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Examination on the Relationship between OVX and Crude Oil Price with Kalman Filter

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Abstract

Chicago Board Option Exchange publishes the Crude Oil Volatility Index (OVX) in 2007, which is regarded as a new barometer to research the variance of oil future prices. This paper explores how OVX changes are influenced by crude oil price returns with time-varying coefficients achieved by Kalman filter. The results indicate a negative and asymmetric contemporaneous relationship between OVX changes and crude oil price returns.

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Keywords: Crude Oil Volatility Index; Crude oil price; Kalman filter; Asymmetric effect

1. Introduction

How to estimate market risk is a widely discussed topic in academia [1-4]. Chicago Board Option Exchange (CBOE) publishes the Crude Oil Volatility Index (OVX), which can be regarded as risk gauge of the market. OVX is an up-to-the-minute market estimate of expected 30-day volatility of crude oil futures prices. The OVX is applied to options on the United States Oil Fund. It uses real-time bid/ask quotes of nearby and second nearby options with at least 8 days to expiration, and weights these options to derive a constant, 30-days measure of expected volatility. It’s believed that OVX provides a new barometer to research the variance of oil future prices. This study examines the information content of OVX regarding the crude oil spot returns.

Academics are interested in the relationship of implied volatility between underlying stock market returns of the underlying stock market. It has been well documented in that there is a strong asymmetric negative contemporaneous relationship between the changes of the implied volatility index and the underlying index

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returns. Fleming, Ostdiek and Whaley [5] ,Whaley [6, 7], Simon [8], Giot [9] and Car and Wu [10] all identify the statistically significant negative and asymmetric contemporaneous relationship between VIX and S&P 100 (or S&P 500) returns. Furthermore, an analogous relationship has also been documented in other equity markets; Simon [8] and Giot [9] study Nasdaq, Dowling and Muthuswamy [11] focus on the Australian stock market, Siriopoulos and Fassas [12] study FTSE 100 and later Chen and Lai [13] investigate the relationship between VHSI and HSI.

Many econometric problems require estimation of the state of a system that changes over time using a sequence of noisy measurements made on the system[4]. Previous research has analyzed the relationship either by using static linear regression model, or by dividing sample periods into several distinct sub-periods. This static method cannot seize the time evolvement of the relationship. Many researchers have applied the Kalman filter to evaluate the dynamic evolvement. Crude oil is the most important raw material in the world. Its prices can impact the development of global economy, therefore it is necessary to examine the relationship between OVX and crude oil returns.

This research concentrates on the dynamic relationship of OVX with both crude oil returns using Kalman filter. Other sections of this study are structured as follows. The second section provides a brief introduction of Kalman filter. The third section examines the relationship between crude oil price returns and changes of OVX. Finally, a brief conclusion is provided.

2. Methodology

Kalman filter can describe a recursive solution to the discrete data linear filtering problem and assumes the posterior density at every time step is Gaussian and can be parameterized by a mean and covariance [14]. The kernel idea of Kalman filter is to estimate the state of a process, in a way that minimizes the mean of the squared error [15].

The basic form of Kalman filter is as follows

$$x_t = A_t x_{t-1} + w_{t-1} \tag{1}$$

$$z_t = H_t x_t + v_t \tag{2}$$

A_t and H_t are known matrices defining the linear functions. The random variables w_{t-1} and v_t represent the process and measurement noise respectively. They are assumed to be independent, random walk and follow normal distributions, in other words, $w_t \sim N(0, Q_t)$ and $v_t \sim N(0, R_t)$.

For the constant R_t, we can take some off-line sample measurements in order to determine the variance of the measurement noise. If R_t is time-varying we can use maximum likelihood estimation in each step to get R_t [16].

If the elements of Q_t are set with very small numbers, it will take more data to achieve an accurate estimation and the state variables cannot provide a rapid response to the new measurement. Q_t can be also estimated by a maximum likelihood estimation step by step [16].

