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# Spin bits at two loops

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**Abstract**

We consider the super-Yang–Mills/spin system map to construct the SU(2) spin bit model at the level of two loops in Yang–Mills perturbation theory. The model describes a spin system with chaining interaction. In the large $N$ limit the model is shown to be reduced to the two loop planar integrable spin chain.


## 1. Introduction

Large $N$ physics [1] gained considerable interest in recent years (see [2] for a recent review and references) due to the AdS/CFT conjecture enlightenment [3,4] and, more recently, to the consideration of various limits for this correspondence [5–11]. These achievements lead to an intensive study of the anomalous dimensions of local gauge invariant composite operators in $\mathcal{N} = 4$ super-Yang–Mills (SYM) model [12]. The major breakthrough in this investigation was the discovery of integrability of the matrix of anomalous dimensions in the planar limit, $N \to \infty$ [13,14]. These results were extended to two and higher loops [15,16].

As it is now clear, there is a one-to-one correspondence between one trace operators in SYM theory and the states in spin chain models. It was enough to consider the planar limit of SYM theory. If the nonplanar contribution is considered, the one trace sector is not conserved anymore and one ends up with trace splitting and joining in the operator mixing [17]. Even in this case one can still consider a one-to-one map between local gauge invariant operators and a spin system [18,19]. In this case one has to introduce a set of new degrees of freedom, beyond the spin states, which describes the chaining state of our spin system. This new field takes values in the symmetry

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group of permutations of spin bits and introduces a new gauge degree of freedom. An alternative approach to the description of the nonplanar contribution is discussed in [20].

In this Letter we extend the analysis of [18,19] to the two loop level of SYM perturbation theory, i.e., we consider the SYM anomalous dimension/mixing matrix to two loops and apply the map to the spin bit system to this matrix.

The plan of the Letter is as follows. In Section 2 we introduce the notations; then in Section 3 we consider the two loop nonplanar anomalous dimension matrix which we map to an operator acting on the spin bit space. Using the properties of the symmetry group we are able to reduce the SU(2) nonplanar two loop Hamiltonian to a remarkably simple form. Finally, in Section 4 we draw some conclusions.

In this Letter we use conventions and notations of [18,19].

## 2. The setup and the one loop result

We consider the SU(2) sector of local gauge invariant SYM operators which are generated by two holomorphic (multi)trace operators built from two complex SYM scalars $\phi = \phi_5 + i\phi_6$ and $Z = \phi_1 + i\phi_2$, with the typical form

$$\mathcal{O} = \mathrm{Tr}(\phi Z \phi\phi Z \ldots)\,\mathrm{Tr}(\phi\phi\phi Z \ldots)\,\mathrm{Tr}(\ldots)\cdots.$$

This trace can be written in the following explicit form using a permutation group element $\gamma \in S_L$:

$$\mathcal{O} = \phi_{i_1}^{a_1 a_{\gamma_1}} \phi_{i_2}^{a_2 a_{\gamma_2}} \cdots \phi_{i_L}^{a_L a_{\gamma_L}} \equiv |\phi_{i_1}, \ldots, \phi_{i_L}; \gamma\rangle,$$

where $L$ is the total number of "letters" $\phi_i = (\phi, Z)$ in $\mathcal{O}$ which are numbered by a label $k = 1, \ldots, L$. The permutation element $\gamma_k$ gives the next multiplier to the $k$th letter

$$\gamma \equiv (\gamma_1\gamma_2\ldots\gamma_k\ldots\gamma_L): \quad \begin{pmatrix} 1 & 2 & \ldots & k & \ldots & L \\ \gamma_1 & \gamma_2 & \ldots & \gamma_k & \ldots & \gamma_L \end{pmatrix} \in S_L.$$

Obviously, the reshuffling of the labels $k \mapsto \sigma_k$ accompanied by a conjugation of $\gamma$ with the same group element $\sigma^{-1}\cdot\gamma\cdot\sigma$ leaves the trace form of $\mathcal{O}$ unchanged. Therefore, the configurations related by such a transformation should be considered as equivalent

$$(\phi_k, \gamma) \sim \left(\phi_{\sigma_k}, \sigma^{-1}\cdot\gamma\cdot\sigma\right). \tag{2.1}$$

Now, we should map the space of such operators to the system of $L$ SU(2) $\frac{1}{2}$-spins (spin bits). The map is completed by associating to each bit the spin value $|-\frac{1}{2}\rangle$, if we find in the respective place the letter $\phi$, and $|+\frac{1}{2}\rangle$, if we find $Z$.

