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Study on the behaviour of rubber aggregates concrete beams using analytical approach

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A B S T R A C T

Concrete is one the most extensively used construction material all over the world. Many scientists and researchers are in quest for developing alternate construction material that are environment friendly and contribute towards sustainable development. Huge amount of rubber tyres waste is being generated day by day which creates the disposal problem and has many environmental issues. As this scrap rubber waste is an elastic material having less specific gravity, energy absorbent material can be used as a replacement material for obtaining lightweight concrete. In present study an attempt is made to partially replace the rubber aggregates by coarse aggregates in concrete and to study its impact on properties of concrete. A modified concrete is prepared by replacing coarse aggregates in concrete with rubber aggregates by varying the replacement proportion from 0% to 20% with increment of 5%. 3 cubes for each percentage of replacement are casted and tested after 28th days of curing. The physiomechanical properties like density, compressive strength and elastic properties of modified concrete are determined from concrete cubes experimentally and further stresses and displacement at every 50 mm depth of beams are determined analytically by method of initial functions (MIF). MIF is an analytical method in which elastic properties and theoretical loads are used to analyse the beams without conducting any experimental programme. The analytical results by MIF are compared with bending theory.

1. Introduction

The scarcity and availability of sand and aggregate at reasonable rates are now giving anxiety to the construction industry. Over years, deforestation and extraction of natural aggregates from river beds, lakes and other water bodies have resulted in huge environmental problems. Hence, to prevent pollution authorities are imposing more and more stringent restrictions on the extraction of natural aggregates and its crushing. The best way to overcome this problem is to find alternate aggregates for construction in place of conventional natural aggregates. In this research rubber aggregates from discarded tyre rubber in size ranges from 10 to 20 mm is partially replaced with natural aggregates in cement concrete. This attempt of replacing the coarse aggregates with rubber aggregates will save the natural aggregates, reduces weight of structure and also helps achieve sustainability.

1.1. Method of initial functions (MIF)

MIF was first proposed by Malieev in 1951 and further developed by Vlasov in 1955. Beams that are built of more than one material are called composite beams. It is difficult to analyse the laminated beams by the bending theory used for ordinary beams. In MIF, equations governing the flexure of composite laminated beams are derived without making any assumption regarding the physical behaviour of beams. The method of initial functions (MIF) has been used for deriving the equations. It is an analytical method of elasticity theory allows us to obtain the exact solutions for certain types of problems without use of hypotheses about the character of the stress strain state of the structural element. In recent years the MIF has been used intensively for the analysis of various problems. For example, three dimensional elasticity equations for circular cylindrical shells are solved by assuming Taylor series expansions for finding stresses and displacements [18].

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Notations

L	length of beam	G	shear modulus of elasticity
H	depth of beam	μ	Poisson's ratio
b	width of beam	v	vertical displacement
d	density of concrete	X	shear stress
E	Young's modulus of elasticity	σ_x	bending stress
F	characteristics compressive strength of concrete		

2. Literature review

Recycled tyre rubber is used to develop deliberately low elastic modulus and highly ductile ECC repair material so as to alleviate repair failure induced by restrained drying shrinkage [1]. Tyre aggregate replaced with coarse aggregate with various percentages and compared with regular concrete. Fresh and hardened concrete strength were identified [2]. Rubber waste additives reduced both static and dynamic modulus of elasticity [3]. Higher content of waste tyre crumb rubber particle used in concrete increases workability of concrete and produce the lightweight concrete [4]. Compressive strength of concrete with 100% replacement of chipped rubber showed 90% reduction. However reduction in strength was 80% when crumb rubber was used as 100% replacement to sand in concrete [5]. The replacement of coarse aggregate by junk rubber in concrete has resulted in reduced compressive strength and densities [6]. Rubber tyre aggregate were added to M35 mix with different percentages. Gradual reduction in the compressive and tensile strength was observed. Upto 8% of rubber aggregate can be added in the concrete without considerable reduction in strength [7]. Replacement of rubber increased water permeability depth in the concrete and increases the water absorption in case of coarse aggregate replacement but reduces the water absorption in case of cement replacement. [8]. Slump value is decreased as the percentage of replacement of scrap tyre rubber increase so decrease in workability. Also decrease in compressive, split tensile and flexural strength. In the rubberized concrete the loss of strength was 45% with 15% replacement of coarse aggregate with rubber particle [9]. Replacing conventional fine aggregate with crumb rubber at 10–30%, the unit-weight of concrete can be reduced from 14% up to 28% depending on the type and the content of the crumb rubber. The concrete exhibits superior thermal and

Fig. 1. Rubber aggregates 10–20 mm.

