim} \frac{\bar{m}_Q^2}{(x+\bar{m}_Q^2)^2} + \overline{D}_B(c,\bar{m}_Q;x). \tag{4.10}
$$

Choosing negative sign in the square root, one finally obtains

$$
\begin{aligned}
&B(\bar{m}_Q;x)\\
&\quad \underset{x,\bar{m}_Q^2\to\infty}{\sim} -\frac{\bar{m}_Q}{(x+\bar{m}_Q^2)}\\
&\qquad\qquad \times\left[1 + \frac{(x+\bar{m}_Q^2)^2}{\bar{m}_Q^2}\overline{D}_B(\bar{m}_Q;x)\right]^{1/2},
\end{aligned}
\tag{4.11}
$$

where $\overline{D}_B(\bar{m}_Q;x)$ does not depend on c and its explicit expression is not important here. Thus, only in the uniform limit $x, \bar{m}_Q^2 \to \infty$ the heavy quark propagator may become the free one up to the composition $\overline{D}_B(\bar{m}_Q;x)$, similar to Eqs. (3.28).

It is instructive to present explicitly a few first terms of the expansion (3.14) for the $B(x)$ function by substituting it directly into the general solution (2.3), on account of the known already expansion for the $A(x)$ function. Omitting all the tedious algebra one obtains

$$
\begin{aligned}
&B_0^2(x) = 1 - e^{2(c-x)},\\
&B_2(x) = \big[-x - 1 + e^{2(c-x)}(c-2)\big] B_0^{-1}(x),
\end{aligned}
\tag{4.12}
$$

and all odd terms are zero. In order to reproduce the free quark propagator case, one has to go again to the limit $x \to \infty$ and fixed c. Neglecting then the exponentially suppressed terms and choosing the negative sign in front of the square root, i.e., $B_0(x) = -1$, one obtains $B_2(x) = x + 1$ and so on. Summing up, one gets the free quark propagator solution (3.28), indeed. However, even in this case there is a solution with opposite sign, corresponding apparently to the free heavy antiquark propagator. Concluding, let us remind that it is a general feature of nonlinear systems, like the initial system (2.2), that the number of independent solutions is not fixed *a priori*.

5. Conclusions

We have shown that the quark propagator in 2D covariant gauge QCD reveals several desirable and promising features, so our conclusions are:

(1) The quark propagator has no poles, indeed (quark confinement).

(2) It also implies DBCS, i.e., a chiral symmetry is certainly dynamically (spontaneously) broken for light quarks, while a chiral symmetry preserving solution is forbidden.

(3) The chiral limit physics (i.e., the Goldstone sector) can be exactly evaluated, since we have found exact solution for the quark propagator in this case.

(4) We develop an analytical formalism, the so-called chiral perturbation theory at the fundamental quark level, which allows one to find solution for the quark propagator in powers of the light quark masses.

Each correction satisfies the differential equation, which can be formally solved exactly.

(5) We develop also an analytical formalism, which allows one to find solution for the quark propagator in the inverse powers of the heavy quark masses. It coincides with the free quark propagator up to terms of order $1/\bar{m}_Q^3$. So this solution automatically possesses the heavy quark flavor symmetry up to terms of order $1/\bar{m}_Q$.

(6) We have proved that the exact solution for the $A(x)$ function coincides with the direct solution of the initial system (2.2), obtained by using the above-mentioned expansions in the inverse powers of the heavy quark masses.

(7) At the same time, the exact solution (4.7) for the heavy quark mass function $B(x)$ by no means can be reduced to the free heavy quark propagator. So it is not coincided with the solution obtained by the expansions in the inverse powers of the heavy quark masses.

(8) There is no doubt left that by using the expansions in the inverse powers of the heavy quark masses from the very beginning, we are loosing some piece of the important information on the structure of the nonlinear system (2.2) itself. So its direct solution, obtained by using the straightforward expansions in the inverse powers of the heavy quark masses, has a particular character. Evidently, such straightforward solution does not take into account the response of the vacuum, which determines the modification of the quark propagator, while the exact solution (4.7) does take this response into the consideration.

(9) The general solution (2.3) does not demonstrate the principal difference in the analytical structure for light and heavy quarks propagators. Also at the fundamental quark level the heavy quark mass limit is not Lorentz covariant. That is why in the case of heavy quarks we will use the general solution (4.7) rather than solutions (3.30) and (3.36). To take into account the vacuum's response is important even for heavy quarks.

(10) Our approach makes it possible to calculate physical observables from first principles. All results will depend only on the renormalized coupling constant (which has the dimensions of mass) and the corresponding constant of integration. A physically well-motivated scale-setting scheme is only needed to fix them.

Our general conclusion is that 2D covariant gauge QCD implies quark confinement and dynamical breakdown of chiral symmetry without explicitly involving some extra degrees of freedom. The only dynamical mechanism responsible for them, which can be thought of in 2D covariant gauge QCD, is the direct interaction of massless gluons [7,8]. This interaction is a main dynamical effect not only in 2D QCD but in 4D QCD as well, i.e., in QCD itself. However, to directly generalize the quark confinement mechanism of 2D covariant gauge QCD to 4D QCD is impossible. The problem is that in former theory the coupling constant, having the dimensions of mass, plays the role of a mass gap, which was introduced and discussed by Jaffe and Witten (JW) in Ref. [12]. In latter theory the coupling constant is dimensionless, and there is none of the characteristic scales in QCD Lagrangian. If QCD confines then such a characteristic scale is very likely to exist, and it is not Λ_{QCD}, of course, which can be considered as responsible for the nontrivial PT dynamics in QCD (scale violation, asymptotic freedom). A possible way how to introduce a mass gap responsible for the nontrivial NP dynamics in QCD has been described in Ref. [13]. Its possible relation to the above-mentioned JW mass gap is also discussed there.

Acknowledgements

A financial support from HAS-JINR Scientific Collaboration Fund (P. Levai) and Hungarian Scientific Fund OTKA T30171 (K. Toth) is to be acknowledged.

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