We define $\hat{x}_t^- \in R^n$ to be a prior state estimate at step *t* given knowledge of the process prior to step *t*, and $\hat{x}_t \in R^n$ to be a posterior state estimate at step *t* given measurement z_t. And we define prior and a posterior estimate errors as $e_t^- \equiv x_t - \hat{x}_t^-$ and $e_t \equiv x_t - \hat{x}_t$.

The prior estimate error covariance is given by $P_t^- = E[e_t^- e_t']$.

And the posterior estimate error covariance is $P_t = E[e_t e_t']$.

For deriving the equations for the Kalman filter, we begin with the goal of finding an equation that computes posterior state estimate $\hat{x}_t$ as a linear combination of prior $\hat{x}_t^-$ and a weighted difference between an actual measurement z_t and a measurement prediction $H\hat{x}_t^-$ as shown below.

$$\hat{x}_t = \hat{x}_t^- + K(z_t - H\hat{x}_t^-) \quad (3)$$

The difference $(z_t - H\hat{x}_t^-)$ is called the measurement innovation, or the residual. The matrix K is chosen to be the gain that minimizes the posterior error covariance. One form K that minimizes P_t is given by [17, 18]:

$$K_t = P_t^- H'(HP_t^- H' + R)^{-1} \quad (4)$$

The aim of filtering is to find the expected value of the state vector x_t conditional upon the information available at time *t*. In a further step, we can use smoothing to get a more accurate estimation of the state vector given the whole set of information. The mean of the distribution of x_t, conditional upon all the samples, may be written as $E(x_t|Y_T)$. The corresponding estimator is called smoother.

In smoother the recursion starts from the last time step T with $\hat{x}_T^s = \hat{x}_T$ and $P_T^s = P_T$. In this research, we focus on the contemporaneous relationship between volatility index changes and the corresponding stock returns. In order to obtain a better and more accurate description of this relationship, we bring smoothing into this research. The smoothing algorithm is the discrete-time Kalman smoother, also known as the Rauch-Tung-Striebel-smoother (RTS), [19-21]. It can be used for computing the smoothing solution for model Equations (1) and (2) given as distribution:

$$P(x_t|y_{1:T}) = N(x_t|\hat{x}_t^s, P_t^s) \quad (5)$$

3. Relationship between OVX and oil price return

3.1. Data

In this study, daily data of OVX and oil prices, from May 2007 to November 2014, are used to examine the relationship. OVX used in this study is downloaded from Yahoo finance. This research uses WTI spot prices to represent the oil price and the data is downloaded from Energy Information Administration.

For the empirical analysis, Siriopoulos and Fassas [22] use the daily changes of the volatility index. Whaley [7] use the daily rate of change of the VIX. Since the daily logarithmic returns approximately equal to the daily rate of index returns and the use of logs is consistent with the positive skewness in IV data [8], this study uses daily logarithmic returns for OVX and oil price. An obvious advantage of using logarithms is the logarithms can avoid negative estimation of volatility and oil price.

Table 1 Descriptive statistics of implied volatility indices and stock index returns

Statistic	OVX	Logarithm returns of OVX	Oil Prices	Logarithm returns of oil prices
Obs.	1906	1905	1906	1905
Mean	35.662	0.0002	88.271	0.0000
Max	100.420	0.425	145.310	0.164
Min	14.500	0.440	30.2800	0.128
Median	32.830	0.003	91.115	0.001
Std	14.712	0.049	18.218	0.024
Skewness	1.638	0.855	0.382	0.045
Kurtosis	6.387	15.026	3.906	9.339
Jarque-Bera	1763.3	1171.1	111.57	3190.2
ADF test (p-value)	0.211	0.001	0.498	0.001
KPSS Test (p-value)	0.010	0.100	0.010	0.100

Descriptive statistics for OVX and their crude oil price are provided in Table 1. The table reports that the daily logarithm changes of OVX are stationary time series with mean approximately equal to zero. The Jarque-Bera test rejects the null hypothesis of a normal distribution for OVX, and ΔOVX. Table 1 also contains summary statistics of daily logarithm returns of oil prices. The means of logarithm returns are very close to zero and their standard deviations are 4.9% and 2.4%, respectively. The Jarque-Bera statistic robustly rejects the normal distribution hypothesis for all logarithm returns. For the unit root test, this research uses both augmented Dickey-Fuller (ADF) test [23] and KPSS test [24] with an intercept. Results of the ADF tests (Table 1) indicate that time series data of OVX and oil prices are non-stationary but their logarithm returns are stationary. Results of KPSS test show that none of the implied volatility indices are stationary in the levels. Combining these two results, we can conclude that time series data of logarithm returns is stationary.

