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Collisional energy loss above the critical temperature in QCD

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ABSTRACT

We compute the collisional energy loss for a heavy quark above the critical temperature in Quantum Chromodynamics (QCD). We work in the semi Quark–Gluon Plasma, which assumes that this region is dominated by the non-trivial holonomy of the thermal Wilson line. Relative to the result of leading order in perturbation theory, at a fixed value of the coupling constant we generically find that collisional energy loss is suppressed by powers of the Polyakov loop, $l < 1$. For small values of the loop, this suppression is linear when the heavy quark scatters off of light quarks, and quadratic when the heavy quark scatters off of gluons, or for Compton scattering.

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Experimentally the collisions of heavy ions at ultra-relativistic energies, such as at the Relativistic Heavy Ion Collider (RHIC), or the Large Hadron Collider (LHC), appear to be determined in large part by the behavior of QCD at nonzero temperature. At low temperatures, the confined phase can be modeled by a hadron resonance gas, while at high temperature, a reasonable approach is to use a resummed perturbation theory. However, for experiments both at the LHC, and especially at RHIC, during most of the collision the temperatures probed are not far from the transition temperature. In QCD, this transition temperature is that for the approximate restoration of chiral symmetry, T_χ .

This intermediate region cannot be treated reliably either by a hadron resonance gas, nor by (resummed) QCD perturbation theory. One approach to this region is the “semi” Quark–Gluon Plasma (QGP), where the ionization of color is incomplete. This region of partial deconfinement is modeled by including a non-trivial holonomy for the thermal Wilson line, by means of a matrix model [1–6]. A non-trivial holonomy implies that the expectation value of Polyakov loop lies between its value in the confined phase, which is small (exactly zero in the pure glue theory) and that in the perturbative QGP, which is near one. On a femtosphere one can show that this is manifestly the appropriate effective theory [7]. In QCD, numerical simulations on the lattice uniformly indicate that the Polyakov loop has such an intermediate value between temperatures of T_χ and a few T_χ [8–11].

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We note that there are other approaches to the semi-QGP. These include quasi-particle models [12], which are indirectly related to a matrix model [13]. There are also Polyakov loop models, which take as variables not the eigenvalues of the thermal Wilson line, but just its trace [14]. Other models include gases of monopoles [15] and dyons [16]. There are also models involving bions, which are another type of matrix model [17].

In this Letter we directly use the results from numerical simulations on the lattice to determine the eigenvalues of the Wilson line. From this we then compute the collisional energy loss for a heavy quark. The computations are a straightforward extension of those in ordinary perturbation theory.

Our results have a simple physical interpretation. A non-trivial holonomy represents the fact that as the temperature decreases, the density of particles with a given color charge decreases. This is obvious in the pure glue theory, where the probability to create a particle with any color charge necessarily vanishes in the confined phase. That is, color is “bleached” in the confined phase. With dynamical quarks, at nonzero temperature there is always some small probability to create particles with nonzero color charge. Nevertheless, numerical simulations on the lattice indicate that this probability is really rather small near the critical temperature, T_χ . In any case, particles in the adjoint representation, such as gluons, are more strongly suppressed than quarks, which lie in the fundamental representation. To a good approximation for three colors, when the Polyakov loop is small the density of gluons is proportional to the square of the loop, while the density of quarks is proportional to a single power of the loop.

We work to leading order in perturbation theory in a fixed background field for the non-trivial holonomy. For collisional energy loss, in the limit of a small value of the loop, we find that the scattering off of light quarks is suppressed by a single factor of the loop, while that for gluons, or for Compton scattering in a thermal bath, is quadratically suppressed. We expect that the suppression of scattering off of quarks and gluons near T_χ holds in any effective theory, although surely the details differ. For the time being we defer a detailed comparison to experiment to future study.

1. Perturbative calculation of the collision energy loss with non-trivial holonomy

1.1. Introduction

To represent non-trivial holonomy for a $SU(N_c)$ gauge group, we expand about a background, classical gluon field A_0^{cl} , where

$$A_0^{\text{cl}} = \frac{i}{g} \text{diag}(Q^1, Q^2, \dots, Q^{N_c}). \quad (1)$$

Here g is the coupling constant for the gauge field, so as the background field is proportional to $1/g$, it is manifestly non-perturbative. Further, the gluon field A_0 is not real, but purely imaginary. We introduce such a mean field to model the effect of non-trivial holonomy, and so it should be understood as arising from an ensemble average over non-perturbative fluctuations. Thus we do not attempt to derive from first principles how this field arises, but simply use results from lattice simulations to determine the Q 's.

