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Procedia Engineering 127 (2015) 1010 – 1017

**Procedia
Engineering**www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

MHD stagnation point flow of a nanofluid with velocity slip, Non-linear radiation and Newtonian heating

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Abstract

The flow of a viscous, incompressible and electrically conducting nanofluid flow over a stretching sheet under the influence of a transverse magnetic field is investigated taking in to account the effect of non-linear thermal radiation, newtonian heating and partial velocity slip. The nanofluid model considered in the paper incorporates the effect of Brownian motion and thermophoresis. The governing equations, in similarity form, are solved using Matlab's in-built boundary value problem solver "bvp4c". The nanofluid flow model discussed in the present paper has significant applications in fluid engineering devices where the boundary surface is subjected to convecting heating and the temperature difference between the ambient fluid and the surface is large.

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Peer-review under responsibility of the organizing committee of ICCHMT – 2015

Keywords: Magnetic field; nanofluid; Newtonian heating; stretching sheet; thermal radiation.

1. Introduction

The boundary layer theory over a stretching sheet has remarkable applications in many industrial and engineering problems. Crane [1] was the first to introduce the study of boundary layer flow on two dimensional elastic stretching sheet. Thereafter, many researchers have pooled contributing their stake on further research over stretching sheet. Dutta et al. [2] examined the viscous incompressible flow temperature distribution with respect to uniform heat flux in a stretching sheet. They observed decreasing temperature profiles with increase Prandtl number. The surface condition effect of a micropolar fluid over a stretching with strong suction or injection was analyzed by Kelson and Deseaux [3]. They identified that suction parameter decreases boundary layer width. Bhargava et al. [4] examined the micropolar fluid flow over a porous stretching sheet with effect of mixed convection. Their study reveals that heat transfer rate decreases with an increase in injection while it increases with an increase in the as Grashof number and suction parameter. Vajravelu et al. [5] studied heat transfer in the MHD flow over a non-isothermal sheet with thermal radiation and heat source. They observed that for enhancing the values of thermal radiation, thermal distribution in the fluid flow region should be reduced. Shaw et al. [6] examined homogeneous and heterogeneous reaction effects on

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micropolar fluid past a porous stretching/shrinking sheet with suction. It was observed that as the stretching/shrinking increases, concentration of reactants increases. Bhattachaya [7] has studied heat transfer in unsteady boundary layer stagnation- region of shrinking/stretching sheet with time dependent surface temperature. He observed that dual solution exist for some numbers of ratio for which first solution is a decrease in temperature with unsteadiness while the second solution gives a decrease in thermal profile followed by an increase at large distance from the sheet.

Thermal and solutal convection has numerous applications in manufacturing processes such as in evaporation at the surface of a water body, distribution of temperature, energy transfer in a wet cooling tower, drying, damage of crops due to freezing and also in geophysics and volcanic systems [8–10]. Nanoparticle-fluid suspensions are termed nanofluids, obtained by dispersing nanometer sized particles in a conventional base fluid like water, oil, ethylene glycol etc. The flow analysis of nanofluids has been the topic of comprehensive research, as it has improved thermal conductivity behavior in heat transfer processes. The energy transport in a nano fluid consists of random motion of suspended nanoparticles, called the Brownian motion. This motion transports energy directly by nanoparticles. This model gives a general expression for effective thermal conductivity of nanofluids. Nanoparticles are made from various materials, such as metallic oxide (Al_2O_3 , CuO), nitride ceramics (AlN , SiN), carbide ceramics (SiC , TiC), metals (Cu , Ag , Au), semiconductors, (TiO_2 , SiC). Single, double or multi walled carbon nanotubes (SWCNT, DWCNT, MWCNT), alloyed nanoparticles ($Al_{70}Cu_{30}$) etc. have been used for the preparation of nanofluids. According to Prodanovi et al. [11], nanofluids containing ultrafine nanoparticle have the capability of flowing in porous media, and these flows can improve oil recovery; and help control/ optimize recovery process. Nanoparticles are also used to determine changes in fluid saturation and reservoir properties during oil and gas production. Many studies on nanofluids are being conducted by scientists and engineers due to their diverse technical and biomedical applications. Examples include nanofluid coolant (electronics cooling, vehicle cooling, and so on), medical applications (cancer therapy and safer surgery by cooling), process industries (materials and chemicals, detergency, food and drink, paper and printing, and textiles). Advances in nanoelectronics, nanophotonics, and nanomagnetism; ultrahigh performance cooling is necessary for many industrial technologies. The nanofluids are more stable and have acceptable viscosity with better wetting, spreading, and dispersion properties on solid surface [12,13].