Fig. 2. Concrete cubes.

Table 1
Physical properties of materials used.

Material	Specific gravity	Bulk density (kg/m^3)
Rubber aggregates	1.10	650
Fine aggregates	2.6	1650
Coarse aggregates	2.8	1720

Table 3
Compressive strength and density of concrete.

% of replacement	Ultimate load (kN)	Compressive strength (N/mm^2)	Weight of cubes in kg	Density in kg/m^3
0%	713.25	31.7	8.15	24141.81
5%	657.67	29.23	7.56	2240.702
10%	576.22	25.61	7.23	2143.14
15%	480.05	21.34	6.51	1928.88
20%	373.72	16.61	6.34	1879.933

Table 2
Mix proportion (kg/m^3).

Replacement %	Cement (kg)	Water in litres (W/c = 0.50)	Fine aggregates (kg)	Coarse aggregates (kg)	Rubber aggregates (kg)
0%	364.81	225.17	610.43	1239.64	–
5%	437.77	224.2	590.03	1177.65	23.30
10%	437.77	224.2	590.03	1115.67	46.73
15%	437.77	224.2	590.03	1053.69	70.101
20%	437.77	224.2	590.03	991.71	93.46

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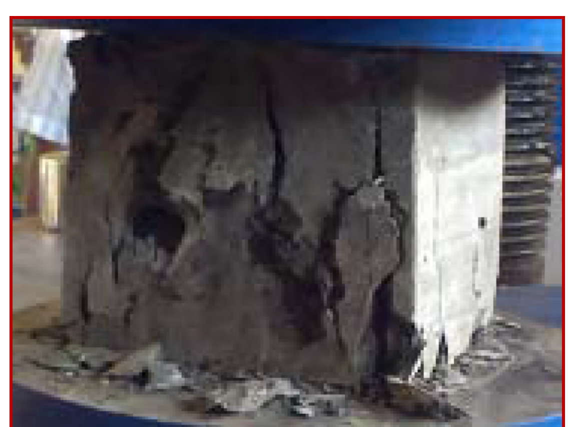
sound properties than plain concrete as measured by the decrease in thermal conductivity coefficient; increase in sound absorption coefficient and noise reduction coefficient [10]. Method of initial functions is used for two dimensional elasto dynamic problems for plain stress and plain strain conditions [11]. MIF has been applied for deriving higher order theories for laminated composite thick rectangular plates [12]. Method of initial function is used for the study of composite beams having two layers of orthotropic material and developed governing equation [13]. Result obtained with MIF are compared with result obtained by FEM based software ANSYS and it is observed that they are comparable [14]. MIF is successfully applied for the analysis of reinforced concrete brick layer [15]. The method of initial function is used to see the effect of elastic properties on the behaviour of the beam [16]. The deflection obtained by MIF is equal to the deflection obtained by other theories. The normal stress equal to the intensity of loading and shear stress equal to zero at the top of beam are obtain, this shows that MIF is successfully applied for the analysis of beam

(a) Cube with normal concrete.

(b) Cube with 5% rubber aggregates.

(c) Cube with 10% rubber aggregates.

(d) Concrete cube with 15% rubber aggregates.



(e) Concrete cube with 20% rubber aggregates.

Fig. 3. Failure of concrete cubes.

[17]. MIF gives correct result for both shallow and deep beams. Also in this method it is not necessary to assume the position of neutral axis; it incorporates the position of neutral axis by itself. Hence it is concluded that analysis done by MIF provides more realistic behaviour of beam sections of any depth.

3. Material and methods

The material used for experimental work is ordinary Portland cement (OPC) of grade 53, river sand with a fineness modulus of 2.40, natural aggregates and rubber aggregates with a size ranges from 10 to 20 mm. The rubbers aggregate shown in Fig. 1 are obtained by shredding the heavy vehicles scrap tyre rubber.