In perturbation theory, the anomalous dimension matrix is given by

$$\Delta(g) = \sum_k H_{2k}\lambda^{2k}, \tag{2.2}$$

with $\lambda^2 = \frac{g_{YM}^2 N}{8\pi^2}$ being the 't Hooft coupling. The coefficients in this expansion are given in terms of effective vertices, i.e., the operators $H_{2k}$. They can be determined, e.g., by an explicit evaluation of the divergencies of two-point function $\langle\mathcal{O}(0)\mathcal{O}(x)\rangle$ Feynman amplitudes.

At the zero, one and two loop level, the SU(2) anomalous dimension matrices are given by the following expressions [21]:

$$H_0 = \mathrm{Tr}(\phi\check{\phi} + Z\check{Z}),$$
$$H_2 = -\frac{2}{N}{:}\mathrm{Tr}\big([\phi, Z][\check{\phi}, \check{Z}]\big){:},$$
$$H_4 = \frac{1}{N^2}\big\{2{:}\mathrm{Tr}\big([Z,\phi]\big[\check{Z},\big[Z,[\check{Z},\check{\phi}]\big]\big]\big){:} + 2{:}\mathrm{Tr}\big([Z,\phi]\big[\check{\phi},\big[\phi,[\check{Z},\check{\phi}]\big]\big]\big){:} + 4N{:}\mathrm{Tr}\big([\phi,Z][\check{\phi},\check{Z}]\big){:}\big\},$$

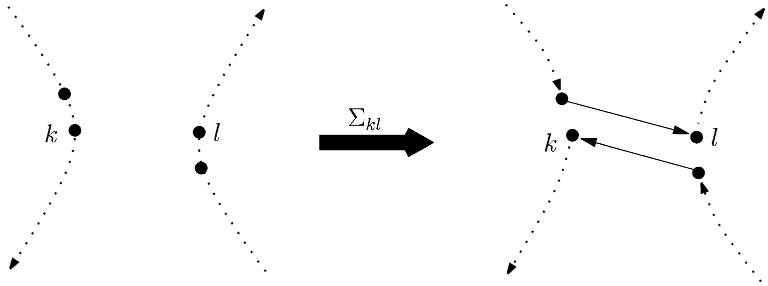


Fig. 1. Splitting and joining of chains by $\Sigma_{kl}$.

where the checked letters $\check{\phi}$ and $\check{Z}$ correspond to derivatives with respect to the matrix elements

$$\check{Z}_{ij} = \frac{\partial}{\partial Z^{ji}}, \qquad \check{\phi}_{ij} = \frac{\partial}{\partial \phi^{ji}}$$

and colons denote the ordering in which all checked letters in the group are assumed to stay on the right of the unchecked ones.

In order to find the "pull back" of the Hamiltonian (2.2) to the spin description, one has to apply it on a (multi)trace operator corresponding to the spin bit state $|s, \gamma\rangle$ and map the result back to the corresponding spin bit state. This can be done term-by-term in the perturbation theory expansion series.