Since the gauge group is $SU(N_c)$, the vector potential is traceless,

$$\sum_{a=1}^{N_c} Q^a = 0. \quad (2)$$

The elements Q^a are real, and we can assume that they are distributed symmetrically about the origin. (This is equivalent to assuming the expectation value of the Polyakov loop, Eq. (5) below, is real. This is true if there is no net baryon density; otherwise it is necessary to generalize the ansatz.)

For three colors, as in QCD, this implies there is only one independent variable, Q :

$$Q^a = (-Q, 0, Q). \quad (3)$$

The Wilson line in the temporal direction is

$$\mathbb{L}(\vec{x}) = \mathcal{P} \exp \left(ig \int_0^{1/T} d\tau A_0(\vec{x}, \tau) \right); \quad (4)$$

T is the temperature, τ is the imaginary time, and \mathcal{P} denotes time ordering. The Wilson line is a unitary matrix, $\mathbb{L}^\dagger \mathbb{L} = \mathbf{1}$. Under a gauge transformation Ω , the Wilson line transforms as $\mathbb{L} \rightarrow \Omega^\dagger \mathbb{L} \Omega$. The trace of the Wilson line is the Polyakov loop,

$$\ell(\vec{x}) = \frac{1}{N_c} \text{tr} \mathbb{L}(\vec{x}), \quad (5)$$

and is gauge invariant. There are also higher loops, $(1/N_c) \text{tr} \mathbb{L}^n$, which are obviously also gauge invariant. For a general field in $SU(N_c)$, there are $N_c - 1$ independent loops.

For three colors, under the mean field ansatz of Eq. (3) there is one independent loop, which we can take to be the simplest,

$$\ell = \frac{1}{3} \left(1 + 2 \cos \left(\frac{Q}{T} \right) \right). \quad (6)$$

While the Wilson line is not gauge invariant, its eigenvalues are. To leading order in weak coupling it suffices to deal with the background A_0^{cl} field. Beyond leading order it is necessary to deal with the eigenvalues of the Wilson line, which are gauge invariant. Typically, lattice simulations do not measure the eigenvalues directly, but only the Polyakov loop, which is a sum over the eigenvalues. The eigenvalues were measured directly in one recent study, [11]. The results, however, agree with measurements of the (bare) Polyakov loop.

From the lattice measurements of the bare Polyakov loop, it is necessary to extract the renormalized Polyakov loop by removing an ultraviolet divergent mass renormalization. In the pure glue theory this has been carefully analyzed by Gupta, Huebner, and Kaczmarek [9]. For QCD, with dynamical quarks, we use the results of Bazavov et al. [10], taking the renormalized Polyakov loop from their Fig. (13), with a $p4$ action, at $N_\tau = 8$.

Physically the background field which generates non-trivial holonomy can be thought of as an imaginary chemical potential for color [4]. The Bose–Einstein/Fermi–Dirac statistical distribution function for a gluons and quarks are given, respectively, by ($E > 0$)

$$n_{ab}^g(E) = \frac{1}{\exp((E - iQ^{ab})/T) - 1}, \quad (7)$$

$$n_a^q(E) = \frac{1}{\exp((E - iQ^a)/T) + 1}. \quad (8)$$

Since quarks lie in the fundamental representation, their distribution function involves only one color index, through Q^a . For gluons in the adjoint representation, a difference of two indices enters, $Q^{ab} = Q^a - Q^b$.

These are the statistical distribution functions for emission into a thermal bath. Those for absorption from a thermal bath are given by

$$\bar{n}_{ab}^g(E) = 1 + n_{ab}^g(E) \quad (9)$$

for gluons, and

$$\bar{n}_a^q(E) = 1 - n_a^q(E) \quad (10)$$

for quarks. The relative minus sign is because quarks obey the Fermi–Dirac exclusion principle.

For a given Q^a , these distribution functions are complex valued. Physical quantities become real after summing over a distribution of Q 's. To illustrate this, consider the following example. Start with the quark statistical distribution function, and sum over all quark colors:

$$\sum_{a=1}^{N_c} n_a^q(E) = \sum_{a=1}^{N_c} \frac{1}{1 + \exp((E - iQ^a)/T)} = \sum_{n=1}^{\infty} (-)^{n+1} e^{-nE/T} \text{tr} \mathbb{L}^n. \quad (11)$$

This is a measure of the total number of quarks.