Table 1 shows the physical properties of material required for preparing the mix proportion of the rubber concrete as shown in Table 2 M25 grade concrete is used. The percentage of replacement is done by volume. For each percentage three specimens of cubes size 150 × 150 × 150 mm are casted and tested after 28 days of curing. The casted concrete cubes are shown in Fig. 2.

3.1. Testing of concrete cubes

The concrete cubes were tested according to IS 516:1959 code for methods of test for strength of concrete. Before testing weight of concrete cubes were noted to determine the density of concrete. The concrete cubes are tested on compressive strength testing machine of 2000 kN capacity. The load is applied without shock and increased continuously at a rate of approximately 140 kg/sq cm/min until the resistance of the specimen to the increasing load breaks down and no greater load can be sustained. The maximum load applied to the specimen shall then be recorded and the appearance of the concrete and any unusual features in the type of failure are noted (see Table 3).

In Fig. 3 for 0% and 5% replacement ductile failure is observed i.e. extensive plastic deformation and energy absorption before the failure. For 10% to 15% replacement elastic failure takes place. When load is applied to the specimen it absorbs more energy but after failure when the applied load was removed due to rubber aggregates in concrete the cracks formed are recovered to some extent. For 20% replacement propagation of crack is very fast. Cracks propagate nearly perpendicular to the direction of the applied stress. The ultimate load carried by these specimens is less but toughness is more as compared to the other concrete mix.

4. Analysis of composite beam by MIF and bending theory

For finding the stresses and displacement MIF is used. As coarse aggregates are replaced with rubber aggregates in concrete it

Table 4
Load and elastic properties.

Material	% of replacement of coarse aggregates	Load P_o (kN)	E (N/mm^2)	G (N/mm^2)	μ
Coarse aggregates replaced with rubber aggregates concrete	0	100	28729.13	1197.47	0.20
	5	98.80	24658.06	10274.19	0.20
	10	97.50	21589.82	8995.75	0.20
	15	93.20	16827.65	7011.52	0.20
	20	85.20	14284.55	5951.89	0.20

Table 5
Beam loaded with point load 100 kN.

	Depth of beam (mm)	0	50	100	150	200	250	300	350	400
MIF	V (mm)	0.512	0.512	0.512	0.511	0.511	0.511	0.511	0.510	0.511
	X (N/mm^2)	0.000	0.530	0.810	0.930	0.960	0.920	0.790	0.510	0.000
	σ_x (N/mm^2)	−6.265	−4.580	−2.870	−1.800	0.481	1.985	3.930	5.740	7.670
Bending theory	V (mm)	0.430	0.430	0.430	0.430	0.430	0.430	0.430	0.430	0.430
	σ_x (N/mm^2)	−7.500	−5.600	−3.750	−1.900	0.000	1.900	3.750	5.600	7.500

Table 6
Beam loaded with point load 98.80 kN.

	Depth of beam (mm)	0	50	100	150	200	250	300	350	400
MIF	V (mm)	0.579	0.579	0.579	0.576	0.576	0.577	0.575	0.575	0.574
	X (N/mm^2)	0.000	0.430	0.730	0.880	0.930	0.850	0.670	0.380	0.000
	σ_x (N/mm^2)	−6.246	−4.430	−2.801	−1.420	0.370	1.875	3.711	5.575	7.530
Bending theory	V (mm)	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
	σ_x (N/mm^2)	−7.410	−5.550	−3.705	−1.852	0.000	1.852	3.705	5.550	7.410

Table 7
Beam loaded with point load 97.50 kN.

	Depth of beam (mm)	0	50	100	150	200	250	300	350	400
MIF	V (mm)	0.648	0.648	0.646	0.646	0.644	0.644	0.641	0.641	0.641
	X (N/mm^2)	0.000	0.400	0.680	0.840	0.880	0.810	0.640	0.350	0.000
	σ_x (N/mm^2)	−6.036	−4.364	−2.620	−1.372	0.350	1.840	3.685	5.485	7.325
Bending theory	V (mm)	0.565	0.565	0.565	0.565	0.565	0.565	0.565	0.565	0.565
	σ_x (N/mm^2)	−7.310	−5.475	−3.650	−1.820	0.000	1.820	3.650	5.470	7.310

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Table 8
Beam loaded with point load 93.20 kN.

