New coupled models for transient pressure analysis in low permeability reservoirs

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Abstract

It is shown that the flow in the low permeability medium is nonlinear, which is quite different from the general Darcy flow. In this paper, we first present a survey of methods that has been used to analyze the low permeability well problems for the past 10 years, then we present two new coupled models that behaves better in complex situations. We assume that there are two different zones near the wellbore and the flow characteristics in the inner zone is different from that in the outer zone, and there is an connecting interface between them, as mentioned before, we called them coupled models.

In this paper, we also give four different types of the typical curves for four different models and use the methods of alternating iteration algorithm to solve the nonlinear differential equation system. These four different types of the curves will cover the typical problems that will possibly be encountered when exploring the low permeability reservoirs.

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Selection and peer-review under responsibility of RIUDS

Keyword: low–permeability reservoirs, well testing analysis, typical curves;

1. Introduction

Well testing for low permeability reservoirs has long been conducted in China. The major Chinese petroleum corporations, such as CNPC, Sinopec, CNOOC, have all been exploring more and more low permeability reservoirs in recent years. Under the high demand of the Chinese market, the low permeability reservoirs production have been taken into considerations more and more seriously.

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A lot of works have been done already in China. Cheng, Shiqing has long been given the solution using the analysis method based on the concept of the threshold pressure\cite{1}. Also, Zheng, Chunfeng, Yao, Jun, draw the curves of the homogenous low permeability reservoirs applying the concept of changing permeability effect\cite{2-3}. Later on, Yao, Jun, Wang, Jianjun gave the typical curves of dual permeability reservoir systems, they concluded that the permeability is pressure sensitive\cite{4}. There are few researches out of China, partly due to the lack of interests in production of low-permeability block. Only recently, there have been a paper written by Yu Shu, Perapon Fakcharoenphol which talks about the non-Darcy displacement problem in composite reservoirs\cite{5}.

2. The physical and mathematical models

We establish three different types of models about the low-permeability reservoirs (the first one has been solved by Cheng, Shiqing & Zheng, Chunfeng, as mentioned in reference 1, 2. the same solution is reached using alternative method), and there are non-Darcy flow, Darcy and non-Darcy coupled flow, dual non-Darcy coupled flow, see fig.1. All of them are homogenous and infinitely great, and there is an interface between two different zones in latter two models.

![Diagram of flow models](image)

Then we establish 3 different mathematical models, according to the fig.1 mentioned above. As for the non-Darcy flow, the permeability $K$ is a function of the $dp/dr$ ($K = K(dp/dr)$). We will use this in the following discussion (the equation of motion):

Non-Darcy flow model:

\[
\frac{\partial P^2}{\partial x^2} + \frac{\partial (dp/dx)}{\partial x} \frac{\partial P}{\partial t} = \phi \mu C r_w e^{2x} \frac{\partial P}{\partial t}
\]

Darcy & non-Darcy coupled flow model:

This model is similar with the pure non-Darcy model. There is an additional inner zone of Darcy flow, and there is also an interface between them, the outer zone pressure and flow rate equals the inner ones.

\[
\frac{\partial P^2}{\partial x^2} = \phi \mu C r_w e^{2x} \frac{\partial P}{\partial t}
\]

\[
K \frac{\partial P^2}{\partial x^2} + \frac{\partial K(dp/dx)}{\partial x} \frac{\partial P}{\partial t} = \phi \mu C r_w e^{2x} \frac{\partial P}{\partial t}
\]

\[
\text{equation of interface condition:} \quad p_1(x,t) = p_2(x,t) \quad \text{at} \quad \frac{\partial v_1(x,t)}{\partial x} = \frac{\partial v_2(x,t)}{\partial x}
\]

dual non-Darcy coupled flow model:

The model here also is similar to the above one. We just change the equation of inner zone.
equation of inner non-Darcy filtering flow:
\[ K_i \frac{dp_i}{dx} \frac{\partial^2 P}{\partial x^2} + \frac{\partial K_i \frac{dp_i}{dx}}{\partial x} \frac{\partial P}{\partial x} = \phi \mu \varepsilon x^2 r_w^2 \frac{\partial P}{\partial t} \]

equation of outer non-Darcy filtering flow:
\[ K_i \frac{dp_i}{dx} \frac{\partial^2 P}{\partial x^2} + \frac{\partial K_i \frac{dp_i}{dx}}{\partial x} \frac{\partial P}{\partial x} = \phi \mu \varepsilon x^2 r_w^2 \frac{\partial P}{\partial t} \]
equation of interface condition:
\[ p_1(x,t) = p_2(x,t) \bigg|_{x=x_w}, \quad v_1(x,t) = v_2(x,t) \bigg|_{x=x_w} \]