Khan and Pop [14] introduced nanofluids in stretching sheet with effects of Brownian motion and thermophoresis forms. It was found that skin friction and Sherwood numbers decreasing with brownian and thermophoresis numbers. Rosmila et al. [15] have studied the MHD convection flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of thermal stratification. It was investigated that velocity and temperature of the fluid increases as the strength of magnetic field increase. Kameswaran et al.[16] examined the problem of steady boundary layer flow in a nanofluid for chemically reacting, stretching or shrinking sheet. They reported that the velocity profile decreases with an increase in nano particle volume fraction, while the temperature and concentration profiles increases. Bhasker Reddy et al. [17] investigated the influence of variable thermal conductivity and partial velocity slip on hydromagnetic two-dimensional boundary layer flow of a nanofluid with Cu nanoparticles over a stretching sheet with convective boundary condition. Nadeem et al. [18] investigated the flow of three-dimensional exponentially stretching sheet for water based nanofluid. They identified that with increase in the stretching parameter, the skin frictions and the heat transfer rate increase.

The aim of this research article is to study the influences of various flow properties and physical parameters such as Brownian parameter, thermophoresis parameter and magnetic parameter on thermosolutal convection characteristics with effects of non-linear thermal radiation, newtonian heating and partial velocity slip.

2. Mathematical Formulation of the Problem

Consider the flow of viscous, incompressible, and electrically conducting nanofluid over a stretching sheet under the influence of a transverse uniform magnetic field B_0 . The x -axis is taken along the length of the stretching sheet in horizontal direction while the y -axis is normal to the sheet. Two equal but opposite forces act on the sheet to stretch it along its length with a velocity $U_w(x)$ keeping the position of the origin as fixed. Also the free stream velocity of the nanofluid is assumed as $U_\infty(x)$. It is assumed that the surface of the sheet is convectively heated from a hot fluid having temperature T_f and heat transfer coefficient h_f and the difference between the ambient fluid temperature T_∞ and temperature within the boundary layer T_w is large enough to create radiation effects in the flow field. It is further assumed that there is a partial velocity slip which takes place at the fluid solid interface. The magnetic

Reynolds number of the fluid is assumed to be very small so that the induced magnetic field effects can be neglected in comparison to the applied one. The nanoparticle volume fraction concentration at the surface and outside the boundary layer region are C_w and C_∞ , respectively.

Under the above assumptions, the boundary layer equations governing the flow, heat and nanoparticle volume fraction of a viscous, incompressible, and electrically conducting nanofluid under the influence of a transverse magnetic field and radiative heat transfer using Rosseland’s diffusion approximation, are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (U_\infty - u), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[\left(\alpha_m + \frac{16\sigma^* T^3}{3\alpha^*} \right) \frac{\partial T}{\partial y} \right] + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right], \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively, α_m is the thermal diffusivity of the fluid, ν is the kinematic viscosity, D_B is the Brownian diffusion coefficient, B_0 is the induced magnetic field, D_T is the thermophoretic diffusion coefficient and $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, c is the specific heat at constant pressure, ρ_p is the density of the particles, σ^* is the Stefan-Boltzmann constant, α^* is the Rosseland mean absorption coefficient.