	Depth of beam (mm)	0	50	100	150	200	250	300	350	400
MIF	V (mm)	0.805	0.805	0.802	0.802	0.800	0.800	0.800	0.800	0.800
	X (N/mm^2)	0.000	0.390	0.660	0.810	0.850	0.780	0.620	0.350	0.000
	σ_x (N/mm^2)	−6.015	−4.080	−2.560	−1.110	0.290	1.755	3.569	5.450	7.068
Bending theory	V (mm)	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694
	σ_x (N/mm^2)	−6.990	−5.240	−3.490	−1.740	0.000	1.740	3.490	5.240	6.990

Table 9
Beam loaded with point load 85.50 kN.

	Depth of beam (mm)	0	50	100	150	200	250	300	350	400
MIF	V (mm)	0.919	0.919	0.916	0.915	0.911	0.911	0.910	0.910	0.910
	X (N/mm^2)	0.000	0.380	0.640	0.790	0.830	0.760	0.600	0.340	0.000
	σ_x (N/mm^2)	−5.660	−4.060	−2.540	−1.060	0.150	1.610	3.500	5.130	6.440
Bending theory	V (mm)	0.747	0.747	0.747	0.747	0.747	0.747	2.652	0.747	0.747
	σ_x (N/mm^2)	−6.390	−4.790	−2.590	−1.590	0.000	1.590	2.590	4.790	6.390