All other equations below are same:
equation of initial condition:
\[ p(x,t) \bigg|_{t=0} = p_e \]
equation of inner boundary condition:
\[ Bq = \frac{2 \pi h K_	ext{well}}{\mu} \left( \frac{\partial P}{\partial x} \right)_{t=0} - 24 C \frac{dp_w}{dt}, \quad P_w = \left[ P_w - S \frac{\partial P_n}{\partial x} \right]_{t=0} \]
equation of external boundary:
\[ \lim_{x \to \infty} p(x,t) = p_e \]

3. The approach of numerical solution

Apparently, the three different mathematical models above are non-linear, so it is applicable to solve the problem using the method of alternating iteration algorithm, then we can draw the typical curves of all the models. We can also discuss the parameters inside according to the curves we have already gotten.

3.1. discretization

We will discuss discretization of three different models comparatively. These parameters below will apply to all of the models:
\[ m = \frac{3.6 k_w d_i}{e^{2s} e^{2s} d^2 \pi^2 r_w^2 \phi \mu C_i}, \quad m_2 = \frac{3.6 d_i}{e^{2s} e^{2s} d^2 \pi^2 r_w^2 \phi \mu C_i}, \quad k_i^{n+1} = (k_i^n + k_i^{n+1}) / 2, \quad k_i^{n+1} = (k_i^n + k_i^{n+1}) / 2 \]

and \( m \) is referred to numerical nodal point of the interface. \( g_{\text{max}} \) is meant to the maximum number of grid.

non-Darcy flow model:
discretized equation of non-Darcy flow motion:
\[ -m_2 k_i^{n+1} p_i^{n+1} + (m_2 (k_i^{n+1} + k_i^{n+1}) + 1) P_i^{n+1} - m_2 k_i^{n+1} p_i^{n+1} = P_i^n \]
discretized equation of initial condition:
\[ p_i^n = p_e \bigg|_{i=1,2,\ldots,g_{\text{max}}} \]
discretized equation of inner boundary condition:
\[ \frac{k_i^{n+1}}{1.842 \times 10^{-3} \mu dx} + \frac{24 C}{dt} p_i^{n+1} - \frac{k_i h}{1.842 \times 10^{-3} \mu dx} p_i^{n+1} = \frac{24 C}{dt} - q \]
discretized equation of external boundary:
\[ p_{g_{\text{max}}} = p_e \]

Darcy&non-Darcy coupled flow model:
discretized equation of inner Darcy flow:
\[ -m P_i^{n+1} + (2m + 1) P_i^{n+1} - m P_i^{n+1} = P_i^n \]
discretized equation of outer non-Darcy flow:
\[ -m_2 k_i^{n+1} P_i^{n+1} + (m_2 (k_i^{n+1} + k_i^{n+1}) + 1) P_i^{n+1} - m_2 k_i^{n+1} p_i^{n+1} = P_i^n \]
discretized equation of initial condition:
\[ p_i^n = p_e \bigg|_{i=1,2,\ldots,g_{\text{max}}} \]
discretized equation of inner boundary condition:
\[
\frac{k_{h}}{1.842 \times 10^{3}} \mu dx \frac{24C}{dt} p^{n+1} + \frac{k_{h}}{1.842 \times 10^{3}} \mu dx \frac{24C}{dt} p^{n+1} = 24C - q
\]

discretized equation of external boundary:

\[
p^{n+1}_{\text{ex}} = p_{e}
\]

discretized equation of external boundary:

\[
-k_{nm-1}^{n+1}p^{n+1}_{nm-1} + (k_{nm-1}^{n+1} + k_{nm+1}^{n+1})p^{n+1}_{nm} - k_{nm+1}^{n+1}p^{n+1}_{nm+1} = 0
\]

dual non-Darcy coupled flow model:

discretized equation of external boundary:

\[
p^{n+1}_{\text{ex}} = p_{e}
\]

discretized equation of interface condition:

\[
-m_{i}k_{j-1/2}^{n+1}p^{n+1}_{j-1} + (m_{i}(k_{j-1/2}^{n+1} + k_{j+1/2}^{n+1}) + 1)p^{n+1}_{j} - m_{i}k_{j+1/2}^{n+1}p^{n+1}_{j+1} = P^{n+1}_{i}
\]