We first compute the number of quarks in the deconfined phase at very high temperature. At very high temperature the theory is essentially perturbative, and we can set all Q 's to vanish. The number of quarks is then

$$N_{\text{deconfined}}^q = \int \frac{d^3k}{(2\pi)^3} \sum_{a=1}^{N_c} n_a^q(E) = N_c \left(\frac{3\zeta(3)}{4\pi} \right), \quad (12)$$

where the Riemann zeta-function $\zeta(3) \approx 1.20206 \dots$. The numerical value of the right hand side is not important, what we wish to emphasize is that as expected in the deconfined phase, the quark density is proportional to the number of quarks, N_c , as one would expect.

Contrast this with the background field in the confined phase of the pure glue theory, where any Polyakov loop with nonzero $Z(N_c)$ charge vanishes. The explicit Q which produces the confined vacuum is

$$Q_{\text{conf}}^a = \frac{\pi T}{N_c} (N_c + 1 - 2a), \quad a = 1 \cdots N_c. \quad (13)$$

For three colors, Eq. (3), $Q = 2\pi T/3$. In the confined phase, all loops with nonzero $Z(N_c)$ charge vanish. The only Polyakov loops which are nonzero are those which are $Z(N_c)$ neutral. These loops wrap around the imaginary time direction by an integral multiple of N_c , for which

$$\frac{1}{N_c} \text{tr} \mathbb{L}_{\text{conf}}^{kN_c} = (-)^{k(N_c+1)}. \quad (14)$$

In the confined phase, then,

$$\sum_{a=1}^{N_c} n_a^q(E) = \frac{N_c}{1 + \exp(N_c E/T)}. \quad (15)$$

For massless quarks, the energy is related to the momentum k by $E = |k|$. Integrating over the momenta, the total number of colored particles in the confined phase is

$$N_{\text{conf}}^q = \int \frac{d^3k}{(2\pi)^3} \sum_{a=1}^{N_c} n_a^q(E) = \frac{1}{N_c^2} \left(\frac{3\zeta(3)}{4\pi} \right). \quad (16)$$

This computation illustrates two points. First, although individual elements $n_a^q(E)$ are complex, the sum over a gives results which are real. The second concerns the magnitude of the sum. In the limit of an infinite number of colors, the number of quarks is $\sim N_c$ in the deconfined phase, and very small, $\sim 1/N_c^2$, in the confined phase. This ratio is strictly zero only in the limit of infinite N_c . For finite N_c , there is a small density of quarks in the “confined” phase. This matters with dynamical quarks, where this density is nonzero. This is the usual observation that there is no strict order parameter for confinement in the presence of dynamical quarks. As mentioned previously, however, in practice the density of quarks in the confined phase is small, at least as measured by numerical simulations on the lattice for three colors and for two or three flavors of quarks [8,10].

1.2. Energy loss in the sQGP

Consider a heavy quark of mass M and energy E , where $E = \sqrt{\vec{p}^2 + M^2}$. We assume that the heavy quark is moving rapidly, with $p \gg M$. The energy loss per unit length x is given by ($v = p/E$)

$$\begin{aligned} \frac{dE}{dx} &= \sum_i \frac{1}{2Ev} \int_k \frac{n_i(k)}{2k} \int_{k'} \frac{\bar{n}_i(k')}{2k'} \\ &\times \int_{p'} \frac{\omega}{2E'd} |\mathcal{M}_i|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K'). \end{aligned} \quad (17)$$

Here $P = (E, \vec{p})$ is the four momentum of the incident heavy quark, $P = (E', \vec{p}')$ that of the outgoing heavy quark, whilst $K = (k, \vec{k})$ and $K' = (k', \vec{k}')$ are the four momenta of the particles which the heavy quark scatters off, and $\omega = E - E'$. The integrals are over spatial momenta of the virtual particles,

$$\int_k = \int \frac{d^3k}{(2\pi)^3}, \quad (18)$$

and similarly for $\int_{k'}$ and $\int_{p'}$. The index i represents a label over the different types of particles, including whether they are bosons or fermions/antifermions, as well as color, flavor, and spin. For the squared amplitude, $|\mathcal{M}_i|^2$ in Eq. (17), one sums over the initial and final spin states, and divides by the degeneracy factor, $d = 2$, of the incoming particle.