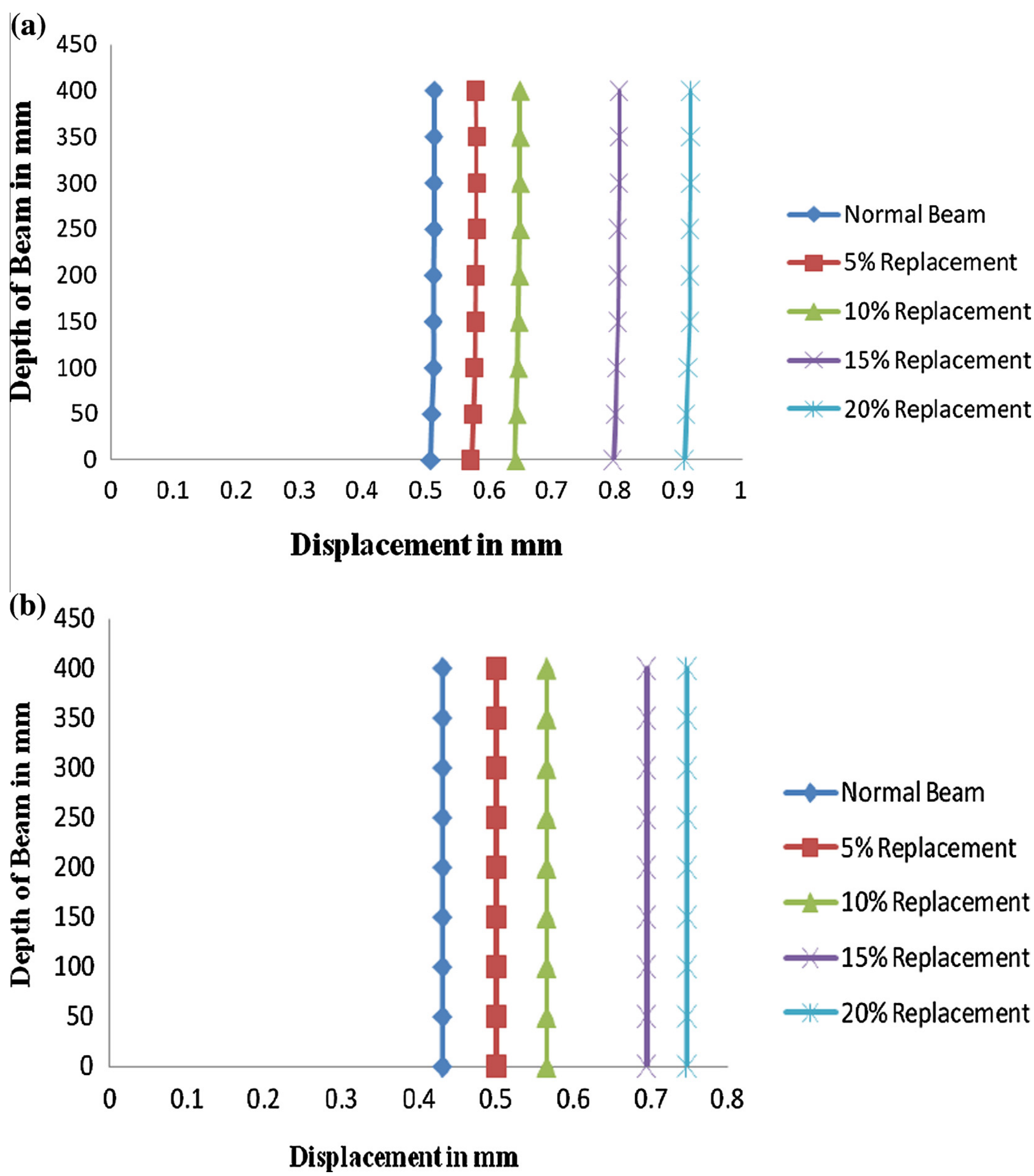

Fig. 4. (a) Displacement Vs depth of beam by MIF. (b) Displacement Vs depth of beam by bending theory.

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forms a composite material. The concrete beams of such composite materials cannot be analysed by the common theories. So in this study MIF is used in which elastic properties and theoretical loads are used to analyse the beams without conducting any flexural test or experimental programme. The elastic properties (E, G, μ) of modified concrete are determined from concrete cubes results. The theoretical loads P_o are calculated depending on characteristics compressive strength of cube after 28th days of curing and according to limit state design of beams. The calculated values theoretical loads P_o are given in Table 4. The modulus of elasticity (E) is determined by using empirical relation $E = 0.043d^{1.5}f^{0.5}$ [01]. This relation depends on characteristic compressive strength and density of concrete after 28 days of curing which is applicable for concretes with densities in the range of 1440–2560 kg/m^3. μ is constant for all cases and G is determined by using the expression $G = \frac{E}{2(1+\mu)}$.

The following values of beam dimensions are chosen for the particular problem.

$H = 400$ mm, $l = 2000$ mm, $b = 150$ mm.

The boundary condition of simply supported edges is given by $X = Y = v = 0$, at $x = 0$ and $x = l$.

A point load is assumed, on top surface of the beam. The expression for point load in sine series is given by

$$p(x) = \frac{2P_o}{l}\sum_{n=1}^{\infty}\sin\frac{\pi x}{l}$$

The stresses and displacement are determined at every 50 mm depth of beam.

4.1. Analytical results and discussion

The following are analytical results of stresses and displacement determined by using the MIF and bending theory. The elastic properties and loads are used to analyse the beams without conducting any experimental programme. The beam is analysed for each percentage of replacement and the comparison of results by MIF is done with bending theory results.

Tables 5–9 show the analytical results of displacement and stress (both bending and shear) which are determined by using MIF and bending theory. These results are discussed below by plotting the graphs for each percentage of replacement across the depth of beam.

Fig. 4 shows variation of displacement across the depth of beam. The variation of displacement (v) is almost linear across the depth. It increases with increase in percentage replacement of coarse aggregates by rubber aggregates in concrete. Less variation is observed 5% and 10% but displacement is more for 15% and 20% as compared to normal beam. The displacement results by MIF shows the exact displacement i.e. displacement at every depth of beam varies as compared to the bending theory results.

Fig. 5 shows variation of shear stress for different percentage of replacement across the depth of beam. There is no need to validate the results of shear stress with bending theory because the physical conditions of beam that shear stress is maximum at centre and zero at support of beam is exactly satisfied. As percentage replacement of coarse aggregates by rubber aggregates in concrete increases the shear stress decreases.