discretized equation of outer non-Darcy flow:

\[
-m_{i}k_{j-1/2}^{n+1}p^{n+1}_{j-1} + (m_{i}(k_{j-1/2}^{n+1} + k_{j+1/2}^{n+1}) + 1)p^{n+1}_{j} - m_{i}k_{j+1/2}^{n+1}p^{n+1}_{j+1} = P^{n+1}_{i}
\]

discretized equation of initial condition:

\[
p^{1}_{i} = p_{e} \bigg|_{s_{i} = 1, 2, \ldots, S_{\text{max}}}
\]

discretized equation of inner boundary condition:

\[
\frac{k_{h}}{1.842 \times 10^{3}} \mu dx \frac{24C}{dt} p^{n+1} + \frac{k_{h}}{1.842 \times 10^{3}} \mu dx \frac{24C}{dt} p^{n+1} = 24C - q
\]

discretized equation of external boundary:

\[
p^{n+1}_{\text{ex}} = p_{e}
\]

discretized equation of interface condition:

\[
-k_{nm-1}^{n+1}p^{n+1}_{nm-1} + (k_{nm-1}^{n+1} + k_{nm+1}^{n+1})p^{n+1}_{nm} - k_{nm+1}^{n+1}p^{n+1}_{nm+1} = 0
\]

All of the three models’ numerical equations can be solved separately by the method of alternating iteration algorithm. We can calculate the distribution of pressure about low permeability reservoirs at any time, then we can get the pressure of wellbore plot vs. time.

3.2. The procedures solving the nonlinear problems in low permeability reservoirs

(1) According to the core infiltration experiment, we draw the curve of flow rate vs. pressure gradient, then we can derive the relationship between permeability&pressure using the equation of

\[
v = -\frac{K(dp/dx)}{\mu} \left[ dp \right] \left[ \frac{dP}{dx} \right].
\]

All of these can be seen in fig.2.

(2) We divide the curves of fig.2.(b) into many linear pieces. The permeability K equals a separate constant value.

(3) Alternating iteration algorithm will be used to solve the problem of the non-Darcy’s nonlinear problem, first we should make an initial value of the permeability in the nonlinear differential equation system, then the pressures will be calculated by the method of alternating iteration algorithm. We calculate the pressure gradient in each grid again, after that we can obtain the permeability of each grid.
which is calculated from the curve of dp/dr v.s. K plot. We use this permeability, then substitute it into the nonlinear differential equation system. With sufficient time of calculation, we can obtain the results with desired accuracy. We can then calculate the exact pressures of the low permeability reservoirs, substitute them into the next differential equation system, and resume the following calculation.

We can clearly see from the fig.2.(a). When the pressure gradients are low, the plot shows apparent nonlinear response. Using the formula of \[ v = \frac{-K(dp/dr)}{\mu} \], we can plot dp/dr vs. K easily, as it is shown in fig.2.(b). We can see that the permeability is changing and it is a function of dp/dr(K=K(dp/dr)). The relevant data we use for numerical calculation are: q = 25m³/d, Pe = 16.5MPa, Ct = 0.0014MPa⁻¹, φ = 0.12, h = 10 m, \( \mu_1 = 1 \text{mPa.s} \), \( \mu_2 = 0.5 \text{mPa.s} \), B = 1.0, \( R_w = 0.062 \text{m} \), \( k_{max} = 1.95 \text{md} \), S = 0.2.

4. The plots of the three non-Darcy models

We draw the typical well-testing plots of the three models mentioned above, and we will discuss the results of these typical curves according to the nonlinear numerical solution.

- Non-Darcy flow model:
  Using the plot of the fig.2.(a), when the dp/dr vs v plot is the type v1, we can get the typical well testing plot below. Similarly, using the type v3 of the fig.2.(a), we can get the same well testing plot (The horizontal axis is the dimensionless time, and the vertical axis is the dimensionless pressure. CD=5000).

![fig.3.(a) Typical curve using v1 model of fig.2.(a); (b) Typical curve using v3 model of fig.2.(a);](image)

From the fig.3, we can clearly see that the degree of the nonlinear non-Darcy filtrating is related to the shape of the typical curves. As we can see from the fig.3., if we assume that we get the K1 changing permeability (as we can see from fig.2.(b)), the typical curve we will get will be like the fig.3.(a), the overlapping length of the PD&dPD is long and we hardly get any useful information using the traditional matching method. Comparatively, when the nonlinear filtrating degree is the K3 (from fig.3.(b)), we will get the curve like the fig.3.(b). We can get that if we had the better non-Darcy filtrating condition, we would obtain better shape of the typical curve. We can clearly see the curves of PD&dPD apart, and we can get more accurate information of the low-permeability reservoirs through the matching.