A simple form for the one-loop nonplanar Hamiltonian was found earlier [18,19] (see also [22] for a related discussion) and reads

$$H_2 = \frac{1}{2N} \sum_{k,l} H_{kl} \Sigma_{k\gamma_l} = \frac{1}{N} \sum_{k,l} (1 - P_{kl}) \Sigma_{k\gamma_l}, \tag{2.3}$$

where the permutation and chain "twist" operators are respectively defined in the following way $(k, l = 1, \ldots, L)$:

$$P_{kl} \big|\{\ldots A_k \ldots A_l \ldots\}\big\rangle = \big|\{\ldots A_l \ldots A_k \ldots\}\big\rangle, \quad \text{with } A_l, A_k \in \{\phi, Z\},$$
$$\Sigma_{kl}|\gamma\rangle = \begin{cases} |\gamma\,\sigma_{kl}\rangle, & \text{if } k \neq l, \\ N|\gamma\rangle, & k = l. \end{cases} \tag{2.4}$$

$\Sigma_{kl}$ acts as a chain splitting and joining operator as illustrated in Fig. 1. Notice that two $\Sigma$'s do not commute if they have indices in common. The factor $N$ in the case $k = l$ in Eq. (2.4) appears because the splitting of a trace at the same place leads to a chain of length zero, whose corresponding trace is $\mathrm{Tr}\,1 = N$. It is important to note that the operator $\Sigma_{kl}$ acts only on the linking variable, while the two-site SU(2) one-loop spin bit Hamiltonian $H_{kl} = 2\,(1 - P_{kl})$ acts on the spin space. Therefore, the two operators commute.

## 3. The two loop Hamiltonian

Let us now consider the two loop Hamiltonian

$$H_4 = \frac{1}{N^2}\big\{2{:}\mathrm{Tr}\big([Z,\phi]\big[\check{Z},\big[Z,[\check{Z},\check{\phi}]\big]\big]\big){:} + 2{:}\mathrm{Tr}\big([Z,\phi]\big[\check{\phi},\big[\phi,[\check{Z},\check{\phi}]\big]\big]\big){:} + 4N{:}\mathrm{Tr}\big([\phi,Z][\check{\phi},\check{Z}]\big){:}\big\}. \tag{3.1}$$

We introduce the operator

$$\mathcal{O}_{B_1,B_2,B_3} = \mathrm{Tr}(\check{A}_k A_{B_1} \check{A}_l A_{B_1} \check{A}_m A_{B_3}),$$

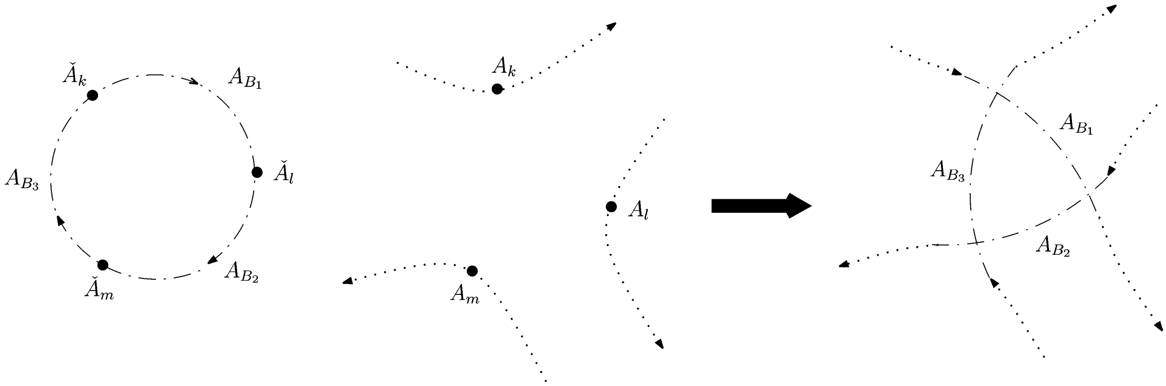


Fig. 2. The action of $\mathcal{O}_{B_1,B_2,B_3}$ on a spin chain state.

with $\check{A}_k, \check{A}_l, \check{A}_m = \check{\phi}, \check{Z}$ acting on the $k$th, $l$th, $m$th sites of the state $|\ldots A_k \ldots A_l \ldots A_m \ldots;\gamma\rangle$, respectively. Here $B_i$ are non-intersecting sequences chosen in the set $\{klm\}$ and $A_{B_i}$ are then monomials in $A_k, A_l, A_m$. For example, the choices $B_1 = \emptyset, k, lm, klm$ correspond to $A_{B_1} = 1, A_k, A_l A_m, A_k A_l A_m$. Indeed, as $A_{B_1}, A_{B_2}, A_{B_3}$ are made of the same number of $\phi$ and $Z$ than in $\check{A}_k, \check{A}_l, \check{A}_m$, any trace of (3.1) can be written with such an $\mathcal{O}_{B_1,B_2,B_3}$ operator.