The appropriate boundary conditions, taking into account the partial velocity slip and Newtonian heating at the surface, are

$$u = U_w + L \frac{\partial u}{\partial y} = cx + L \frac{\partial u}{\partial y}, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h(T_f - T), \quad C = C_w \text{ at } y = 0$$

$$u \rightarrow U_\infty = ax, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \text{ as } y \rightarrow \infty \tag{5}$$

where $U_w = cx(c > 0)$ is the velocity of the stretching sheet, L is slip length and k is the thermal conductivity of the nanofluid. The free stream velocity of the nanofluid is $U_\infty = ax(a > 0)$.

In order to transform the governing equations (1)-(4) to similarity form, we introduce the following transformation

$$\psi(x, y) = \sqrt{av}xf(\eta), \quad \theta = \frac{(T - T_\infty)}{(T_f - T_\infty)}, \quad \theta_r = \frac{T_w}{T_\infty}, \quad \phi = \frac{(C - C_\infty)}{(C_w - T_\infty)} \text{ where } \eta = \sqrt{\frac{a}{\nu}}y, \tag{6}$$

where θ_r is temperature ratio parameter, η is the dimensionless stream function, and the above transformation is chosen in such a way that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

Using the above transformation, the equation of continuity (1) is automatically satisfied and Eqs. (2), (3) and(4), reduces to

$$f''' + ff'' - f'^2 - Mf' + 1 + M = 0, \tag{7}$$

$$\left[1 + \frac{4}{3N_r} \{1 + (\theta_r - 1)\theta\}^3 \right] \theta'' + Prf\theta' + PrNb \theta' \phi' + \left[PrNt + \frac{4}{N_R} (\theta_r - 1)\{1 + (\theta_r - 1)\theta\}^2 \right] \theta'^2 = 0, \tag{8}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0. \tag{9}$$

The boundary conditions (5) reduce to

$$f(\eta) = 0, \quad f'(\eta) = \alpha + Af''(\eta), \quad \theta'(\eta) = -Bi[1 - \theta(\eta)], \quad \phi(\eta) = 1 \text{ at } \eta = 0,$$

$$f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \tag{10}$$

In the above equations, primes denote differentiation with respect to η and the nine parameters are defined by

$$Pr = \frac{\nu}{\alpha_m}, Nb = \frac{(\rho c)_p D_B (c_w - c_\infty)}{(\rho c)_f \nu}, M = \frac{\sigma B_0^2}{\rho a}, A = L \sqrt{\frac{a}{\nu}}, \alpha = \frac{c}{a},$$

$$Le = \frac{\nu}{D_B}, Nt = \frac{(\rho c)_p D_T (T_f - T_\infty)}{(\rho c)_f T_\infty \nu}, N_r = \frac{k \alpha_*}{4 \sigma_* T_\infty^3}, Bi = \frac{h}{k} \sqrt{\frac{\nu}{a}}, \tag{11}$$

where Pr is the Prandtl number, Nb is the Brownian motion parameter, M is the magnetic parameter, A is velocity slip parameter, α is the ratio of rates of stretching velocity and free stream velocity, Le is the Lewis number, Nt is the thermophoresis parameter, N_r is a radiation parameter, and Bi is the Biot number.

The physical quantities of engineering interest are the skin friction coefficient C_f , the local Nusselt number Nu_x and the local Sherwood number Sh_x which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2(x)}, Nu_x = \frac{x q_w}{k(T_f - T_\infty)}, Sh_x = \frac{x q_m}{D_B(C_w - C_\infty)}$$

where the surface shear stress τ_w , the local heat flux q_w , and the local mass flux q_m are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_{y=0}, q_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}.$$

Using the similarity variables (6), the coefficients of skin-friction, heat transfer, and mass transfer, are given by

$$(Re_x)^{1/2} C_f = f''(0), \tag{12}$$

$$(Re_x)^{-1/2} Nu_x = - \left[1 + \frac{4}{3 N_r} \{1 + (\theta_r - 1) \theta(0)\}^3 \right] \theta'(0), \tag{13}$$

$$(Re_x)^{-1/2} Sh_x = -\phi'(0), \tag{14}$$

where $Re_x = \frac{u_w(x)x}{\nu}$ is the local Reynolds number based on the stretching velocity $u_w(x)$.