A lot of the works have already been done about the the pure low permeability non-Darcy model, if interested, please look for the paper written by Cheng, Shiqing, Zheng, Chunfeng (as I put them in reference [1] & [2]).

- Darcy & Non-Darcy coupled flow model:

When we develop the low-permeability reservoirs, usually we will adopt the water-flood program to explore them. The flow of the areas near the wellbore can be assumed to be Darcy flow, because this area near the wellbore gets more flushing of the injection water, and this will in turn change the condition of the reservoirs. The physical model can be seen from the fig.1.(b). Using the alternating iteration algorithm, we will get the typical curves of this model with different range of interface. As can been seen from fig.4.
As clearly shown in the Fig.4., the curves from top to bottom represent the range of interface rD 20, 150, 400, 8100, 22000 correspondingly. rD represents the dimensionless interface range (rD = r_inter/ r_w, refer to the appendix for all of the dimensionless parameters). The permeability K of inner zone is 1.95 md, the plot of the filtrating of the outer zone is the K1 in fig.2.(b).

We can conclude that the curve of this model can be divided into three parts, the overlapping head, the hump in the middle, and the upward bending rear. The non-Darcy flow of the outer zone will cause the upward bending in the typical curves, and the longer distance of the interface, the later the curve will bend upward. When the rD is 22000, which we can see this range is long enough to be assumed infinite, the later part of the typical curve equals 0.5, which we can assume it to be pure Darcy flow.

Also, when the range interface is quite short (just like the curve Rd=20), All the things we can see is the overlapping part, which indicates that the length of the interface has a strong relation with the shape of the curve. The closer near the wellbore, longer the overlapping part.

We then draw the curves which use the changing permeability mode land threshold pressure model, as shown in fig.5..
body from the geology point of view. This time we will introduce two different dual non-Darcy coupled
flow model curves, the first one is the one that have inner zone condition better than outer zone, and the
second one is the one that have worse inner zone condition. Both of the zones are non-Darcy flow. The
physical model can be seen at the fig.1.(c).

In the same way, we use the alternating iteration algorithm to obtain the results, and we can plot the
typical curves of two different model. As shown in Fig.6.

![Fig.6.](image)

**Fig.6.(a)** dual non-Darcy coupled flow model typical curves (CD=5000, rD=20,150,400,8100,22000,K2 from fig.2(b) inner
zone,and K1 is outer zone);

**Fig.6.(b)** dual non-Darcy coupled flow model typical curves (CD=5000, rD=22000,8100, 400,150, 20,K2 from fig.2(b) inner
zone, and K3 is outer zone);

In the fig.6.(a), K2 filtrating rule represents the inner zone and K1 represents the outer zone, the curves
from top to bottom are rD=20,150,400,8100,22000 correspondingly, while in the fig.6.(b), K2 is inner
zone and K3 is outer zone, on the contrary, the curves from top to bottom are rD=22000,8100,400,150,20.

Now we can discuss the data inside fig.6.. Base on fig.6.(a), we can conclude that: when the the
filtrating rule of inner zone is better than outer zone, we still get curves that are divided into three parts:
the overlapping head; the hump in the middle; and the upward bending in the rear. The upward bending of
the last part is caused by the non-Darcy flow of the outer zone. The longer the interface range, later
upward bending will occur, when the range of interface is long enough, just as rD=8100,22000, we will
not see the upward bending part, maybe more time will be needed for the occurrence of upward bending.
If the rD is fairly short, just as rD=20, all the curves overlap as a whole.

There are observable differences in fig.6.(b), the curves from top to bottom are rD=22000,8100,400,150,20 separately. But it is clear and understandable: when outer zone filtrating rule
is better than inner zone, the closer to the wellbore, the faster it takes to see curve to bend downward.
There are only two parts in the curves, the overlapping part and the hump part, and the downward
bending is caused by the better filtrating rule of outer zone. The shorter of the interface range, steeper the
hump we will get.