The computations of energy loss for a heavy quark in hot QCD were first carried out by Braaten and Thoma [18]. They showed that the integral over phase space can be simplified considerably; see also Appendix A of Peigné and Peshier [19]. Using these simplifications,

$$\frac{dE}{dx} = \frac{1}{16\pi^2 E p} \int_k \frac{n(k)}{2k} \int_{t_{\min}}^0 dt \int_{\omega_{\min}}^{\omega_{\max}} d\omega \bar{n}(k + \omega) \frac{\omega}{d\sqrt{\gamma}} \sum_i |\mathcal{M}_i|^2. \quad (19)$$

Here s , t , and u are the usual Mandelstam variables,

$$s = (P + K)^2, \quad t = (P - P')^2, \quad u = (P - K')^2, \quad (20)$$

and we introduce the quantity γ ,

$$\gamma = -\alpha^2 \omega^2 + \beta \omega + \delta, \quad (21)$$

where

$$\alpha = \frac{1}{p}(s - M^2), \quad (22)$$

$$\beta = -\frac{2t}{p^2} [E(s - M^2) - k(s + M^2)], \quad (23)$$

$$\delta = -\frac{t}{p^2} \{t[(E + k)^2 - s] + 4p^2 k^2 - (s - M^2 - 2Ek)^2\}. \quad (24)$$

Since the square root of γ enters into Eq. (19), γ must be positive. Requiring that $\gamma > 0$ fixes the limit of integration over t and ω , with $t: t_{\min} \rightarrow 0$, and $\omega: \omega_{\min} \rightarrow \omega_{\max}$.

In this article we only compute the energy loss to leading logarithmic order. In this instance the distribution function $\bar{n}(k + \omega)$ can be replaced with 1 [18,19]. Thus we obtain

$$\frac{dE}{dx} = \frac{1}{16\pi^2 E p} \int_k \frac{n(k)}{2k} \int_{t_{\min}}^0 dt \int_{\omega_{\min}}^{\omega_{\max}} d\omega \frac{\omega}{d\sqrt{\gamma}} \sum_i |\mathcal{M}_i|^2. \quad (25)$$

1.3. Coulomb scattering

The amplitude for Coulomb scattering, of a heavy quark off of a light quark in the thermal bath, is illustrated in the left hand side of Fig. 1. This amplitude involves the color trace

$$(T^{cd})_{ab} (T^{cd})_{ef} (T^{d'c'})_{ba} (T^{d'e'})_{fe} = \frac{N_c^2 - 1}{4N_c}. \quad (26)$$

The summation in this expression must be performed with an open color index e , because the energy loss depends on background field through Q^e . The amplitude reduces to

$$\begin{aligned} \sum_i |\mathcal{M}_i|^2 &= \frac{8N_f g^4}{N_c} \left(\frac{N_c^2 - 1}{4N_c} \right) \\ &\times \left(\frac{2(s - M^2)^2 + (u - M^2)^2 + 2M^2 t}{t^2} \right), \end{aligned} \quad (27)$$

where N_f is the number of light quark flavors, and thus

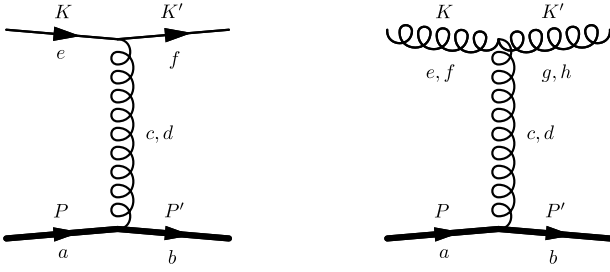


Fig. 1. Scattering of a heavy quark (thick line) off of a light quark (left) and a gluon (right) in the t -channel. Historically, the diagram on the left hand side is referred to as Coulomb scattering, while that on the right hand side is Compton scattering, off of a gluon, in the t -channel.

$$\frac{dE}{dx}\Big|_Q^{\text{qk}} = \frac{1}{16\pi^2 E p} \sum_{e=1}^{N_c} \int_k \frac{n^{\text{q}}(k - iQ^e)}{2k} \int_{t_{\min}}^0 dt \times \int_{\omega_{\min}}^{\omega_{\max}} d\omega \frac{\omega}{d\sqrt{\gamma}} \sum_i |\mathcal{M}_i|^2. \quad (28)$$

Here the subscript on dE/dx refers to the dependence on the background field A_0^{cl} through Q^e .