3. Results and Discussion

The highly nonlinear coupled ordinary differential Eqs. (7)-(9) subject to the boundary conditions (10) were solved numerically using `bvp4c` routine of Matlab. In order to investigate the effects of velocity slip on the nanofluid velocity, nanofluid temperature and nanoparticle volume fraction, the profiles of these physical quantities are displayed in Fig. 1 while the effects of non-linear thermal radiation, temperature ratio parameter, Brownian motion, thermophoresis and convective heating on the nanofluid temperature and nanoparticle concentration are presented in Figs. 2 to 6.

The effect of velocity slip on the nanofluid velocity, nanofluid temperature, and nanoparticle volume fraction is depicted in Fig. 1. An increase in the velocity slip parameter (A) corresponds to a decrease in the relative velocity of the fluid and the stretching sheet. It is found that the nanofluid velocity is a decreasing function of A while the nanofluid temperature and nanoparticle volume fraction are increasing functions of the velocity slip parameter A . Thus the decrease in the relative velocity between the fluid and the stretching sheet implies an increase in the nanofluid velocity and corresponding decrease in nanofluid temperature and nanoparticle volume fraction.

The effect of thermal radiation parameter N_r on the nanofluid temperature and nanoparticle volume fraction is displayed graphically in Fig. 2. It is worthwhile to note that the radiation effect is inversely proportional to the radiation parameter N_r , which implies an increase in radiation effect with decrease in N_r . We may observe from 2 that the nanofluid temperature decrease with increasing N_r which helps to conclude that the temperature of the nanofluid increase with increasing radiation effects. The nanoparticle volume fraction increases near the wall within the boundary layer region and changes its characteristics away from the wall. Fig. 3 presents the effect of temperature ratio parameter $\theta_r (= \frac{T_f}{T_\infty} > 1)$ on the fluid temperature $\theta(\eta)$ and nanoparticle volume fraction $\phi(\eta)$. A value of $\theta_r > 1$ reflects higher surface temperature as compared to the ambient fluid and it is found that the increasing values of the temperature ratio parameter causes the nanofluid temperature to increase. Thus, as the sheet temperature increases, the

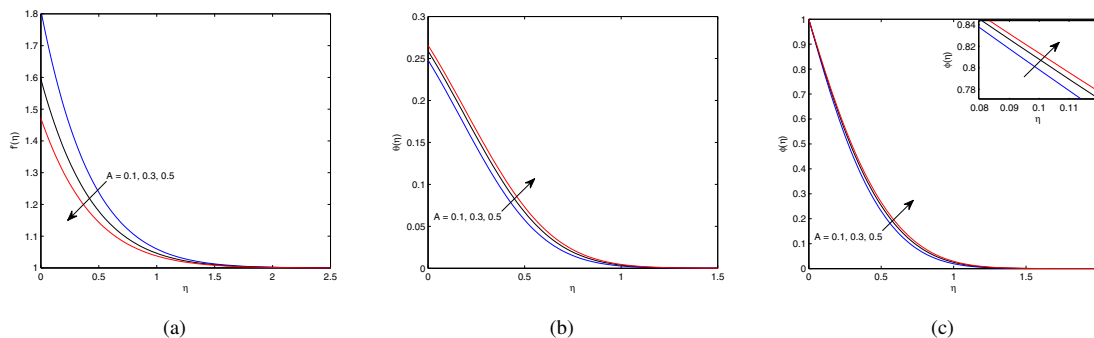


Fig. 1: The effect of momentum slip parameter A on (a) the fluid velocity f' , (b) the fluid temperature θ and (c) nanoparticle volume fraction ϕ when $M = 2, N_r = 5, \theta_r = 2, Le = 5, Pr = 6.7850, Nb = 0.1, Nt = 0.1, \alpha = 2$ and $Bi = 0.5$.