Acting with $\mathcal{O}_{B_1,B_2,B_3}$ on a spin chain state specified by $\gamma$, one finds

$$\mathcal{O}_{B_1,B_2,B_3}:\gamma = \begin{pmatrix} \gamma_k^{-1} & k & \gamma_l^{-1} & l & \gamma_m^{-1} & m \\ k & \gamma_k & l & \gamma_l & m & \gamma_m \end{pmatrix} \to \begin{pmatrix} \gamma_k^{-1} & B_1 & \gamma_l^{-1} & B_2 & \gamma_m^{-1} & B_3 \\ B_1 & \gamma_l & B_2 & \gamma_m & B_3 & \gamma_k \end{pmatrix}.$$

A pictorial view of the action of $\mathcal{O}_{B_1,B_2,B_3}$ is given in Fig. 2. The three relevant cases are

$$\begin{aligned}
\mathcal{O}_{klm,\emptyset,\emptyset}|\gamma\rangle &= \left|\begin{pmatrix} \gamma_k^{-1} & k & l & m & \gamma_l^{-1} & \gamma_m^{-1} \\ k & l & m & \gamma_l & \gamma_m & \gamma_k \end{pmatrix}\right\rangle = P_{lm}\Sigma_{l\gamma_m}\Sigma_{\gamma_k m}|\gamma\rangle, \\
\mathcal{O}_{m,\emptyset,kl}|\gamma\rangle &= \left|\begin{pmatrix} \gamma_k^{-1} & m & \gamma_l^{-1} & \gamma_m^{-1} & k & l \\ m & \gamma_l & \gamma_m & k & l & \gamma_k \end{pmatrix}\right\rangle = P_{km}\Sigma_{l\gamma_m}\Sigma_{\gamma_k\gamma_l}|\gamma\rangle, \\
\mathcal{O}_{kl,\emptyset,m}|\gamma\rangle &= \left|\begin{pmatrix} \gamma_k^{-1} & k & l & \gamma_l^{-1} & \gamma_m^{-1} & m \\ k & l & \gamma_l & \gamma_m & m & \gamma_k \end{pmatrix}\right\rangle = P_{km}\Sigma_{km}\Sigma_{l\gamma_m}|\gamma\rangle.
\end{aligned} \tag{3.2}$$

In order to write the operators in terms of $P$ and $\Sigma$, we used the fact that permutations can also be viewed as operators acting on $\gamma$ rather than on spin states. From such a viewpoint, the action of $P_{kl}$ on $\gamma$ is

$$P_{kl}\left|\begin{pmatrix} . & . & . & k & . & l & . & . & . \\ . & k & . & . & . & . & . & l & . \end{pmatrix}\right\rangle = \left|\begin{pmatrix} . & . & . & l & . & k & . & . & . \\ . & l & . & . & . & . & . & k & . \end{pmatrix}\right\rangle,$$

while the action of $\Sigma_{kl}$ is given explicitly by

$$\Sigma_{kl}\left|\begin{pmatrix} . & . & . & . & . \\ . & k & . & l & . \end{pmatrix}\right\rangle = \left|\begin{pmatrix} . & . & . & . & . \\ . & l & . & k & . \end{pmatrix}\right\rangle.$$

All other contributions in (3.1) can be written as permutations and/or relabeling of indices in (3.2). Collecting the sixteen terms coming from (3.1), one finds

$$\begin{aligned}
H_4 = \frac{2}{N^2}\sum_{k\neq l\neq m}\big[&(P_{km}P_{lm} + P_{kl}P_{lm} - P_{kl} - P_{km})(\Sigma_{l\gamma_m}\Sigma_{\gamma_k\gamma_l} + \Sigma_{kl}\Sigma_{\gamma_l m}) \\
&+ (2P_{lm} + 2P_{kl} - 2 - P_{kl}P_{lm} - P_{lm}P_{kl})\Sigma_{k\gamma_l}\Sigma_{l\gamma_m}\big] + \frac{4}{N}\sum_{k,l}(P_{kl}-1)\Sigma_{k\gamma_l}.
\end{aligned} \tag{3.3}$$