The integration with respect to ω is

$$\int_{\omega_{\min}}^{\omega_{\max}} d\omega \frac{\omega}{\sqrt{\gamma}} = \pi \frac{\beta}{2\alpha^3}. \quad (29)$$

Integration with respect to the spatial momentum k is done by expanding the quark distribution function into a series which starts with the Boltzmann term,

$$n^{\text{q}}(k - iQ^e) = - \sum_{n=1}^{\infty} (-)^n e^{-n(k+iQ^e)/T}. \quad (30)$$

Keeping only the terms to leading logarithmic order, we find a very simple result: the expression in the semi-QGP is an overall factor, which depends upon the Q 's, times that for the perturbative QGP:

$$\frac{dE}{dx}\Big|_Q^{\text{qk}} = S^{\text{qk}}(Q) \frac{dE}{dx}\Big|_{Q=0}^{\text{qk}}, \quad (31)$$

where the result in the perturbative QGP is

$$\frac{dE}{dx}\Big|_{Q=0}^{\text{qk}} = \alpha_s^2 T^2 N_f \frac{N_c^2 - 1}{12N_c} \pi \ln\left(\frac{ET}{m_D^2}\right); \quad (32)$$

N_f is the number of light quark flavors. We regulate the infrared logarithmic divergence of the integral over the Mandelstam variable t by the gluon Debye mass.

We note that in the semi-QGP that the gluon Debye mass depends upon the background field through the Q 's. We can neglect this dependence because it only enters beyond leading logarithmic order.

The Q -dependent factor in Eq. (31) is given by

$$S^{\text{qk}}(Q) = \frac{12}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^2} \left(\frac{\text{tr} \mathbb{L}^n}{N_c} \right). \quad (33)$$

The superscript in $S^{\text{qk}}(Q)$ denotes that it is due to scattering off of light quarks.

In the perturbative regime this suppression factor equals unity, as

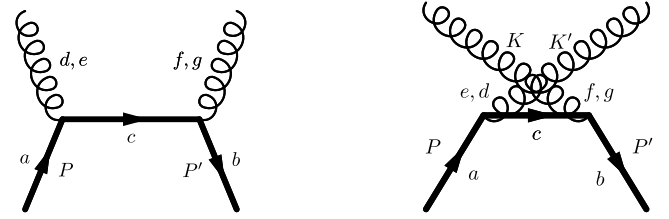


Fig. 2. Compton scattering of a heavy quark (thick line) off of gluons in the s -channel (left) and u -channel (right). Only the diagram on the right hand side generates a logarithm.

$$S^{\text{qk}}(0) = \frac{12}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^2} = \frac{6}{\pi^2} \zeta(2) = 1. \quad (34)$$

In the confined phase of the pure glue theory, only loops with $n = kN_c$ contribute, so that by Eq. (14),

$$S^{\text{qk}}(Q_{\text{conf}}) = \frac{12}{\pi^2} \frac{1}{N_c^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^2} = \frac{1}{N_c^2}. \quad (35)$$

For physically relevant case, $N_c = 3$, and under the mean-field ansatz of Eq. (3), by using the identity

$$\sum_{n=1}^{\infty} \frac{(-)^n}{n^2} \cos(2\pi nx) = \pi^2 \left(x^2 - \frac{1}{12} \right) \quad (36)$$

the suppression factor $S^{\text{qk}}(Q)$ can be calculated analytically,

$$S_{N_c=3}^{\text{qk}}(Q = 2\pi Tq) = \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^2} (1 + 2\cos(2\pi nq)) = 1 - 8q^2. \quad (37)$$

Using Eq. (6), we have for the nearly confining and perturbative background field

$$S_{N_c=3}^{\text{qk}}(Q \rightarrow Q_{\text{conf}}) = \frac{1}{9} + \frac{8}{\pi\sqrt{3}} \ell + \mathcal{O}(\ell^2); \quad (38)$$

$$S_{N_c=3}^{\text{qk}}(Q \rightarrow 0) = 1 - \frac{6}{\pi^2} (1 - \ell) + \mathcal{O}((1 - \ell)^2), \quad (39)$$

respectively.

1.4. Compton scattering

There are three diagrams which contribute to what is termed Compton scattering. There is scattering off of a gluon in the t -channel, which is illustrated by the diagram on the right hand side in Fig. 1. There are also two diagrams for scattering off of a gluon in the s -channel and u -channel, as illustrated in Fig. 2. Among them, only the t -channel and u -channel generate leading logarithmic contributions through small angles scattering. Furthermore, all cross terms between different channels do not lead to leading logarithmic contributions, thus we will focus on the squared amplitude of t -channel and u -channel diagrams.

1.4.1. Compton scattering in the t -channel

For Compton scattering in the t -channel, the relevant diagram is that on the right hand side of Fig. 1. The color structure which enters for this diagram is

$$(T^{cd})_{ab} f^{cd,ef,gh} (T^{d'c'})_{ba} f^{d'c',fe,hg} = \frac{N_c}{2} \left(1 - \frac{1}{N_c} \delta^{ef} \right). \quad (40)$$

Again, there is no summation over the color indices e and f . They correspond to those for the gluon in the initial state, which the heavy quark scatters off of.