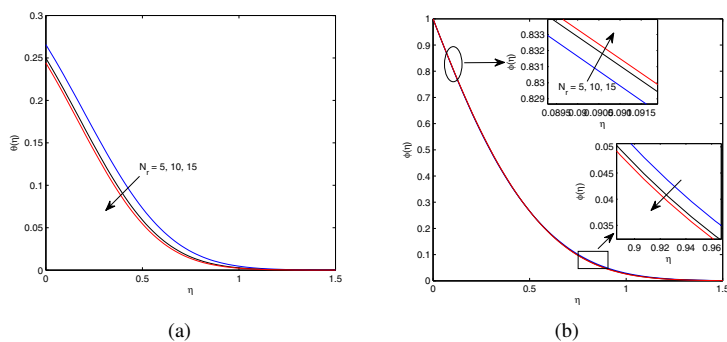


Fig. 2: The effect of thermal radiation parameter N_r on (a) the fluid temperature θ and (b) nanoparticle volume fraction ϕ when $M = 2, \theta_r = 2, Le = 5, Pr = 6.7850, Nb = 0.1, Nt = 0.1, \alpha = 2, A = 0.5$ and $Bi = 0.5$.

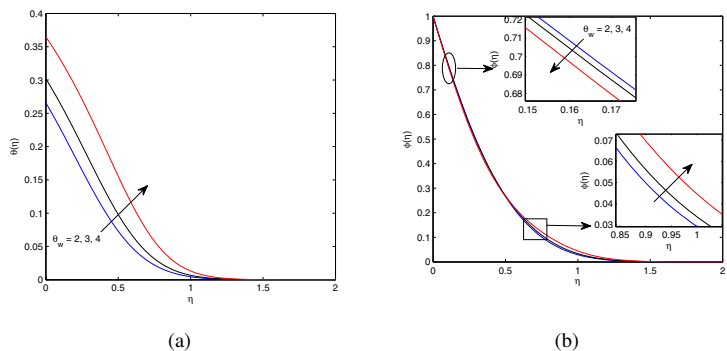


Fig. 3: The effect of temperature ratio parameter θ_r on (a) the fluid temperature θ and (b) nanoparticle volume fraction ϕ when $M = 2, N_r = 5, Le = 5, Pr = 6.7850, Nb = 0.1, Nt = 0.1, \alpha = 2, A = 0.5$ and $Bi = 0.5$.

temperature of the fluid gets increases and heat transfer towards the ambient fluid takes place. The nanoparticle volume fraction shows the decreasing tendency near the wall and increasing tendency away from the wall within the boundary layer region with increasing sheet temperature. The effect of Brownian motion Nb on the nanofluid temperature (θ) and nanoparticle volume fraction $\phi(\eta)$ is displayed in Fig. 4. The strength of the Brownian diffusion due to the

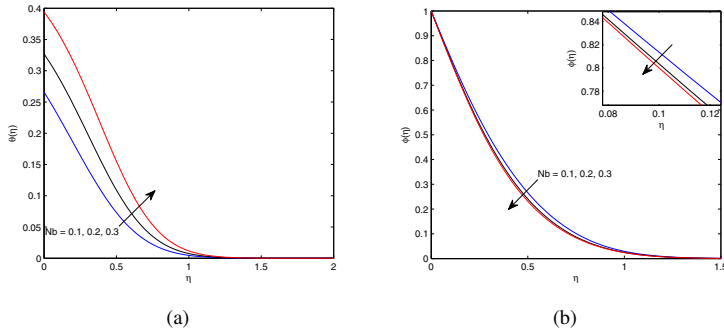


Fig. 4: The effect of Brownian motion parameter Nb on (a) the fluid temperature θ and (b) nanoparticle volume fraction ϕ when $M = 2$, $N_r = 5$, $\theta_r = 2$, $Le = 5$, $Pr = 6.7850$, $Nt = 0.1$, $\alpha = 2$, $A = 0.5$ and $Bi = 0.5$.