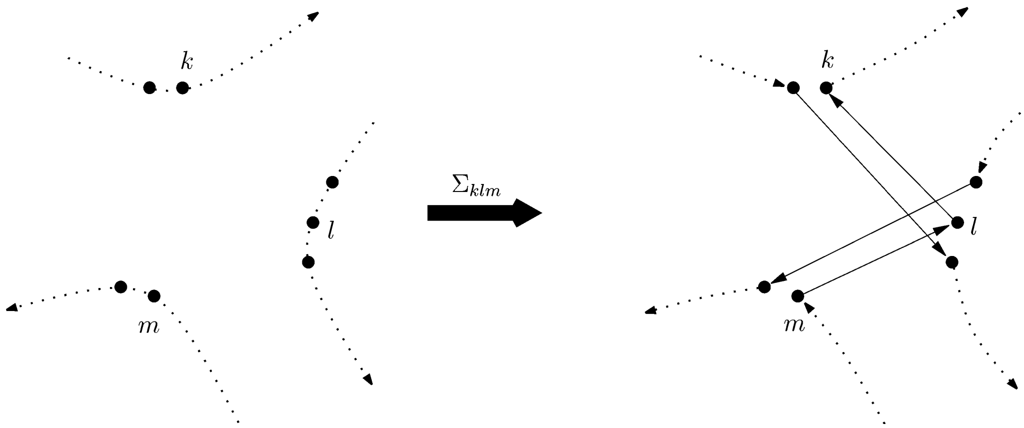


Fig. 3. Splitting and joining of chains by $\Sigma_{klm} \equiv \Sigma_{k\gamma_l} \Sigma_{l\gamma_m}$.

Using the relation

$$1 - P_{kl} - P_{km} - P_{lm} + P_{km} P_{lm} + P_{kl} P_{lm} = 0 \tag{3.4}$$

valid for $k \neq l \neq m$ on $\frac{1}{2}$-spin states, one can rewrite the two loop $\mathrm{SU}(2)$ spin bit Hamiltonian (3.3) as

$$\begin{aligned} H_4 = {} & \frac{2}{N^2} \sum_{k \neq l \neq m} \big[ (P_{lm} - 1)(\Sigma_{l\gamma_m} \Sigma_{\gamma_k\gamma_l} + \Sigma_{kl} \Sigma_{\gamma_l m}) + (P_{lm} + P_{kl} - P_{km} - 1) \Sigma_{k\gamma_l} \Sigma_{l\gamma_m} \big] \\ & + \frac{4}{N} \sum_{k,l} (P_{kl} - 1) \Sigma_{k\gamma_l}. \end{aligned} \tag{3.5}$$

Next, we would like to write the Hamiltonian in terms of $P$'s and a single (two loop) joining–splitting operator

$$\Sigma_{klm} \equiv \Sigma_{k\gamma_l} \Sigma_{l\gamma_m}.$$

Its action is depicted in Fig. 3. Because a one-operator trace vanishes and a two-operators trace is fully symmetric, the joining–splitting $\Sigma_{klm}$ operator satisfies

$$\Sigma_{llm} = \Sigma_{kll} = (P_{kl} - 1) \Sigma_{klk} = 0. \tag{3.6}$$

Noticing that[1]

$$\begin{aligned} & \Sigma_{l\gamma_m} \Sigma_{\gamma_k\gamma_l} = \Sigma_{\gamma_k\gamma_l} \Sigma_{l\gamma_m} = \Sigma_{\gamma_k lm} \quad \text{for } l \neq \gamma_k \\ & \Sigma_{kl} \Sigma_{\gamma_l m} = \Sigma_{m\gamma_l} \Sigma_{lk} = \Sigma_{ml\gamma_k^{-1}} \quad \text{for } k \neq \gamma_l, \end{aligned}$$