The matrix element for scattering in the t -channel is

$$\sum_i |\mathcal{M}_i|^2 = -\frac{8g^4}{N_c} \left(\frac{N_c}{2}\right) \left(1 - \frac{1}{N_c} \delta^{ef}\right) \left(\frac{(s-M^2)(u-M^2)}{t^2}\right). \quad (41)$$

The expression for energy loss in the t -channel is

$$\begin{aligned} \frac{dE}{dx} \Big|_Q^{\text{gl}} &= \frac{1}{16\pi^2 E p} \sum_{e,f=1}^{N_c} \int_k \frac{n^g(k-i[Q^e-Q^f])}{2k} \int_{t_{\min}}^0 dt \\ &\times \int_{\omega_{\min}}^{\omega_{\max}} d\omega \frac{\omega}{d\sqrt{\gamma}} \sum_i |\mathcal{M}_i|^2. \end{aligned} \quad (42)$$

Performing the integrals over k , t , and ω as before, again the result is a Q -dependent factor times the perturbative result:

$$\frac{dE}{dx} \Big|_Q^{\text{gl,t-ch}} = S^{\text{gl}}(Q) \frac{dE}{dx} \Big|_{Q=0}^{\text{gl,t-ch}}. \quad (43)$$

The perturbative result is

$$\frac{dE}{dx} \Big|_Q^{\text{gl,t-ch}} = \alpha_s^2 T^2 (N_c^2 - 1) \frac{\pi}{6} \ln\left(\frac{ET}{m_D^2}\right). \quad (44)$$

The modification of the perturbative result in the semi-QGP is given by a factor

$$S^{\text{gl}}(Q) = \frac{1}{N_c^2 - 1} \left(\frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{|\text{tr} \mathbb{L}^n|^2}{n^2} - 1 \right). \quad (45)$$

In the perturbative QGP this factor is unity,

$$\begin{aligned} S^{\text{gl}}(0) &= \frac{1}{N_c^2 - 1} \left(\frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{N_c^2}{n^2} - 1 \right) \\ &= \frac{1}{N_c^2 - 1} \left(\frac{6}{\pi^2} \zeta(2) N_c^2 - 1 \right) = 1, \end{aligned} \quad (46)$$

as it must be. The superscript in $S^{\text{gl}}(Q)$ denotes that it is due to scattering off of a gluon. We show in the next subsection that to leading logarithmic order, the suppression factor for scattering in the u -channel is the same as in the t -channel, Eq. (54).

In a confining background field, the suppression factor for t -channel scattering is found to vanish,

$$S^{\text{gl}}(Q_{\text{conf}}) = \frac{1}{N_c^2 - 1} \left(\frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} - 1 \right) = 0. \quad (47)$$

For three colors, $N_c = 3$, using the ansatz Eq. (3) and the identity

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \cos(2\pi n x) = \frac{\pi^2}{12} \left(x^2 - x + \frac{1}{6} \right), \quad (48)$$

we obtain

$$S_{N_c=3}^{\text{gl}}(Q = 2\pi T q) = 1 - 3q(2 - 3q). \quad (49)$$

In the limiting cases of a confining and perturbative background field we get

$$S_{N_c=3}^{\text{gl}}(Q \rightarrow Q_{\text{conf}}) = \frac{27}{4} \ell^2 (1 - \ell) + \mathcal{O}(\ell^4); \quad (50)$$

$$\begin{aligned} S_{N_c=3}^{\text{gl}}(Q \rightarrow 0) \\ = 1 - \frac{3\sqrt{3}}{\pi} \sqrt{1 - \ell} + \frac{3\sqrt{3}}{4\pi^2} (1 - \ell) + \mathcal{O}((1 - \ell)^2). \end{aligned} \quad (51)$$

1.4.2. Compton scattering in the u -channel

The only diagram which generates a logarithm at leading order is that on the right hand side of Fig. 2. The color structure for this diagram is

$$(T^{fg})_{ac} (T^{de})_{cb} (T^{fg})_{ac'} (T^{de})_{c'b} = \frac{C_f}{2} \left(1 - \frac{1}{N_c} \delta^{de}\right). \quad (52)$$

Here $C_f = (N_c^2 - 1)/(2N_c)$ is the Casimir for the fundamental representation.