5. Summary of the three different models

This time we will discuss the shared phenomenon in the three models. We will compare four different
models, they are the pure non-Darcy flow, the dual non-Darcy coupled flow, the Darcy&non-Darcy
coupled flow and the pure Darcy flow. The typical curve of 4 different models and their pressure profile
can be seen in fig.7.
We can see from the fig.7(a) (The dual non-Darcy flow model curve is from the fig.6.(a), the pressure profile is the same), the curves from top to bottom are pure non-Darcy flow model, dual non-Darcy coupled flow model, Darcy&non-Darcy flow coupled flow, pure Darcy flow correspondingly. The existence of the non-Darcy phenomenon cause the longer time of the overlapping, in pure non-Darcy condition, all we can see is the overlapping part. The non-Darcy filtrating rule can cause the upward bending in the later part of the typical curve. As the inner zone condition get better, we can get curves that have their humps appeared early and clear. The last curve we get is for the pure Darcy flow model, it is the lowest, and its later derivative curve equals 0.5. From top to bottom, the filtrating condition is getting better, and this indicates that the lower the curve in the plot, the better reservoir condition it is.

Fig.7.(b) give the pressure profile plot of the four different models, we can see that the worse condition of the reservoir, the higher injection pressure we will need. In pure non-Darcy flow model, we must make the injection pressure as high as 40Mpa, and this pressure will dramatically drop to only 23Mpa in pure Darcy flow. The other two coupled flow models injection pressure are in between, the exact values are 37Mpa & 28Mpa respectively. This plot partially explains the fact that the better condition of the reservoir can cause the lower injection pressure, as shown in Fig.7.(b). The existence of the non-Darcy flow can cause the high injection pressure, and this may cause the injection water to open the cleft that exists, and in turn cause the oil well near the injection well to be flooded quickly. This has been seen many times in the Chinese low permeability reservoir block, and is still a major problem to the development engineers.

6.Conclusions

In conclusion, we give the general conclusions about these three models. Many low-permeability reservoir have the actual curves resemble the typical plots of the three models. So our models are reasonable and acceptable in practical use. Here are the conclusions we have reached.

1. We considered three different non-Darcy models, as shown in fig.1, alternating iteration algorithm was used to solve the equations. We can get the pressure of all grids with desired accuracy, then we can generate the typical well-testing plot. The algorithm we used is steady and reasonable.

2. As for the well-testing typical plots of the three models, we can sum up some general rules about them:

For the pure non-Darcy flow model, the filtrating rule is related to the shape of the curve, when the filtrating condition is worse, we will get plots with longer overlapping, and humps will be seen on the plot when the condition gets better.

For the Darcy&non-Darcy coupled flow, the interface range is strongly related to the shape of the plots. When the interface range is short, all we can see on the plot are the overlapping curves, as the interface
range grows, we can gradually see the curves with three parts: overlapping head, hump in the middle and upward bending in the rear, and the upward bending in the last part is caused by the non-Darcy flow. When the interface range is long enough, we can get the derivative curve which equals 0.5, and this means simple Darcy flow.

For the dual non-Darcy coupled flow, we divide the situation into two categories, the one inner zone filtrating condition is better than outer zone and the one inner zone filtrating condition is worse. In the first model, the shape of the plot is similar to the Darcy&non-Darcy coupled flow model, the upward bending at the last part is caused by the non-Darcy flow and when the interface range is long enough, we can not get the derivative curve which equals 0.5. All we can get is the humped part. For the second model, we will get the plot with only the humped part, the closer of the interface range, the steeper of the hump.

3. We give the comparison of the four different models: the pure non-Darcy flow model, the dual non-Darcy coupled flow model, the Darcy&non-Darcy coupled model and the pure Darcy flow model. We give the comparison of the typical well testing plot and pressure profile separately. We can see from the fig.7., The existence of the non-Darcy phenomenon cause the longer time of the overlapping. The non-Darcy filtrating rule can cause the upward bending in the later part of the typical curve. As the inner zone condition gets better, we can get the curves with their hump part appeared early and clear. The pressure profile plot of the four different models show that the worse condition of the reservoir, the higher injection pressure we will need, this will partly explain the quick-flooded problem in low permeability reservoir block.

4. New non-Darcy flow models should be used in the analysis of low permeability block, if we used the traditional way to analyze the actual low permeability block well testing plot, we will get higher value of the wellbore storage constant C and higher initial pressure P_i. In the traditional way, radial composite model are usually used to analyze. This model has long be questioned by many engineers, because the parameters we get from this model often appear unreasonable. The upward bending of the later part of the well testing curve can be also caused by non-Darcy flow, not just the change of the reservoir condition.

References