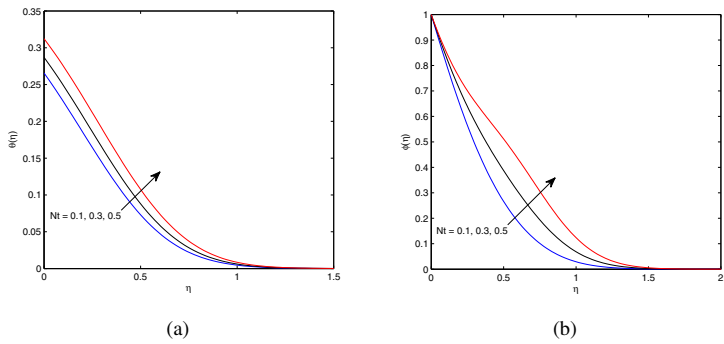


Fig. 5: The effect of thermophoresis parameter Nt on (a) the fluid temperature θ and (b) nanoparticle volume fraction ϕ when $M = 2$, $N_r = 5$, $\theta_r = 2$, $Le = 5$, $Pr = 6.7850$, $Nb = 0.1$, $\alpha = 2$, $A = 0.5$ and $Bi = 0.5$.

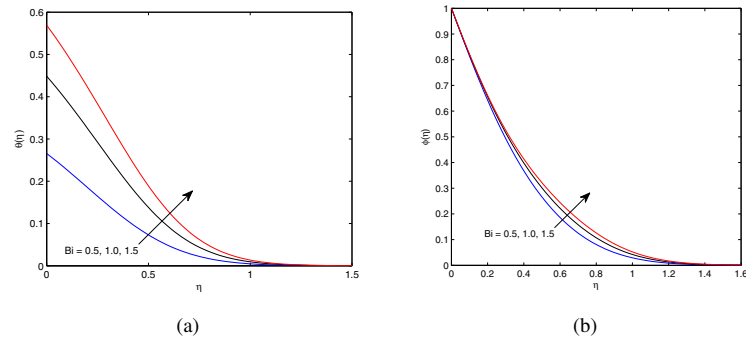


Fig. 6: The effect of Biot number Bi on (a) the fluid temperature θ and (b) nanoparticle volume fraction ϕ when $M = 2$, $N_r = 5$, $\theta_r = 2$, $Le = 5$, $Pr = 6.7850$, $Nb = 0.1$, $Nt = 0.1$, $\alpha = 2$ and $A = 0.5$.

presence of nanoparticles in the flow field is measured with the help of the Brownian motion parameter Nb . It is found that the increase in Brownian motion of the nano sized particles causes an enhancement in the nanofluid temperature and a opposite effect on the nanoparticle volume fraction within the boundary layer region. Due to the collision of small nanoparticles in the flow field a thermal energy is generated which enhances the fluid temperature. Due to

Table 1: Effects of various parameters on coefficient of skin-friction, Nusselt number and Sherwood number when $Pr = 6.7850$ and $Le = 5$.