The matrix element for scattering in the u -channel becomes

$$\sum_i |\mathcal{M}_i|^2 = -\frac{4g^4}{N_c} \left(\frac{C_f}{2}\right) \left(1 - \frac{1}{N_c} \delta^{de}\right) \left(\frac{s-M^2}{u-M^2}\right). \quad (53)$$

After integrating over k , t , and ω , once again the result is a Q -dependent factor times the perturbative result:

$$\frac{dE}{dx} \Big|_Q^{\text{gl,u-ch}} = S^{\text{gl}}(Q) \frac{dE}{dx} \Big|_{Q=0}^{\text{gl,u-ch}}. \quad (54)$$

The result in the perturbative limit is

$$\frac{dE}{dx} \Big|_{Q=0}^{\text{gl,u-ch}} = \alpha_s^2 T^2 C_f^2 \frac{\pi}{6} \ln\left(\frac{ET}{M^2}\right). \quad (55)$$

We find that the color dependent factor in the semi-QGP is the same in the u -channel as in the t -channel, given by Eq. (45).

2. Complete result

2.1. Extracting the loop from the lattice

Besides non-perturbative contributions in the semi-QGP, the Polyakov loop also receives contributions from ordinary perturbation theory,

$$\ell(Q=0) = 1 + \delta\ell(Q=0). \quad (56)$$

To order $\sim g^4$ [20],

$$\begin{aligned} \delta\ell(Q=0) &= +\frac{g^2 C_f m_E}{8\pi T} + \frac{g^4 C_f}{(4\pi)^2} \left[-\frac{N_f}{2} \ln 2 \right. \\ &\quad \left. + N_c \left(\ln \frac{m_E}{T} + \frac{1}{2} \right) \right] + \mathcal{O}(g^5). \end{aligned} \quad (57)$$

Notice that the leading contribution is positive. This implies that the expectation value of the loop exceeds unity. While this cannot be true classically, it occurs because of renormalization.

In Eq. (57),

$$g^2 = 6f_g, \quad (58)$$

$$m_E^2 = (2N_c + N_f) f_m T^2, \quad (59)$$

and

$$f_{g,m} = \frac{4\pi^2}{(11N_c - 2N_f)(\ln(4\pi T/\Lambda_{\overline{MS}}) - \gamma_E + c_{g,m})}, \quad (60)$$

where $\Lambda_{\overline{MS}}$ is the renormalization mass scale in the modified minimal subtraction scheme. Lastly, the coefficients for c_g and c_m are

$$c_g = \frac{2N_f(4\ln 2 - 1) - 11N_c}{2(11N_c - 2N_f)}, \quad (61)$$

$$c_m = \frac{4N_f \ln 2}{11N_c - 2N_f} - \frac{5N_c^2 + N_f^2 + 9\frac{N_f}{2N_c}}{(11N_c - 2N_f)(2N_c + N_f)}. \quad (62)$$

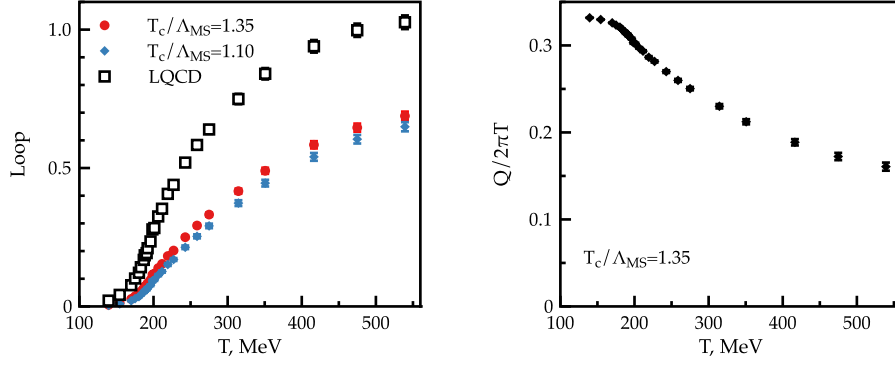


Fig. 3. The left hand figure shows the Polyakov loop from the lattice, Fig. (13) of Ref. [10], and its value after removing perturbative corrections, as in Eq. (63). The result depends upon the value of the renormalization mass scale, $\Lambda_{\overline{MS}}$. The corresponding value of Q , Eq. (6), is given in the figure on the right hand side.

We assume that these perturbative corrections exponentiate,

$$\ell^{\text{total}}(Q) = \exp[\delta\ell(Q=0)]\ell(Q). \quad (63)$$

Even with $Q=0$, exponentiating the leading order corrections is an assumption about those to higher order. Further, the corrections to $\sim g^3$ and $\sim g^4$ will certainly change in the semi-QGP, when the Q 's are nonzero. We do not include this effect for the time being.

Thus we first compute $\ell(Q)$ from Eq. (63) and by using Eq. (6) determine $Q(T)$. The results are shown in Fig. 3.