the first term of the first sum in (3.5) can then be rewritten as

$$\begin{aligned} X \equiv {} & \sum_{k \neq l \neq m} (P_{lm} - 1)(\Sigma_{l\gamma_m} \Sigma_{\gamma_k\gamma_l} + \Sigma_{kl} \Sigma_{\gamma_l m}) \\ = {} & \sum_{\gamma_{k'}^{-1} \neq l \neq m} \Sigma_{k'lm}(P_{lm} - 1) + \sum_{k \neq l \neq \gamma_{m'}} \Sigma_{klm'}(P_{kl} - 1) + \sum_{l,m} (P_{lm} - 1)(\Sigma_{l\gamma_m} \Sigma_{l\gamma_l} + \Sigma_{\gamma_l l} \Sigma_{\gamma_l m}), \end{aligned}$$

[1] Here $k \neq l \neq m$ is understood.

where the terms with $l = \gamma_k$ or $k = \gamma_l$ were explicitly written apart. Relaxing the restrictions in the sums by subtracting the exceptional terms and using $\Sigma_{kk} = N\mathcal{I}$, one gets

$$X = \sum_{k,l,m} \Sigma_{klm}(P_{lm} + P_{kl} - 2) - 2N \sum_{l,m}(P_{lm} - 1)\Sigma_{l\gamma_m}$$

$$- \sum_{l,m}(P_{lm} - 1)[\Sigma_{\gamma_m\gamma_l}\Sigma_{l\gamma_m} - \Sigma_{l\gamma_m}\Sigma_{l\gamma_l} + \Sigma_{m\gamma_l}\Sigma_{lm} - \Sigma_{\gamma_l l}\Sigma_{\gamma_l m}].$$

Finally, using the relations $\Sigma_{\gamma_m\gamma_l}\Sigma_{l\gamma_m} = \Sigma_{l\gamma_m}\Sigma_{l\gamma_l}$ and $\Sigma_{m\gamma_l}\Sigma_{lm} = \Sigma_{\gamma_l l}\Sigma_{\gamma_l m}$ and plugging $X$ in (3.5), one finally finds the surprisingly simple result of this Letter

$$H_4 = \frac{2}{N^2} \sum_{k,l,m} (2P_{lm} + 2P_{kl} - P_{km} - 3)\Sigma_{klm}. \tag{3.7}$$

Notice that in (3.7) one can chose to put or not the restrictions $k \neq l \neq m$, as equaling two indices always gives a term of the form (3.6).

It is also instructive to rewrite the Hamiltonian in terms of SU(2) spin operators $\vec{s} = \frac{1}{2}\vec{\sigma}$, where $\vec{\sigma}$ are usual Pauli matrices, using the identity (see, e.g., [23])

$$P_{kl} = \frac{1}{2} + \frac{1}{2}\vec{s}_k \cdot \vec{s}_l. \tag{3.8}$$

After the substitution of permutation operators, the one-loop Hamiltonian (2.3) takes the following form:

$$H_4 = \frac{8}{N^2} \sum_{k,l,m} \big((\vec{s}_k - 2\vec{s}_l + \vec{s}_m)\big)^2 \Sigma_{klm} = \frac{8}{N^2} \sum_{k,l,m} \big((\vec{s}_k - \vec{s}_l) - (\vec{s}_l - \vec{s}_m)\big)^2 \Sigma_{klm}. \tag{3.9}$$