3.2. Empirical Analysis

Implied volatility index is regarded as “investor fear gauge” since it spikes during periods when the market is in turmoil. Associated with this proposition, the asymmetric effect was also tested by the researchers. The asymmetric effect assumes that the change in implied volatility rises at a higher absolute rate when the stock market falls than when it rises. Siriopoulos and Fassas [22] test the relationship between stock market returns and implied volatility by a regression analysis of daily changes of the volatility index ($\Delta IV_t = IV_t - IV_{t-1}$) against the daily positive returns (ΔS_t^+) and negative returns (ΔS_t^-) of the corresponding underlying stock index.

This section analyzes the relationship between logarithm returns of OVX and oil price with Kalman filter which assumes the coefficients are time-varying instead of static. The simple regression model that refers to Siriopoulos and Fassas [22], is as follows:

$$\Delta IV_t = \alpha + \beta_1 \Delta S_t^+ + \beta_2 \Delta S_t^- + \varepsilon_t \tag{6}$$

Where ΔS_t^+ equals to the daily return when the daily return is positive, and equals to 0 when the daily return is negative. ΔS_t^- goes just the opposite.

Time varying modification with Kalman filter method, the transition function is given by

$$\begin{bmatrix} \alpha_{t+1} \\ \beta_{1,t+1} \\ \beta_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_{1,t} \\ \beta_{2,t} \end{bmatrix} + w_t \tag{7}$$

The measurement function is given by

$$\Delta IV_t = \begin{bmatrix} 1 & \Delta S_t^+ & \Delta S_t^- \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_{1,t} \\ \beta_{2,t} \end{bmatrix} + v_t \tag{8}$$

As Harvey [25] indicated, when the process equation is non-stationary the initial distribution of the state variables should be specified in terms of diffuse prior. Here we consider the case when Q and R are determined by the first four hundred data points. We try to obtain a much accurate estimate of the relationship between contemporaneous logarithm volatility changes and logarithm stock index returns by Kalman filter, so Kalman smoothing is used after filtering in this section.

Estimates of α_t, $\beta_{1,t}$ and $\beta_{2,t}$ are described in Fig.1. This paper also takes VIX and S&P 500 as a comparison and the results are shown in Fig.2. For VIX and S&P 500, both $\beta_{1,t}$ and $\beta_{2,t}$ are negative, which prove the negative relationship between volatility changes and equity market returns. For OVX and crude oil spot price, although $\beta_{2,t}$ is always negative, $\beta_{1,t}$ fluctuates around zero and sometimes it is positive which indicate the positive relationship between OVX changes and crude oil spot returns.

For VIX and S&P 500, the absolute value of $\beta_{2,t}$ is always larger than the absolute value of $\beta_{1,t}$, which confirms the asymmetric effect that the change in volatility indices rises at a higher absolute rate when the stock market falls than when it rises. However, for OVX and crude oil spot price, the absolute value of $\beta_{2,t}$ is not always larger than the absolute value of $\beta_{1,t}$ according to time-varying coefficients, the asymmetric effect is not significant.