Fig. 6 shows variation of bending stress across the depth of beam for different percentage of replacement. The profile of graphs shows that results from MIF and bending theory are nearly same with less difference. According to bending theory the bending stress should be zero at neutral axis and maximum at top fibres, but the results by MIF shows that the bending stress is not exactly zero it has some non-zero value. The bending stress decreases as the percentage replacement of coarse aggregates by rubber aggregates in concrete increases. Reduction of bending stress is less up to 15% but suddenly decreases by 12% for 20% replacement.

4.2. Comparison of MIF and bending theory

Figs. 7 and 8 show results for 5% replacement of coarse aggregate with rubber aggregates in concrete. From Fig. 7 it is clear that for bending stress the MIF results are nearly equal to bending theory results with difference less than 10%. Fig. 8 shows the difference in results of MIF and bending stress which is less than 15%. The graph of displacement results by bending theory is a straight line but the in case of MIF the shape of line is irregular. The analytical results calculated by MIF considering the elastic properties of

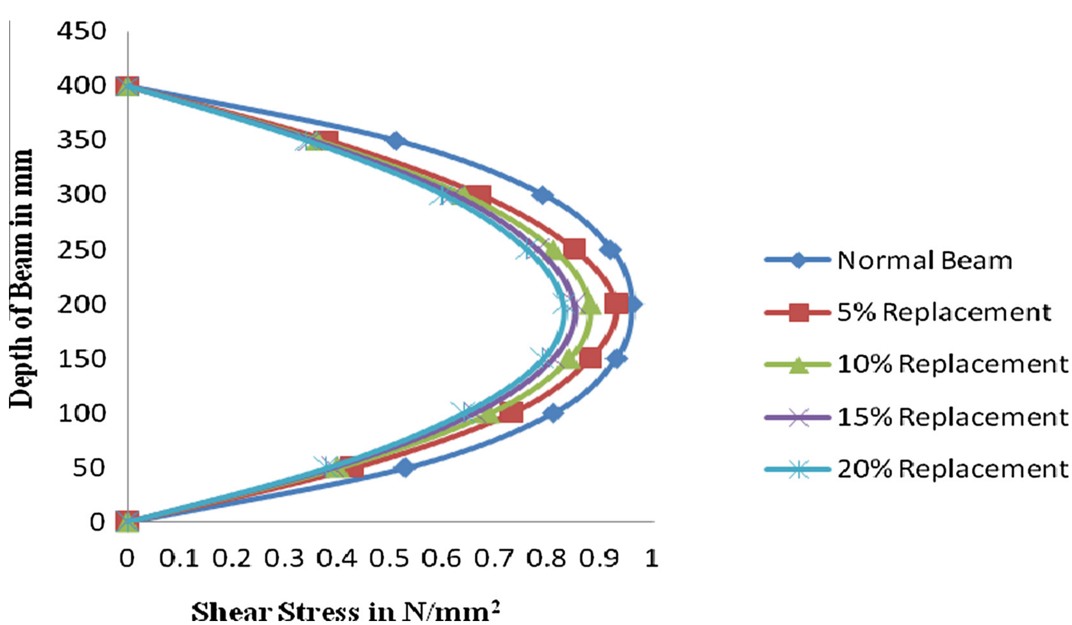

Fig. 5. Shear stress Vs depth of beam by MIF.