This Hamiltonian has a simple meaning (see Fig. 3): $\Sigma_{klm}$ cyclically exchanges the incoming and outgoing ends of the chains adjacent to the bits $k, l$ and $m$; at the same time the spin part acts as the discrete second derivative along the new chain. After knowing that the one-loop Hamiltonian has the similar structure

$$H_2 = \frac{4}{N} \sum_{k,l} (\vec{s}_k - \vec{s}_l)^2 \Sigma_{k\gamma_l},$$

it is very tempting to conjecture that at the arbitrary $n$-loop level the Hamiltonian is given by the discrete derivative of the order $n$ squared times the splitting that cyclically exchange the chain ends

$$H_n \sim \frac{4n}{N^n} \sum_{k_1,\dots,k_n} \left( \sum_{i=0}^{n} \frac{n!}{(n-i)!i!}(-1)^i \vec{s}_{k_i} \right)^2 \Sigma_{k_0\gamma_{k_1}}\Sigma_{k_1\gamma_{k_2}} \cdots \Sigma_{k_{n-1}\gamma_n}.$$

This is compatible with the BMN conjecture [5] but it implies that the Hamiltonian can be written linearly in pair permutation operators $P_{kl}$ at *any loop level*, which unfortunately is probably not the case.

### 3.1. The planar limit

$N \to \infty$ affects just the "twist" operator in the following way:[2]

$$\lim_{N\to\infty} \frac{1}{N} \Sigma_{kl} = \delta_{kl}.$$

The two loop nonplanar SU(2) spin bit Hamiltonian (3.7) gives then the correct known expression [21] in the planar limit $N \to \infty$:[3]

$$\lim_{N\to\infty} H_4 = 2\sum_{k=1}^{L}(4P_{k,k+1} - P_{k,k+2} - 3) = 2\sum_{k=1}^{L}(-4 + 6P_{k,k+1} - P_{k,k+1}P_{k+1,k+2} - P_{k+1,k+2}P_{k,k+1}), \tag{3.10}$$

where in the last passage we used the identity (3.4).

## 4. Conclusion

In this Letter we considered the anomalous dimension and mixing of composite operators of $\mathcal{N} = 4$ SYM theory in the SU(2) symmetric sector. Using the isomorphic map between gauge invariant composite operators and the spin bit states, we computed the spin bit Hamiltonian corresponding to two-loop corrections to the anomalous dimension/mixing matrix. The resulting Hamiltonian at this level has two important properties:

(i) The Hamiltonian shows at the two-loop level an explicit full factorization in the spin and chain splitting parts similar to the one-loop level.
(ii) Its action is given by a three-point spin interaction and a cyclic exchange (hopping) of the chain ends.

The first property is expected to hold at any loop order since the Hilbert space of the spin bit model is always the direct product of the spin space and the linking variable $\gamma$-space. The second property has a natural generalization to $n+1$ interacting points appearing at $n$ loops: cyclic exchange of the chain ends multiplied by the square of the $n$th discrete derivative of spin operators $\vec{s}_k$. In the continuum limit, this is in perfect agreement with the BMN conjecture which gives a term $\sim \lambda^{2n}(\partial^n \phi)^2$ as the $n$ loop contribution.

A strong consequence of this higher loop conjecture is the requirement that at any loop level the spin part of the Hamiltonian should always be linear in permutation operators. This implies strong restrictions on the planar limit too. Of course, there is a very rich set of identities involving permutation operators which could be used to prove such a property. Our attempts to check this at the three-loop level with the expressions for planar Hamiltonians given by [21] so far failed.

Similar results giving the Hamiltonian at three and more loops in terms of spin-bit would give more insight, allowing one to give a conjecture for generalization. In fact, there is enough data and technique at this stage to produce the three loop Hamiltonian. The problem being only algebraic difficulty, it seems hopefully superable by the use of computer algebra.

Finally, we notice that it would be interesting to extend this analysis to other sectors of $\mathcal{N} = 4$ SYM; unfortunately, only SU(2) anomalous dimension operators are known beyond one-loop.

[2] In fact from Eq. (2.4) the following decomposition of $\Sigma_{kl}$ holds

$$\Sigma_{kl} = N\delta_{kl} + (1 - \delta_{kl})\tilde{\Sigma}_{kl},$$

where $\tilde{\Sigma}_{kl}$ is the joining–splitting operator spoiled of its degeneracy in the case of coinciding sites.

[3] We assume here a single trace, so that $\gamma_k \equiv k+1$, with the identification $L+1 \equiv 1$.

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