2.2. Suppression factors in the semi-QGP

Summing up the contributions from Coulomb scattering, Eq. (31), from Compton scattering in the t -channel, Eq. (43) and from Compton scattering in the u -channel, Eq. (54), gives a total result for energy loss which is

$$\frac{dE}{dx} = \left(S^{\text{qk}}(Q) \alpha_s^2 T^2 \pi \frac{N_f(N_c^2 - 1)}{12N_c} \ln\left(\frac{ET}{m_D^2}\right) + S^{\text{gl}}(Q) \left(\frac{(N_c^2 - 1)}{6} \ln\left(\frac{ET}{m_D^2}\right) + \frac{C_f^2}{6} \ln\left(\frac{ET}{M^2}\right) \right) \right). \quad (64)$$

We can then use the results for the temperature dependence of Q to plot the suppression factors in the semi-QGP, versus the perturbative results. These are illustrated in Fig. 4.

For temperatures near T_χ , where the expectation value of the Polyakov loop is small, we find that the suppression of the gluon terms, $S^{\text{gl}}(Q)$, is much stronger than for the quark term, $S^{\text{qk}}(Q)$. This is obvious from the corresponding expressions, Eq. (45) for gluons, and Eq. (33) for quarks. For simplicity, neglect corrections which are suppressed by factors of $1/N_c^2$ in Eq. (45), since those are numerically small. Then it is easy to see that for small values of the loop, that

$$S^{\text{qk}}(Q) \sim \ell; \quad S^{\text{gl}}(Q) \sim \ell^2. \quad (65)$$

Physically this is evident. For small values of the loop, the density of quarks is $\sim \ell$, while that of gluons is $\sim \ell^2$. This is simply because the quarks are in the fundamental representation, and the gluons, in the adjoint. In another way, in the double line notation (which is useful at large N_c , but can be used at any N_c), quarks have one line, and gluons, two lines.

We have only illustrated the suppression factors, and leave it for later analysis to make a detailed comparison to experiment. However, our study shows that for temperatures which are probed at both RHIC and even at the LHC, that the scattering off of light quarks completely dominates over scattering off of gluons. This is directly a manifestation of the “bleaching” of color in the semi-QGP, as the density of colored particles decreases.

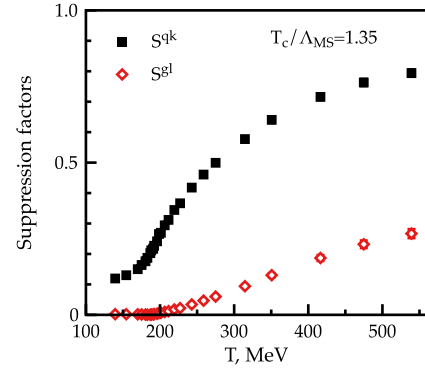


Fig. 4. Suppression factors for quarks, S^{qk} , and gluons, S^{gl} , using the values of the Q 's in Fig. 3. Notice that the suppression is much greater for gluons than for quarks.

3. Conclusions and outlook

In this Letter we computed the suppression of collisional energy loss in the semi-QGP. We find, on elementary and very general grounds, that scattering off of gluons is strongly suppressed in the semi-QGP, while that of quarks is only moderately suppressed.

It is interesting that when the dust settles, we obtain rather simple expressions for the collisional energy loss, as simple suppression factors times the usual perturbative result. This suggests that the same will be true for other electromagnetic probes. We have computed the effects of the semi-QGP upon both dilepton production, and on the production of real photons, and will present these results shortly.

These computations represent the first attempt to extend perturbative computations of quantities in thermal QCD to phenomenologically relevant temperatures of interest, building crucially upon results from numerical simulations on the lattice.

We conclude by noting that it is possible to include perturbative corrections in the present approach. Our model is admittedly phenomenological, in that there is no rigor in choosing the non-perturbative potential of the Q 's. However, once an ansatz for the non-perturbative potential is chosen, computing perturbative corrections about that potential is a well defined procedure. For example, corrections to the (perturbative) potential for an arbitrary distribution of Q 's has been computed at next to leading order by Dumitru, Guo, and Korthals-Altes [6]. It would then be necessary to compute corrections to the same order for the non-perturbative potential in the Q 's, and for the process in question.

More to the point, the results of the present computation suggest that near but above the critical temperature the dominant effect is not from perturbative corrections *per se*, but rather from suppression of colored degrees of freedom by powers of loop. This

is especially true for the quadratic suppression of gluons, versus the linear suppression of the quarks. In the end, only detailed comparison to experiment will decide between ours and competing models.

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