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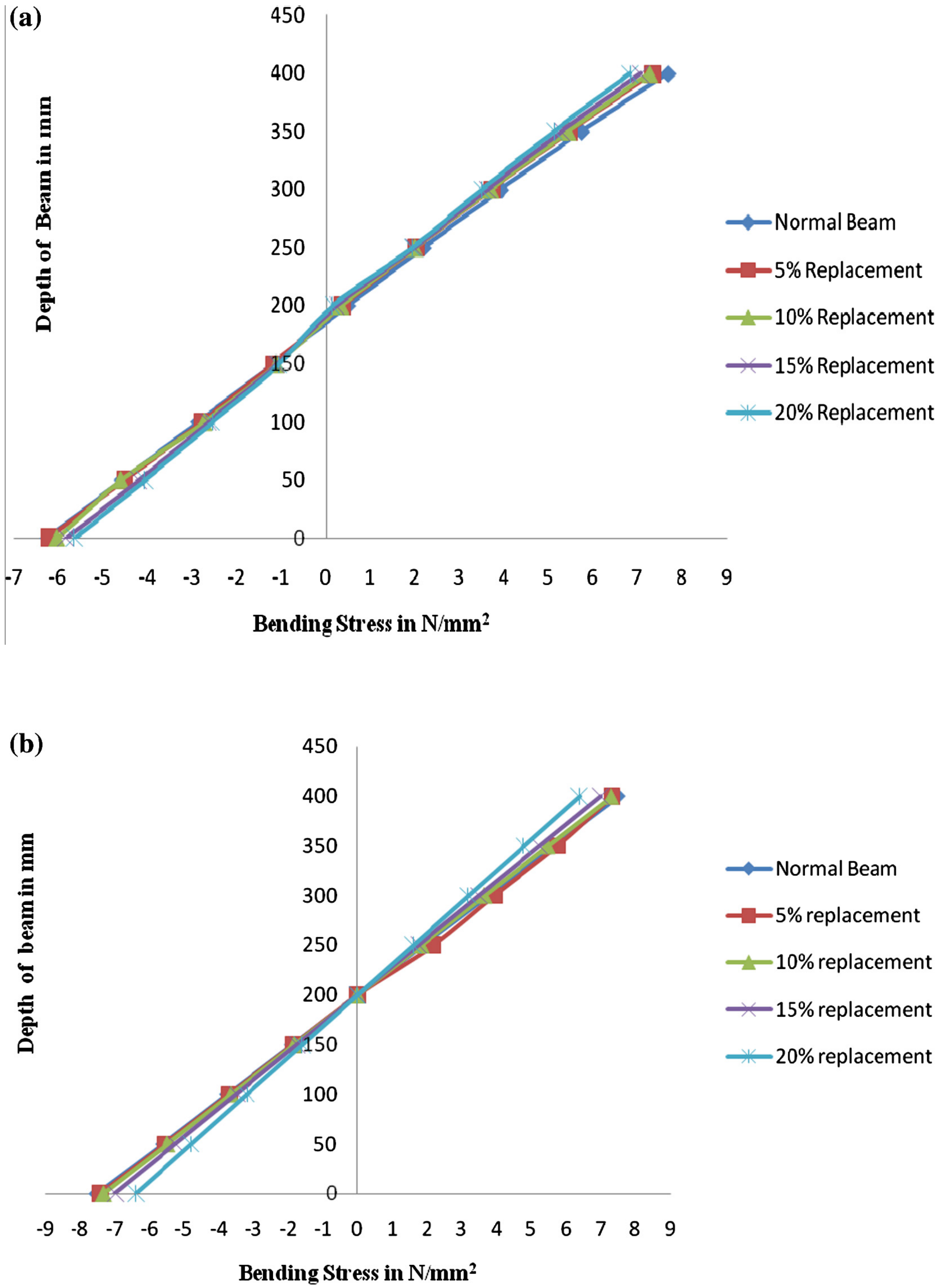

Fig. 6. (a) Bending stress Vs depth of beam by MIF. (b) Bending stress Vs depth of beam by bending theory.

material due to this the exact variation of stresses and displacements across the depth of beam are obtained. While bending theory is based on section properties and assumptions.

5. Conclusions

The following conclusions are made based on the above work.

It is observed that the specific gravity and bulk density of rubber aggregates are less as compared to natural coarse aggregates. The density of concrete decreases when use of rubber aggregates in concrete increases. Due to this the lightweight concrete is obtained which helps to reduce the weight of structure. But the compressive strength decreases and toughness of concrete increases if use of rubber aggregates increases.

It is found that optimum percentage of replacement of rubber aggregates can be up to 15%. It is concluded that such type of concrete cannot be used in structural elements where high strength is required. It can be used in other construction elements like partition walls, road barriers, pavements, sidewalks, etc. which has high demand in construction industries.