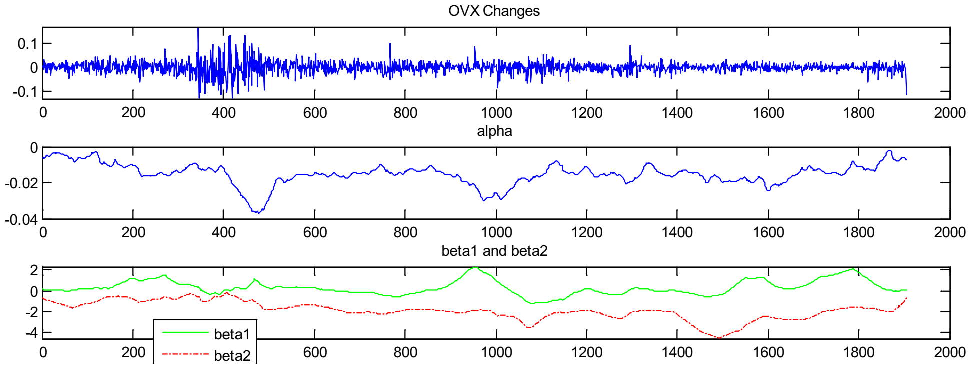

Fig.1. Time-varying coefficients between daily OVX changes and crude oil spot returns

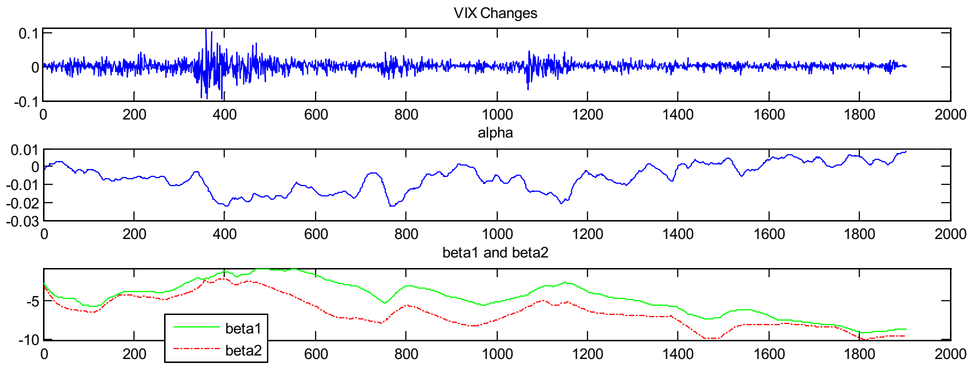

Fig.2. Time-varying coefficients between daily VIX changes and S&P 500 returns

Numerical results (Table 2) provide a comparison between Kalman filter estimation and Ordinary Least Square estimation. Mean squared error and correct directional forecasting percentage present Kalman filter estimation dominant OLS estimation. It indicates that the mean value of $\beta_{1,t}$ for the first data set is positive, which is different from previous researches. The results achieved from Ordinary Least Square suggest that β_1 is not significant which is consistent with $\beta_{1,t}$ which fluctuates around zero.

Table 2 Kalman filter estimation and the comparison with OLS estimation

		Coefficients α	β_1	β_2	MSE	Correct Directional Prediction
OVX and Crude Oil Spot Price						
Kalman Filter	mean	-0.0153	0.2871	-1.9234	0.0016	68.70%
	Std. dev.	0.0060	0.6966	0.8724		
OLS		-0.0098	0.0543	-1.1766	0.0021	63.46%
		(0.0000)	(0.4718)	(0.0000)		
VIX and S&P 500						
Kalman Filter	mean	-0.0060	-4.7472	-6.5640	0.0012	82.97%
	Std. dev.	0.0075	2.1750	1.9688		
OLS (p-value)		-0.0030	-3.3639	-4.1495	0.0022	81.29%
		(0.0328)	(0.0000)	(0.0000)		

4. Conclusion

Implied volatility (IV) is a new measure of markets' expected risk derived from the price of a market traded option and it has attracted much attention in recent years because of its importance for financial markets. This research examines the relationship between OVX and crude oil spot returns. The results indicate that, firstly, the time-varying coefficients only confirm the negative relationship between negative crude oil spot returns and OVX changes. Secondly the asymmetric effect is not as significant as the results got from VIX and S&P 500 index. Finally, the relationship between positive crude oil spot returns and OVX changes is not significant, although the coefficient is positive.

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