M	N_r	θ_r	Nb	Nt	α	A	Bi	$-C_f \sqrt{Re_x}$	$Nu/\sqrt{Re_x}$	$Sh/\sqrt{Re_x}$
2	5	2	0.1	0.1	2	0.5	0.5	1.0559954	0.56564827	1.9085476
6								1.19783053	0.56360081	1.85031759
10								1.28411476	0.56236343	1.81727632
	5							1.05599540	0.56564827	1.90854760
	10							1.05599540	0.47273451	1.89617746
	15							1.05599540	0.44257262	1.89169667
		2						1.05599540	0.56564827	1.90854760
		3						1.05599540	0.73300040	1.94109901
		4						1.05599540	1.09573572	1.98983165
			0.1					1.05599540	0.56564827	1.90854760
			0.2					1.05599540	0.54620893	2.00630565
			0.3					1.05599540	0.52186269	2.03933398
				0.1				1.05599540	0.56564827	1.90854760
				0.3				1.05599540	0.55916364	1.69574713
				0.5				1.05599540	0.55107685	1.57321616
					1.5			0.52197609	0.56129753	1.79227818
					2.0			1.05599540	0.56564827	1.90854760
					2.5			1.60080045	0.56927362	2.01863199
						0.1		1.87844309	0.57087930	2.07184081
						0.3		1.34862035	0.56768153	1.96865917
						0.5		1.05599540	0.56564827	1.90854760
							0.5	1.05599540	0.56564827	1.90854760
							1.0	1.05599540	0.99831813	1.86710996
							1.5	1.05599540	1.31203155	1.85838165

the Brownian diffusion the nanoparticles tend to move away from the surface of the sheet and as result a decrease in nanoparticle volume fraction is encountered within the boundary layer region. The effect of thermophoresis parameter Nt which measures the effect of thermophoretic force on the nanofluid temperature and nanoparticle volume fraction is captured in Fig. 5. Thermophoretic force is the force in which a nanoparticle applies physical force on another nanoparticle to move it away from the surface when the preceding gets heated due to the temperature of the sheet. An increase of Nt increases the thermophoretic force which tends to move the nanoparticles from a region in the boundary layer having higher temperature to a region which has lower temperature and correspondingly and increase in the nanofluid temperature and nanoparticle volume fraction is encountered. The surface of stretching sheet is heated by a hot fluid convectively through the Newtonian heating process. The non dimensional parameter Bi measures the effect of this heating which is shown in Fig. 6. It is recorded that with the increase in convective heating of the surface, the nanofluid temperature increases as well as the nanoparticle volume fraction increases. However the observed effect in nanoparticle volume fraction is minimal.

The effects of pertinent flow parameters viz. the magnetic parameter M , thermal radiation parameter N_r , temperature ratio parameter θ_r , Brownian motion parameter Nb , thermophoresis parameter Nt , stretching parameter α , velocity slip parameter A , and Biot number Bi on the flow field is tabulated in Table 1. It is perceived from Table 1 that the coefficient of skin-friction decreases with increasing magnetic field whereas it increases with the increase in velocity slip and stretching of the sheet. The coefficient of heat transfer is a decreasing function of magnetic field, Brownian diffusion, thermophoretic diffusion, and velocity slip whereas it is an increasing function of thermal radiation, temperature ratio parameter, stretching of the sheet, and convective heating at the surface. The magnetic field, thermal radiation, thermophoretic diffusion, velocity slip, and convective heating tend to decrease the nanoparticle Sherwood number whereas the temperature ratio parameter, Brownian diffusion, and stretching of the sheet has reverse effect on it.

4. Conclusions

The paper presents the study of the flow of a viscous, incompressible, and electrically conducting nanofluid over a stretching sheet in the presence of a transverse magnetic field, non-linear thermal radiation, newtonian heating and partial velocity slip. The study reveals that the momentum boundary layer associated with the nanofluid velocity gets thinner with increasing effects of partial velocity slip. The thickness of the nanofluid thermal boundary layer increases with increasing magnetic field, velocity partial slip, thermal radiation, Brownian and thermophoretic diffusion, and convective heating while the stretching velocity decreases the thickness of the thermal boundary layer. Nanoparticle volume fraction behaves as an increasing function of magnetic field, velocity partial slip, thermophoretic force, and convective heating while it is decreasing function of stretching velocity and Brownian diffusion.

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