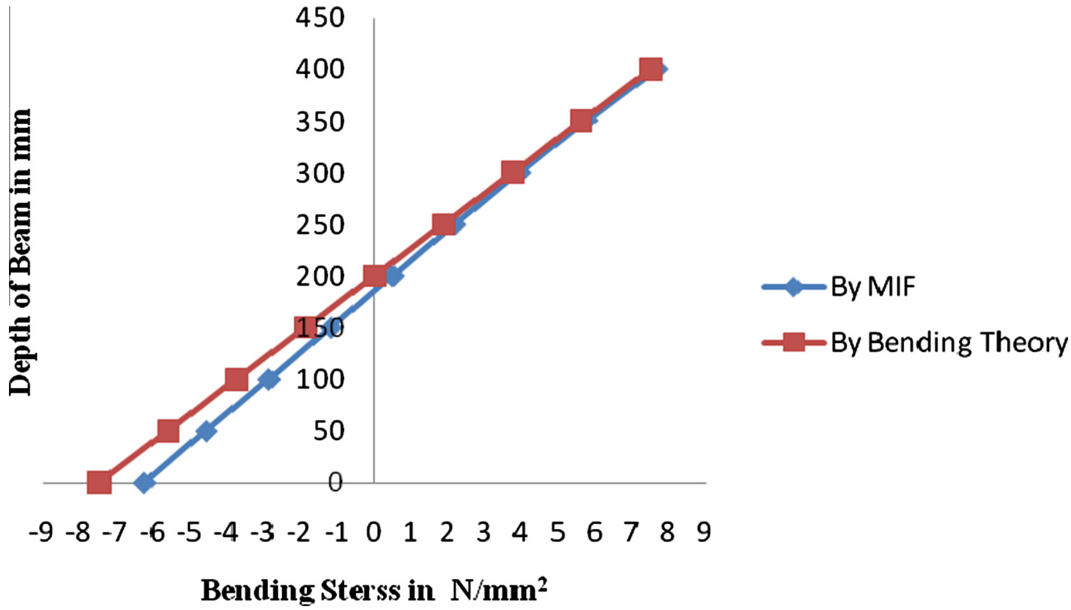

Fig. 7. Comparison of bending stress.

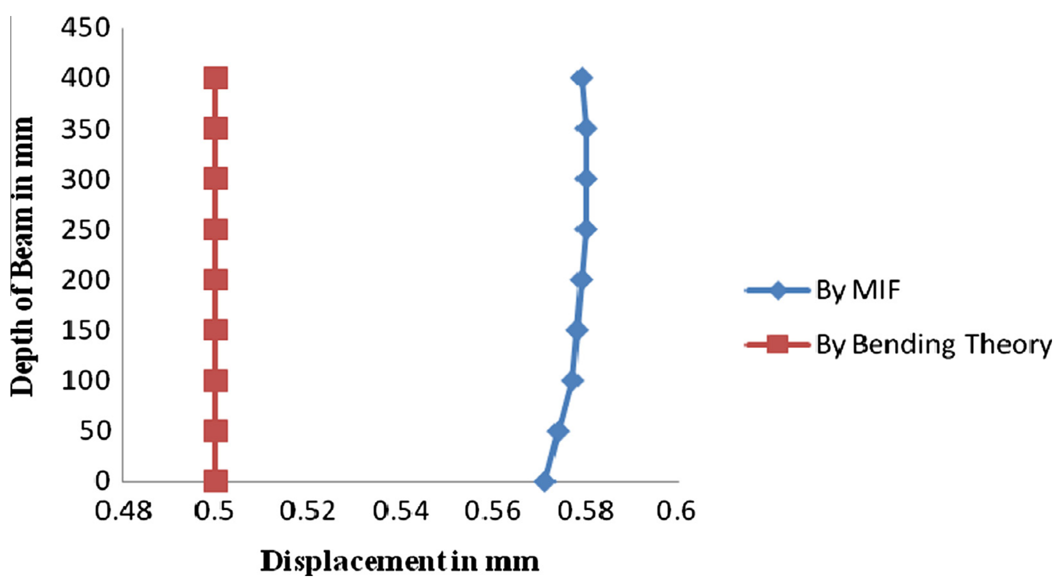

Fig. 8. Comparison of displacement.

By using MIF it is possible to obtain exact results of stresses and displacements. The analytical results calculated by MIF considering the elastic properties of material due to this the exact variation of stresses and displacements across the depth of beam are obtained. While bending theory is based on the assumptions. Hence validation of MIF results with bending theory shows that MIF gives close to exact results.

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